Moor Nook CP School

Year 6

Medium Term Plans

February 2021

Overview of Year

Autumn Term	Number				Geometry and Measures	
	1. Investigating Number Systems	2. Pattern Sniffing	3. Exploring Calculation	4. Generalising Arithmetic	5. Exploring Shape	6. Reasoning with Measures

Spring Term	Number			Statistics
	7. Discovering Equivalence	8. Reasoning with Fractions	9. Solving Number Problems	10. Investigating Statistics

Summer Term	Geometry	Number		Geometry and Measures	
	11. Visualising Shape	12. Exploring Change	13. Proportional Reasoning	14. Describing Position	15. Measuring and Estimating

Year 6 Overview:		
Unit	**Approx Learning Hours**	**Summary of Key Content**
1. Investigating Number Systems	11	Recognise place value; compare and order numbers up to 10 000 000 and to 3 decimal places; Negative numbers; Rounding
2. Pattern Sniffing	9	Generate and describe linear sequences Identify common factors, common multiples and prime numbers
3. Exploring Calculation	15	Mental calculations; Solve addition and subtraction problems; Formal multiplication up to 4dx2d; Multiply a decimal with up to 2 dp by a single digit; Estimation; Use simple formulae (Please refer to Moor Nook's Mental & Written Calculations Policies)
4. Generalising Arithmetic	13	Mental calculations; Formal division up to 4d / 2d; Divide a decimal by an integer; Order of operations; Solve problems involving all 4 operations (Please refer to Moor Nook's Mental & Written Calculations Policies)
5. Exploring Shape	11	Name parts of circles Angles in polygons; angles at a point, on a straight line and vertically opposite Compare and classify geometric shapes
6. Reasoning with Measures	10	Area of parallelograms and triangles; appreciation of range of shapes with same area or perimeter; appreciation of formulae for area; volume of cuboids
7. Discovering Equivalence	8	Compare and order fractions <1; Simplify fractions and express them with a common denominator; Solve percentage calculation problems; FDP equivalence
8. Reasoning with Fractions	8	Add and subtract fractions with different denominators Multiply and divide proper fractions
9. Solving Number Problems	6	Multiply and divide numbers (inc decimals) by powers of 10; revisit formal methods for multiplication and division; solve calculation problems for 4 operations (inc rounding) (Please refer to Moor Nook's Mental & Written Calculations Policies)
10. Investigating Statistics	8	Interpret and construct pie charts and line graphs Calculate and interpret the mean as an average
11. Visualising Shape	8	Draw 2D shapes accurately; recognise, describe and build 3D shapes
12. Exploring Change	4	Coordinates – 4 quadrants
13. Proportional Reasoning	8	Solve problems involving measures. Solve scaling problems; solve similar shapes problems; solve unequal sharing problems (Please refer to Moor Nook's Mental & Written Calculations Policies)
14. Describing Position	4-8	Coordinates – 4 quadrants; Translations
15. Measuring and Estimating	8	Solve problems involving calculation and conversion of measures and units. (Please refer to Moor Nook's Mental & Written Calculations Policies)

Year 6	Unit 1: Investigating Number Systems	
11 learning hours	This unit introduces the number systems and structures that we use at different levels of the curriculum. At KS1 children are working on the place value system of base 10 with the introduction of Roman Numerals as an example of an alternative system in KS2. Negative numbers and non-integers also come in at this stage and progress into KS3. At KS3 and KS4 we start to look at other ways of representing numbers, including standard form, inequality notation and so on.	
Prior Learning	**Core Learning**	**Learning Leads to...**
➢ read, write, order and compare numbers to at least 1 000 000 and determine the value of each digit ➢ read Roman numerals to 1000 (M) and recognise years written in Roman numerals	➢ **read, write, order and compare numbers up to 10 000 000 and determine the value of each digit**	➢ understand and use place value (e.g. when working with very large or very small numbers, and when calculating with decimals)
➢ read, write, order and compare numbers with up to three decimal places	➢ **identify the value of each digit in numbers given to three decimal places and multiply and divide numbers by 10, 100 and 1000 giving answers up to 3 decimal places**	➢ order positive and negative integers, decimals and fractions
➢ read, write and interpret negative numbers in context	➢ **use negative numbers in context, and calculate intervals across zero**	
➢ round any number up to 1 000 000 to the nearest 10, 100, 1000, 10 000 and 100 000 ➢ round decimals with two decimal places to the nearest whole number and to one decimal place	➢ **round any whole number to a required degree of accuracy**	➢ round numbers and measures to an appropriate degree of accuracy (e.g. to a specified number of decimal places or significant figures)

Exemplification	Vocabulary	
1. a) Write the number **three million, six hundred and seven thousand, two hundred and eighty** in numerals b) Write the number **7 346 109** in words 2. a) State the value of the **6** in i) 0.609 ii) 0.456 b) Find the value of Δ in each of these statements: i) 0.16 × 100 = Δ ii) Δ × 10 = 3.4 iii) Δ ÷ 100 = 4.5 iv) 56 ÷ 1000 = Δ 3. a) Ami is a playing a game where you score 10 points for a correct answer and -5 points for a wrong answer. Ami has answered three questions and has a score of -15. How many questions has she got right so far? b) The temperature outside is -3^{o}. If the temperature goes up 10 degrees, what will the temperature outside be now? 4. Round **6 156 827** to a) the nearest 1000 b) the nearest million	Hundred Thousand Ten thousand Million Tenths Hundredths Thousandths Place Value Partition Million Billion	Positive Negative Integer Decimal Decimal Point Decimal Place(s) Round Greater than, > Less than, <

Representation	Fluency	Probing Questions
Representing Integers • Building numbers from place value counters – whole numbers • Using overlapping partitioning cards to construct and deconstruct numbers • Comparing two numbers by constructing, partitioning and analysing place by place.	1. Read, write and understand the place value of numbers up to 10 million • state the value of a given digit in a number • record a number given in words in numerals (no 0 digits) • record a number given in words in numerals (including one or more 0 digits) • write a number given in numerals in words (no 0 digits) • write a number given in numerals in words (including one or more 0 digits) • compare two integers of the same magnitude using < or > • order positive integers of the same magnitude • compare two integers of different magnitudes using < or > • order positive integers of mixed magnitudes	Show me a number that is ten greater than 16,548,891 - a thousand greater? - ten thousand greater? - ten million greater? What's the same and what's different? one, thousand, million, billion Convince me that 85,635,147 is less than one hundred million
Multiplying/Dividing by Powers of 10 • Exploring multiplication by 10 by using place value counters or equivalent to exchange each counter for one worth ten times as much (then x 100, x1000 etc) • Using number cards on a place value grid to abstract the process above e.g. we have 5 in the tens column (which is like having 5 ten counters). We will swap these for 5 hundred counters so there will be a 5 in the hundreds column.	2. Multiply whole numbers by 10, 100 and 1000 • numbers not ending in 0 multiplied by 10 (e.g. 67 x 10) • numbers not ending in 0 multiplied by 100 (e.g. 132 x 100) • numbers not ending in 0 multiplied by 1000 (e.g. 765 x 1000) • numbers ending in 0 multiplied by 10 (e.g. 540 x 10) • numbers ending in 0 multiplied by 100 (e.g. 130 x 100) • numbers ending in 0 multiplied by 1000 (e.g. 40 x 1000) • numbers ending in multiple 0s multiplied by 10, 100 or 1000 (e.g. 1300 x 10 or 71 000 x 100)	Always, Sometimes, Never? When you multiply a number 1000, you add three zeroes on the end. Convince me that 7530 x 100 = 753 000 True or False? Any number can be multiplied by 10.
Multiplying/Dividing by Powers of 10 • Exploring division by 10 by using place value counters or equivalent and exchanging each counter for one worth ten times less (then /100, /1000 etc.)	3. Divide whole numbers by 10, 100 and 1000 (where the answer is a whole number) • integers ending in (one) 0 divided by 10 (e.g. 650 ÷ 10) • integers ending in more than one 0 divided by 10 (e.g. 4500÷10) • integers ending in two 0s divided by 100 (e.g. 16 500 ÷ 100) • integers ending in three+ 0s divided by 100 (e.g. 46 000 ÷ 100) • integers ending in three 0s divided by 1000 (e.g. 1 284 000 ÷ 1000) • integers ending in more than three 0s divided by 1000 (e.g. 1670000 ÷ 1000)	Convince me that 34 000 ÷ 100 = 340 True or False? Any number can be divided by 10.

<table>
<tr>
<td>Representing Decimals
Building numbers from place value counters - decimals
Representing decimals using tenth strips and hundredth squares to show why, for example, 32 hundredths is the same as 3 tenths and 2 hundredths.
Exploring the position of numbers on a scale (e.g. on geogebra)
Positioning integers and decimals on paper strips using paper clips (and adjusting the scale of the strip)</td>
<td>4. Read, write and understand the place value of numbers with up to 3 decimal places
state the value of a given digit in a decimal
record a decimal given in words in numerals (partitioned into tenths, hundredths etc)
record a decimal given in words in numerals (combined place values e.g. sixty-four hundredths)
write a decimal given in numerals in words (no 0 digits)</td>
<td>Convince me that 0.048 > 0.0084

How many numbers are there with exactly 2 tenths?</td>
</tr>
<tr>
<td>Multiplying by Powers of 10
Exploring multiplication by 10 by using place value counters or equivalent to exchange each counter for one worth ten times as much (then x 100, x1000 etc)
Using number cards on a place value grid to abstract the process above e.g. we have 5 in the hundredths column (which is like having 5 hundredths counters). We will swap these for 5 tenth counters so there will be a 5 in the tenths column.</td>
<td>5. Multiply a decimal by 10, 100 and 1000
decimal with one decimal place x 10 (e.g. 3.2 x 10)
decimal with 0 integer and one decimal place x 10 (e.g. 0.8 x 10)
decimal with more than one decimal place x 10 (e.g. 7.14 x 10 or 2.356 x 10)
decimal with two decimal places x 100 (19.87 x 100)
decimal with 0 integer and two dp x 100 (e.g. 0.63 x 100)
decimal with three decimal place x 100 (e.g. 12.356 x 100)
decimal with one decimal place x 100 (e.g. 7.9 x 100)
decimal with three decimal places x 1000 (e.g. 1.863 x 1000)
decimal with 0 integer and three decimal places x 1000 (e.g. 0.519 x 1000)
decimal with fewer than three decimal places x 1000 (e.g. 1.89 x 1000 or 7.6 x 1000)</td>
<td>Always, Sometimes, Never?
When you multiply a number 1000, you add three zeroes on the end.

What’s the same and what’s different?
4.152 and 41.52</td>
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<tr>
<td>Dividing by Powers of 10
Exploring division by 10 by using place value counters or equivalent and exchanging each counter for one worth ten times less (then /100, /1000 etc.)</td>
<td>6. Divide decimals and integers by 10, 100 and 1000
decimal with 0 integer and one decimal place divided by 10 (e.g. 0.8 ÷ 10)
decimal with one decimal place divided by 10 (e.g. 1.8 ÷ 10)
decimal with two or three decimal places divided by 10 (e.g. 2.87 ÷ 10 or 0.123 ÷ 10)
whole number not ending in 0 divided by 10 (e.g. 675 ÷ 10)
decimal with 0 integer and one decimal place divided by 100 (e.g. 0.3 ÷ 100)</td>
<td>Always, Sometimes, Never?
Dividing a decimal by 10 gives you a decimal answer.

Always, Sometimes, Never?
To divide a number by 100, just remove two 0s.</td>
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	• decimal with one decimal place divided by 100 (e.g. 24.1÷ 100) • decimal with two or three decimal places divided by 100 (e.g. 8.45 ÷ 100 or 1.613 ÷ 100) • whole number ending in one or no zeroes divided by 100 (e.g. 12 435 ÷ 100) • decimal with 0 integer and one decimal place divided by 1000 (e.g. 0.2 ÷ 1000) • decimal with one decimal place divided by 1000 (e.g. 6.7 ÷ 1000) • decimal with two or three decimal places divided by 1000 (e.g. 1.45 ÷ 1000 or 3.541 ÷ 1000) • whole number with two, one or no zeroes divided by 1000 (e.g. 4 356 780 ÷ 1000)	
Negative Numbers - Temperature • Using thermometers as a marked number lines to explore negative numbers as positions as well as finding the difference between two temperatures	7. Use negative numbers in the context of temperature • mark a positive, zero or negative temperature on a marked scale • mark a positive, zero or negative temperature on a blank scale • state the highest and lowest temperatures from a set • find the difference between two temperatures i. both positive ii. both negative iii. one zero, one non-zero iv. one positive, one negative	Show me two numbers with a difference of 3 Show me two negative numbers with a difference of 3? Show me two numbers, one negative and one positive, with a difference of 3? Convince me that -73 is less than -1
Negative Numbers - Other • Exploring accounts of businesses etc. to investigate the meaning of negative numbers further. Which number is lower? Larger? Less? Greater? • Playing a game where you score a negative (lose points) for an outcome. E.g. Roll a dice, score 1 point for a 1, 2 points for a 2 and so on up to 5 but lose 10 points for a 6.	8. Use negative numbers in other contexts: • as above with i. money ii. scores/games/points iii. displacement (e.g. tug of war)	Always, Sometimes, Never? 0 is a positive number What's the same and what's different? -730, -73, -37, -7.3 Convince me that there are infinite pairs of negative numbers with a difference of 5
Rounding • Positioning number on marked (and then unmarked) number line to identify neighbouring rounding options • Using number line to investigate when a	9. Round a whole number to the nearest 10, 100, 1000, 10 000, 100 000, 1 000 000 • state two closest multiples of 10, 100 etc • position number and closest multiples on a number line and identify the closest multiple	Convince me that 23 456 rounds to 23 000 to the nearest 1000. Always, Sometimes, Never? Numbers ending in 9 round up.

number is closer to the lower end than the upper end	• identify last required digit and inspect next digit to decide whether to round up or not	Convince me that 3 568 121 rounds to 4 million to the nearest million
	10. Round a whole number to a given degree of accuracy • nearest 20 • nearest 50 • nearest 200 etc.	Show me a length that rounds 430m to the nearest 10 metres. Show me 84,684,152 rounded to - the nearest integer - the nearest 1000 - the nearest million
Choosing a degree of accuracy • Exploring calculations with money involving numbers with 3dp e.g. sharing £10 between 3 people or £1 between 8 people (what is a sensible action to take?) • Exploring contexts such as surveying ¼ of the year group when there are 51 children in the year group	11. Decide the level of accuracy that best suits the question • automatically round answers to money questions to pounds and pence • automatically round answers to questions about objects that cannot be divided to the nearest whole number • identify and match the level of rounding in the question	Always, Sometimes, Never? -35 rounds to -40 to the nearest 10 True or False? It is better to give an exact answer rather than rounding

Further Extension	**Rich and Sophisticated Tasks**
1. Miss Wong, the teacher, has four cards. On each card is a number: 59 996 | 59 943 | 60 026 | 62 312 She gives one card to each pupil. The pupils look at their card and say a clue. Anna says, 'My number is 60 000 to the nearest 10 thousand.' Bashir says, 'My number has exactly 600 hundreds in it.' Charis says, 'My number is 59900 to the nearest hundred.' David says, 'My number is 60 000 to the nearest 10.' Can you work out which card each pupil had? Explain your choices. 2. Eduardo says, 'The the population of Mexico City is 11 million (to the nearest million) and the population of New York is 11·2 million (to the nearest hundred thousand).' He says, 'The population of New York must be bigger than the population of Mexico City because 11·2 million is bigger than 11 million.' Do you agree with him?	**Directed Numbers** STANDARDS UNIT: N8 Using Directed Numbers in Context **Up, Down, Flying Around** ✓ **Strange Bank Account** ✓ **Magic Letters** ✓ **First Connect Three** ✓ **Estimating/Rounding** **Place Your Orders*** ✓ **Thousands and Millions*** ✓ **Round the Four Dice**

<table>
<tr><td>3.
A scientist measured the temperature each day for one week at 06:00.

On Sunday the temperature was 1·6°C.
On Monday the temperature had fallen by 3°C.
On Tuesday the temperature had fallen by 2·1°C.
On Wednesday the temperature had risen by 1·6°C.
On Thursday the temperature had risen by 4·2°C.
On Friday the temperature had fallen by 0·9°C.
On Saturday the temperature had risen by 0·2°C.

What was the temperature on Saturday?</td><td></td></tr>
<tr><th>Misconceptions</th><th>Teacher Guidance and Notes</th></tr>
<tr><td>Pupils may forget about the existence of ten thousands and hundred thousands prior to a million.
They may also then think that the next place from 1 million is a billion.

Children struggle to understand place value of decimals when expressed in numbers greater than 10. For example, they understand 2 tenths and 4 hundredths better than 24 hundredths.

Children often see the size of a negative number rather than its sign – therefore, they may wrongly say that “-25 > -1)

When rounding, some children look at the wrong column to decide whether to leave the stem as it is or to round up - they may also begin a chain of rounding from the end of the number, instead of simply looking at the next number after the required degree of accuracy.

Children think that longer decimals are greater e.g. that 0.099 is bigger than 0.1, as they see the numbers 99 and 1.

Children reduce multiplying and dividing by powers of 10 to adding and removing zeroes and hence struggle when decimals are involved.

Pupils are often misled by the idea of ‘rounding down’, thinking this means that, for example, 0.72 will round to 0.6 and not 0.7 (to 1 decimal place).

Children round from the end of the number, not just the adjacent number (so they end up with a chain of rounding)</td><td>It is important to secure a strong understanding of place value for both integers and decimals before moving on to rounding or multiplying/dividing. It can be useful to introduce some real life problems to show how misinterpreting place value can lead to large errors, for example in administering anaesthetics or in designing a building.

Ensure children are secure with place value to 1 million and possibly beyond - pay particular attention to the columns used more rarely e.g. ten thousands or hundred thousands.

When working with decimals, be careful to model correct language i.e. read decimals digit by digit. Do not read them as HTU e.g. 0.36 is Nought-point-three-six and never Nought-point-thirty-six.

Similarly, be precise with the use of greater than for > (rather than ‘bigger than’ which can be misleading with negative numbers particularly)

When rounding, ensure students can explain visually why numbers are closer to one option rather than another as well as applying the ‘5 and up’ rule. Note that the decision to round the half way point of 5 upwards is a convention (i.e. an agreed way of doing things) rather than an innate feature of the concept.

Link rounding to estimation to help children get a sense of the size of number they are expecting - this will encourage them to use 0s as placeholders also. Try using a visual technique of 'chopping' the number at the required point for rounding and 'circling - looking at' the next number to decide whether you need to round up or not. Avoid the language of rounding down if possible as this is</td></tr>
</table>

When rounding a large number, pupils forget to use 0s as placeholders to maintain the order of magnitude e.g. they round 456,879 to 457 to the nearest thousand rather than 457 000.	misleading. When working with decimals also use the place value apparatus that would have been used for whole numbers earlier e.g. place value counters to help children see decimals as just another form of unitisation in our system. e.g. if they know that 4 and 3 make seven then they also know that 4 tenths and 3 tenths make 7 tenths. Ensure children fully understand negative numbers and can place them securely on a number line or equivalent before calculating intervals between negative numbers or other integers. At this level, use number lines to show the calculation of intervals as this is an area of great confusion later - avoid learning 'rules' about, for example, what happens when you have two negatives.

Key Assessment Checklist

1. I can read, write, order and compare numbers up to 10,000,000.
2. I can talk about the place value of each digit up to 10,000,000.
3. I can multiply and divide numbers by 10, 100 and 1000.
4. I can use negative numbers in context.
5. I can calculate intervals across zero.
6. I can round any number to a required degree of accuracy.

<table>
<tr><th>Year 6</th><th colspan="2">Unit 2: Pattern Sniffing</th></tr>
<tr><td>9 learning hours

Sequences (4)
Number Properties (5)</td><td colspan="2">This unit explores pattern from the early stages of counting and then counting in 2s, 5s, and 10s up to the more formal study of sequences. This sequence work progresses through linear sequences up to quadratic, other polynomial and geometric for the most able older students. For children in KS1, this unit is heavily linked to the following one in terms of relating counting to reading and writing numbers.

Also in this unit children and students begin to study the properties of numbers and to hone their conjecture and justification skills as they explore odd/even numbers, factors, multiples and primes before moving onto powers/indices and their laws.</td></tr>
<tr><th>Prior Learning</th><th>Core Learning</th><th>Learning Leads to...</th></tr>
<tr><td>➢ count forwards or backwards in steps of powers of 10 for any given number up to 1 000 000</td><td>➢ generate and describe linear number sequences</td><td>➢ generate terms of a sequence from a term-to-term rule
➢ recognise and use sequences of triangular, square and cube numbers, simple arithmetic progressions</td></tr>
<tr><td>➢ identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers
➢ know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers
➢ establish whether a number up to 100 is prime and recall prime numbers up to 19</td><td>➢ identify common factors, common multiples and prime numbers</td><td>➢ use the concepts and vocabulary of prime numbers, factors (divisors), multiples, common factors, common multiples, highest common factor and lowest common multiple</td></tr>
<tr><td>➢ recognise and use square numbers and cube numbers, and the notation for squared (^{2}) and cubed (^{3})</td><td></td><td>➢ use positive integer powers and associated real roots (square, cube and higher), recognise powers of 2, 3, 4, 5</td></tr>
</table>

<table>
<tr><th>Exemplification</th><th colspan="2">Vocabulary</th></tr>
<tr><td>1. a) Find the next two terms of these sequences: i) 11, 15, 19, 23, ,,,,, ii) 20, 17, 14, 11, ...
b) A sequence starts on 7 and goes up in 3s. Write down the first 5 terms of the sequence

2. a) Find two common factors of 28 and 20
b) Find two common multiples of 12 and 10
c) Find two prime numbers between 50 and 60</td><td>Sequence
Pattern
Term
First term etc.
Rule
Difference</td><td>Prime (Number)
Factor
Multiple
Common factor
Common multiple</td></tr>
</table>

Representation	Fluency	Probing Questions
Continuing Sequences: • Make the given pattern practically using objects and produce the next term in the pattern • Draw patterns and find the next term, explaining how the pattern grows each time	1. Continue and describe a pattern representing a sequence • find next term • find next three terms • describe how pattern is growing • convert pattern to number sequence	Show me the next three terms of two different sequences that could start like this:  Always, Sometimes, Never? The number of items in a sequence goes up by the same amount each time.
Continuing Sequences: • Colour in the numbers of a sequence on a 100-square to notice the visual pattern that is emerging. Reverse this process to start with a repeating pattern and turn it into a sequence.	2. Continue and describe rule for a linear number sequence • continue and describe an ascending whole number sequence • continue and describe a descending whole number sequence • continue and describe an ascending sequence involving simple decimals • continue and describe a descending sequence involving simple decimals • continue and describe an ascending whole number/decimal sequence with large numbers • continue and describe a descending whole number/decimal sequence with large numbers • continue and describe descending sequences that cross 0 • continue and describe ascending sequences that begin below 0	Show me a sequence starting 1, 4, ... Show me ... an increasing sequence ... a decreasing sequence Convince me that the next term of the sequence 7, 7.5, 8, 8.5, ... will be 9 Convince me that there will be a negative number in the sequence 49, 46, 43, 40, ... What's the same and what's different? 1, 4, 7, 10, … 7, 10, 13, 16, … -7, -4, -1, 2, … 17, 14, 11, 8, ….
Generating Sequences • Build number patterns using objects (from a given rule or vice versa) e.g. start with 3 blocks, add 2 blocks each time e.g. start with 3 blocks and double the number of blocks each time Encourage pupils to use different patterns to represent the same number sequence. • Draw images of sequences of numbers (from a given rule or vice versa) • Give first two terms of a sequence and	3. Generate a linear sequence from a simple rule • integer first term + small positive integer • integer first term – small positive integer • integer first term + large positive integer • simple decimal first term $\pm$ positive integer • integer first term + simple fraction/decimal • integer first term – simple fraction/decimal	Show me a number sequence with a rule of +2 Now show me a pattern to represent this sequence visually True or False? There are an infinite number of sequences with the term-to-term rule +4

then build/draw a pattern to continue to the sequence in as many ways as possible.		
Finding Missing Terms • Build patterns using objects (e.g. shapes, number cards, match sticks, cubes, counters, place value counters etc) and convert into numbers to find the missing terms required visually and numerically	4. Find missing terms in a sequence • find next term • find next three terms • find later term by continuing e.g. 10^{th} term • find missing term between given terms • find first term when given later terms	Convince me that the 10th term of the sequence 3, 7, 11, 15, … is 39 Always, Sometimes, Never? The 5th term of a sequence is double the 10th term of a sequence
Factors • Building arrays to show all factor pairs e.g. take 24 counters and arrange as various arrays to show all the different factor pairs (what happens when the number is a square number?) (what happens when the number is a prime number?) (why can you stop trying to find arrays when you pass the half-way point or, better still, the square root?)	5. Define and find factors of a number • by dividing systematically by 1, 2, 3, 4, etc. • by using factor pairs to find all the factors more efficiently • by discounting multiples of already tested and failed factors • by testing for factors only up to the square root of the number • prime numbers • square numbers	Show me ... all the factors of 28 ... a number with 6 factors ... a number with 2 factors ... a number with 3 factors What's the same and what's different? a) 4, 8, 24, 12 b) 1, 2, 3, 5 Always, Sometimes, Never? A number has an even number of factors
Common Factors • Using arrays to show common factors i.e. finding arrays with a matching length or width for the different numbers (interesting challenge: what happens when the numbers are co-prime?) • Practical/Visual Venn diagrams to represent common factors	6. Identify common factors of two numbers • by listing • by using a Venn diagram • where one number is a factor of the other • where the only common factor is 1 (defined as coprime)	Show me a common factor of 24 and 40 Convince me that 12 is a common factor of 24 and 60 Always, Sometimes, Never? Two numbers have at least one common factor.
Primes • Practical Sieve of Eratosthenes to remove multiples and leave only prime numbers i.e. which numbers are in no times table except their own?	7. Test whether a number up to 120 is prime • by finding all factors and checking to see if there are two • by testing for divisibility by all possible factors (2, 3, 4, etc) • by testing for divisibility more efficiently (by discounting multiples of already failed factors and/or by stopping at the square root of the number)	Show me ... a prime number ... a prime number greater than 50 Convince me that - 2 is a prime number - 1 is not a prime number What's the same and what's different? 7, 9, 13, 17

<table>
<tr><td></td><td></td><td>Always, Sometimes, Never?
Pick a number, multiply by 6, add 1.
The answer is a prime number.

Always, Sometimes, Never?
Prime numbers are odd

Always, Sometimes, Never?
Prime numbers can be a multiple of 4</td></tr>
<tr><td>Multiples
Use arrays to build representations of multiples of a number (by adding an extra row each time).</td><td>8. Define and find multiples of a number
by listing the ‘times table’ of the number
by multiplying systematically by 2, 3, 4, etc.
for large numbers (beyond times table)
for simple decimals
for simple fractions</td><td>What’s the same and what’s different? factor and multiple

Always, Sometimes, Never?
A number has an even number of multiples</td></tr>
<tr><td>Common Multiples
Practical/Visual Venn diagrams to represent common factors and multiples</td><td>9. Identify common multiples of two numbers
by listing
by using a Venn diagram
by using known facts
where one of the numbers is a multiple of the other
by using the product of the original numbers</td><td>Show me a common multiple of 5 and 8

Convince me that if you multiply your original numbers together you will get a common multiple</td></tr>
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Further Extension	Rich and Sophisticated Tasks
• Plot the numbers from a sequence onto a graph. What shapes are the graphs? What do you notice about the graphs of ascending sequences? And descending? • A perfect number is one whose factors sum exactly to the number itself. Find as many perfect numbers as you can! • How many possible factors do you need to test when testing if a number is prime? • Co-prime numbers – two numbers are called co-prime if they have no common factors except 1. Can you find some pairs of co-prime numbers? Now, find some common multiples of your numbers. What do you notice?	Abundant Numbers’, an activity from Nrich requires children to explore factors of numbers. ‘Factors and multiples’ is another Nrich game, perfect for practicing skills. http://nrich.maths.org/1011 http://nrich.maths.org/5468 Generate and describe number sequences **NRICH: Domino Sets * P I** **NRICH: Break it Up! * P I** **NRICH: Button-up Some More ** I** **NRICH: Holes * P I** Identify common factors, common multiples and prime numbers **NRICH: Mystery Matrix ** P I** **NRICH: Factor Lines ** P I** **NRICH: Factor-multiple Chains ** P** **NRICH: The Moons of Vuvv * P** **NRICH: Round and Round the Circle ** P I** **NRICH: Counting Cogs ** P**

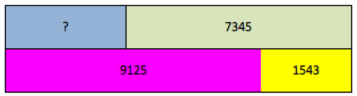
Misconceptions	Teacher Guidance and Notes
Pupils confuse a visual pattern with the corresponding number sequence. They do not realise that there is more than one way to represent the same number sequence. Pupils sometimes do not produce a genuine pattern with objects or images that reflect the number sequence (they do not apply the same rule each time) Pupils think that sequences are always increasing. Pupils think that sequences can only concern integers. Pupils frequently believe that the 2nth term of a sequence will be double the nth term. Pupils struggle to articulate the difference between factors and multiples. Pupils forget to include 1 and the numbers itself when finding factors. Similarly, they forget that the number itself is a multiple (the first multiple) Pupils think that 1 is a prime number and 2 is not. They believe that all prime numbers are odd.	• Note that this unit is only concerned with informal term-to-term rules (position-to-term and nth term rules do not appear until Stage 8) • Ensure you explore non-numerical sequences as well as numerical examples in this unit. • Ensure you are defining a prime number as 'a number with exactly two factors'. • Factors and multiples lie in the Stage 4 and 5 curriculums so, if this an issue, use the guidance in these sections to support. • Get children to consider how many factors and multiples there are of a number - the idea that factors are finite but multiples are infinite is important. • In this stage, you should still be finding common factors and multiples by listing - however, you can extend thinking by asking children about whether knowing that 2 and 3 are both factors of a number gives you any extra info (i.e. that 6 will be too). • It is worth discussing how you know if you've got all the factors - you can get more able children to discover that you only have to test the numbers up to the square root of the number • Guidance for teachers on divisibility tests from nrich at http://nrich.maths.org/1308

Key Assessment Checklist
1. I can generate a linear sequence using a term-to-term rule
2. I can describe a linear sequence using a term-to-term rule
3. I can find missing terms in a sequence
4. I can solve problems involving sequences
5. I can identify factors and multiples of numbers
6. I can identify the common factors of 2 or more numbers
7. I can identify the common multiples of 2 or more numbers
8. I can identify whether any number (of reasonable size) is a prime number

<table>
<tr><th>Year 6</th><th colspan="2">Unit 3: Exploring Calculation</th></tr>
<tr><td>14 learning hours</td><td colspan="2">This unit explores the concepts of addition and subtraction at KS1 building to wider arithmetic skills including multiplication at KS2. It is strongly recommended that teachers plan this unit for KS1/KS2 with direct reference to the calculation policy! At KS3 students are developing calculation into its more general sense to explore order of operations, exact calculation with surds and standard form (which have been introduced in Inv Number Systems briefly) as well developing their skills in generalising calculation to algebraic formulae. They need to substitute into these formulae and calculate in the correct order to master this strand. The formulae referenced are examples of the types of formula they will need to use, but the conceptual understanding for these formulae will be taught elsewhere in the curriculum.</td></tr>
<tr><th>Prior Learning</th><th>Core Learning</th><th>Learning Leads to...</th></tr>
<tr><td>➢ add and subtract numbers mentally with increasingly large numbers
➢ add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)</td><td>➢ perform mental calculations, including with mixed operations and large numbers (addition, subtraction and multiplication)
➢ solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why</td><td>➢ apply the four operations, including formal written methods, to integers, decimals and simple fractions (proper and improper), and mixed numbers</td></tr>
<tr><td>➢ multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers</td><td>➢ multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication
➢ multiply one-digit numbers with up to two decimal places by whole numbers</td><td></td></tr>
<tr><td>➢ use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy</td><td>➢ use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy</td><td>➢ estimate answers; check calculations using approximation and estimation, including answers obtained using technology</td></tr>
<tr><td></td><td>➢ use simple formulae
➢ recognise when it is possible to use formula for area and volume of shapes (Measurement duplicate)</td><td>➢ substitute numerical values into formulae and expressions
➢ understand and use standard mathematical formulae</td></tr>
</table>

Exemplification	Vocabulary
1. Calculate mentally: a) 20 x 600 b) 3040 + 2080 c) 12 000 – 301 2. Two numbers have a difference of 468. One of the numbers is 2356. What could the other number be? .... or .... 3. Calculate: a) 364 x 28 b) 5247 x 34 4. Calculate: a) 0.6 x 7 b) 2.13 x 4 5. Lianne is given the calculation 614 782 – 213 294. She estimates that the answer is 400 000. Do you agree with Lianne? Explain your answer. 6. The area of a kite can be calculated as $A = a \times b \div 2$ where a and b are the length and width of the kite. Using the formula, calculate the area of a trapezium with $a = 16$ cm and $b = 6$ cm	calculate mentally formal method column method integer decimal add, plus, sum, total subtract, take away, difference multiply, product represent estimate round approximate formula term

Representation	Fluency	Probing Questions
Mental Calculations • Using number lines to add and subtract by jumping in partitioned amounts, jotting the interim points on the line • Representing multiplication using table facts using place value counters and changing the 1s for 10s, 100s and so on to see the effect • Similarly for division – comparing the arrays for $24 \div 3$ and $24\,000 \div 3$	1. Calculate mentally using four operations • addition and subtraction of large numbers e.g. 112 000 – 24 000 • multiplication of large numbers using table facts e.g. 200 x 80 • division of large numbers using table facts e.g. $24\,000\,000 \div 3$ • combinations of the operations above	Show me a four digit number and two digit number that can be multiplied without using long multiplication Convince me that 9 000 x 800 = 7 200 000
Addition – Integers • Building numbers from place value counters and adding these, exchanging ten 1s for one 10 etc as required so that no column contains more than 9	2. Add positive integers using a formal method • whole numbers up to 6 digits no exchange • whole numbers up to 6 digits exchange needed from 1s to 10s • whole numbers up to 6 digits only one exchange needed • whole numbers up to 6 digits with multiple exchanges	Show me … an addition that is easy … an addition that is hard What makes it easy or hard? Convince me that 374 999 + 26 718 = 401 717
Addition – Decimals • Using place value counters to build numbers including 0.1 and 0.01 counters etc. As above, adding these	3. Add positive decimals using a formal method • decimals of the same length (same steps as above if needed) • decimals of different lengths	Show me a calculation that is connected to 234 567 + 157 892 = 392 459 (and another....)

column by column, exchanging 10 of a smaller amount for 1 of the next size up wherever necessary		What's the same and what's different? Adding whole numbers; adding decimals True or False? You cannot add decimals that are different lengths
Subtraction • Building a number from place value counters and removing (subtracting) the second amount to see what is left, exchanging one 10 for ten 1s etc as required	4. Subtract positive integers using a formal method • whole numbers up to 6 digits no exchange • whole numbers up to 6 digits only one exchange 10s to 1s • whole numbers up to 6 digits only one exchange needed • whole numbers up to 6 digits multiple exchanges • whole numbers up to 6 digits containing repeated zeroes leading to rolling exchange e.g. 120 004 – 95 786	Show me … a subtraction that is easy … a subtraction that is hard What makes it easy or hard? Convince me that 714 543 – 89 714 = 624 829
Subtraction – Decimals • Using place value counters to build the first number including 0.1 and 0.01 counters etc. As above, removing the second amount from this number to see what is left, column by column. To include exchanging 10 of a larger amount for 1 of the next size down wherever necessary	5. Subtract positive decimals using a formal method • decimals of the same length (same steps as above if needed) • decimals of different lengths	Show me two numbers less than 10 with a difference of 3.56 What's the same and what's different? subtracting whole numbers; subtracting decimals True or False? You cannot subtract decimals that are different lengths
Addition and Subtraction • Representing problems using a bar model to help decide which operations to carry out For example: There are 7345 adults at a football match. Altogether at the match, 9125 people are male and 1543 are female. How many of the people at the football match are children? 	6. Solve addition and subtraction multi-step problems • mentally • from a word problem • using a given bar model representation • missing number problems	Always, Sometimes, Never? Addition makes a number larger Always, Sometimes, Never? Subtraction makes a number smaller
Multiplication – Whole Numbers • Building and drawing (and even simply describing) arrays to represent multiplication (for a x b build an array of	7. Multiply a four-digit number by a single digit using a formal method • no exchange e.g. 2131 x 3 • exchange only from ones to tens e.g. 3216 x 2	What's the same and what's different? 243 x 7 and 247 x 3 Always, Sometimes, Never?

a objects across, copied into b rows) using counters and place value counters for bigger numbers. Use place value counters to represent large numbers in arrays e.g. 234 x 5 as 2 hundreds, 3 tens and 4 ones repeated over 5 rows. Generalising to grid method as an 'undrawn' array. • Making the link between the grid method and an expanded column method	• exchange once only anywhere e.g. 1272 x 4 • two exchanges e.g. 2317 x 3 • exchange anywhere e.g. 6243 x 7	A 4-digit number multiplied by single digit gives another 4-digit number.
	8. Multiply a three-digit number by a two-digit number using a formal method (recap) • three-digit number multiplied by a multiple of 10 e.g. 286 x 40 • three-digit number multiplied by a 2-digit number e.g. 286 x 42	Always, Sometimes, Never? Long multiplication is needed to multiply 3-digit numbers by 2-digit numbers What's the same and what's different? 453 x 28; 453 x 20 + 453 x 8; 400 x 28 + 50 x 28 + 3 x 28
	9. Multiply up to a four-digit number by a two-digit number using a formal method • four-digit number multiplied by a multiple of 10 e.g. 4186 x 30 • four-digit number multiplied by a 2-digit number e.g. 4186 x 34	What's the same and what's different? 4563x14, 3212x20, 4158x27, 6389x50 What's the same and what's different? 214 x 79 and 200 x 80
Multiplying – Decimals (by transforming) • Building arrays using place value counters for decimals also. (Putting the decimal along the top and then creating the appropriate number of rows) • Exploring the connection to the array of the same shape with 1s instead of 0.1s or 0.01s etc.	10. Multiply a one-digit number with up to 2 decimal places by a single digit by transforming to a multiplication of whole numbers (mental or written) • one decimal place, no bridging e.g. 4 x 0.2 • one decimal place, with bridging e.g. 4 x 0.6 • one decimal place, number greater than 1, no bridging e.g. 4 x 1.2 • one decimal place, number greater than 1, with bridging e.g. 4 x 1.6 • two decimal places, no bridging e.g. 4 x 0.21 • two decimal places, with bridging e.g. 4 x 0.36 • two decimal places, number greater than 1, no bridging e.g. 4 x 2.12 • two decimal places, number greater than 1, with bridging e.g. 4 x 5.37	What's the same and what's different? 7 x 3 0.7 x 3 7 x 0.3 Convince me that 6 x 1.3 = 7.8
	11. Multiply a one-digit number with up to 2 decimal places by a two-digit number by transforming to a multiplication of whole numbers • one decimal place, less than 1 e.g. 0.3 x 23 • two decimal places, less than 1 e.g. 0.35 x 23 • one decimal place, greater than 1 e.g. 4.3 x 23 • two decimal places, greater than 1 e.g. 4.35 x 23	Convince me that if I know 23 x 45 I can also find 2.3 x 4.5 Show me a calculation that is connected to 23 x 37 = 851 (and another...)
Multiplying – Decimals – Columnar	12. Multiply a one-digit number with up to 2 decimal places by a single digit by using a columnar method	Convince me that 4.9 x 7 = 34.65

• Building an array to represent the multiplication and converting to grid method, before exploring how to compact this into columnar method	• one decimal place, less than 1 e.g. 0.7 x 6 • one decimal place, greater than 1 e.g. 3.7 x 6 • two decimal places, less than 1 e.g. 0.57 x 6 • two decimal places, greater than 1 e.g. 3.57 x 6	What's the same and what's different? 234 x 5; 2.34 x 5; 2340 x 5 What's the same and what's different? 753 x 1.8; 7.53 x 1800; 750 x 1.8 + 3 x 1.8; 75.3 x 3 x 6
	13. Multiply a one-digit number with up to 2 decimal places by a two-digit number by using a columnar method • one decimal place, less than 1 e.g. 0.7 x 62 • one decimal place, greater than 1 e.g. 3.7 x 62 • two decimal places, less than 1 e.g. 0.57 x 62 • two decimal places, greater than 1 e.g. 3.57 x 62	Always, Sometimes, Never? Multiplication makes a number larger True or False? If there is one number missing from a column method multiplication, I can work it out.
Estimation • Use place value counters or other place value equipment to represent a number and then round it to the nearest 1000, 100 (or even 10) to allow easy mental addition or subtraction or multiplication	14. Estimate the value of a calculation • integers, addition and subtraction, by rounding appropriately • integers, multiplication, by rounding appropriately • integers, any of the three operations, by rounding appropriately • decimal examples using the three operations	Convince me that 7089 x 39 ~ 280 000 Show me how you could estimate the result of 378 115 – 120 981
Formulae • Building a formula from algebra tiles or cards • Replacing (substitute) tiles with their values once known – this works well when children write on the back of their tiles and turn them over to ensure the variables really are replaced completely.	15. Substitute into a given formula to find values • formulae involving one letter e.g. A = x - 3 • formulae involving multiple single letters e.g. A = x + y – 7 • formulae involving multiples of letters e.g. A = 2x + 3y	x and y are two whole numbers less than 1000 Always, Sometimes, Never? • the difference between x and y is 1000 • the difference between x and y is a whole number • the difference between x and y is greater than x

Further Extension	Rich and Sophisticated Tasks
1. Can you use five of the digits 1 to 9 to make this number sentence true? □ □.□ + □.□ = 31.7 Can you find other sets of five of the digits 1 to 9 that make the sentence true?	Perform mental calculations, including with mixed operations and large numbers **NRICH: Exploring Number Patterns You Make ** P I** **NRICH: Become Maths Detectives * P I** Calculate with integers **Making a Difference** ✓ **Playing Connect Three** ✓ **Weights** ✓ **Consecutive Negative Numbers** ✓ **Number Daisy**

2.
A shop sells boxes of chocolates costing £2·60. The shop also sells packets of sweets. One packet costs £1·39. Ramesh has a £10 note and he wants to buy one box of chocolates.

Sara says that Ramesh can work out how many packets of sweets he can buy using the number sentence 10 – 2·60 ÷ 1·39.

Do you agree or disagree with Sara?

If you disagree, what number sentence do you think Ramesh should use?

Explain your reasoning.

3.
Find numbers to complete these number sentences.

736 ÷ 23 = ☐	☐ × 100 = 2400	☐ × 100 = 10 × ☐
7360 ÷ 230 = ☐	25 × ☐ = 200	25 × ☐ = 4 × ☐
230 × 24 = ☐	23 × ☐ = 161	23 × ☐ = 161 × ☐
240 × 23 = ☐	24 × ☐ = 168	24 × ☐ = 168 × ☐
1668 ÷ 8 = ☐	161 ÷ ☐ = 23	161 ÷ ☐ = 23 × ☐
208·5 × 8 = ☐	☐ ÷ 25 = 9	☐ ÷ 25 = 9 × ☐

4.
Fill in the missing numbers to make these number sentences true.

☐ × ☐ = 864

☐ × ☐ × ☐ = 864

5.
Work out:

- 8·4 × 3 + 8·4 × 7
- 6·7 × 5 – 0·67 × 50
- 93 × 0·2 + 0·8 × 93
- 7·2 × 4 + 3·6 × 8

Got It ✓

Solve problems involving addition, subtraction, multiplication and division
NRICH: Always, Sometimes or Never? Number * P

Use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy
NRICH: Four Go * G

Misconceptions	Teacher Guidance and Notes
When adding and subtracting numbers of different magnitude (including decimals of different lengths), children often misalign these in column addition and subtraction (and multiplication). Children can struggle to understand why they 'add a zero' when multiplying by the tens digit during column multiplication. They also make errors in adding up the results of grid methods. Times tables weaknesses will cause errors in calculations and should be addressed asap to minimise the impact. When doing long multiplication, children sometimes forget to multiply all the parts together - they struggle to record each separate multiplication within one line, particularly where there are lots of numbers carried across following exchanges. Children often want to write out an expanded multiplication (which is longer!) but don't realise that this isn't proper long multiplication, where the steps need to be compacted. Children forget to put in a place holder of 0 when multiplying by a tens digit. When working with formulae, students often think that letters represent an object rather than a number e.g. a is an antelope rather than the number of antelope in the zoo. Students sometimes think that algebra is an alphabet code e.g. a =1, b = 2, c = 3, ... When substituting, students forget that the order of operations applies - this learning for number is not connected to algebraic situations. This is why these things are together in this unit!	• This unit is focused on revising addition and subtraction of integers and decimals and the development of fluent formal multiplication of integers and decimals. (Division will be covered directly in Unit 5: Generalising Arithmetic as will Order of Operations, but feel free to touch on this now if required – link to distributive law) • The pitch of Stage 6 is in adding and subtracting large numbers and decimals (including of different lengths) and in multiplying integers of up to 4 digits by one or two digits as well as multiplying decimals of the form a.bc by a whole number. • Stage 5 and below contain guidance and teaching prompts for the calculation work that precedes this. **Ref the Calculation Policy for Four Operations St1-6** • Ensure children are secure with column addition and subtraction before teaching long multiplication and division as these methods depend on the ability to use these skills. • Consider teaching an expanded method first as a precursor to long multiplication to see how the different parts are put together in long multiplication. Initially try to minimise the need to exchange and carry numbers across. • This is the first real reference to algebra that children will have had – try not to get too absorbed in the rules of notation etc at this stage and instead concentrate on the use of a letter to standard for a number. Therefore, explore real problems that lead to rules that can be summarised using algebra and use these rules to find missing information. • As always, when working with decimals, be careful to model correct language i.e. read decimals digit by digit. Do not read them as HTU e.g. 0.36 is Nought-point-three-six and never Nought-point-thirty-six.

Key Assessment Checklist

1. I can perform mental calculations including mixed operations
2. I can add and subtract large numbers and decimals
3. I can solve problems involving addition and subtraction
4. **I can multiply a four digit number by a two digit number using long multiplication**
5. **I can multiply a one digit number with up to 2dp by a whole number**
6. I can solve problems involving any or all of the four operations and use estimation to check answers
7. I can use simple formulae

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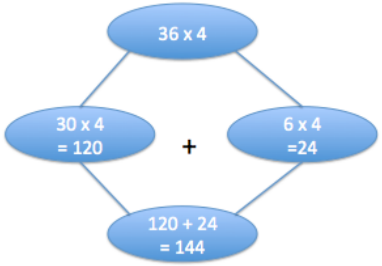
Year 6	Unit 4: Generalising Arithmetic
14 learning hours	This unit is focused on developing fluency in the manipulation of number. At primary level this is focused on arithmetic itself and the methods for four operations particularly; however, this is naturally generalised to thinking about rules of arithmetic more widely at secondary level i.e. algebra. These aspects have been paired together intentionally to help teachers describe algebra as simply a generalisation of number. It is expected that teachers will go back to arithmetic to help students see where the 'rules' of algebra come from.

Prior Learning	Core Learning	Learning Leads to...
	➢ **perform mental calculations, including with mixed operations and large numbers**	
➢ divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context ➢ multiply and divide whole numbers and those involving decimals by 10, 100 and 1000	➢ **divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context** ➢ **use written division methods in cases where the answer has up to two decimal places**	➢ formal written methods, to integers, decimals and simple fractions (proper and improper), and mixed numbers
	➢ **use their knowledge of the order of operations to carry out calculations involving the four operations** ➢ **solve problems involving addition, subtraction, multiplication and division**	➢ use conventional notation for priority of operations, including brackets ➢ recognise and use relationships between operations, including inverse operations (e.g. cancellation to simplify calculation" ➢ recognise and use relationships between operations, including inverse operations (e.g. cancellation to simplify calculations and expressions

Exemplification	Vocabulary	
1. Calculate mentally a) 70×400 b) 62×4 c) $36\,000 \div 90$ d) $420 \div 3$ e) $72\,000 + 6\,500$ 2. a) Calculate $8211 \div 23$ b) Calculate, leaving any remainder as a fraction c) There are 2342 children going to a book festival. They travel on coaches. Each coach can carry 53 people. What is the minimum number of coaches that will be necessary? d) 350 pizzas are shared between 17 people. How many pizzas does each person get?	divide dividend divisor remainder round fraction exchange decimal places	order of operations precedence addition subtraction multiplication division brackets balance

3. Calculate $7211 \div 32$ to 2 decimal places 4. Calculate a) $25 - 4 \times 2$ b) $(35 + 415) \div 5 + 1$ 5. Find the missing value: $567 \div 3 = \blacksquare \div 34$		equal solve

Representation	Fluency	Probing Questions
Using known multiplication/division facts • Building the original fact using place value counters and then adapting the array produced to reflect the new fact • For example, you might replace 1s counters in the 4 x 3 with 100s and 10s to explore 400 x 30 • Alternatively, you might double the width of the array to explore 26×7 versus 13×7	1. Use a known multiplication/division fact to find other related facts • multiple of 10/100/1000 x multiple of 10/100/1000 e.g. 3000×40 • multiple of 10/100/1000 ÷ multiple of 10/100/1000 e.g. $32000 \div 40$ • decimal multiplication e.g. 0.3×8 • decimal division e.g. $7.2 \div 6$ • doubling/tripling/quadrupling first number of a multiplication/division fact e.g. If $13 \times 7 = 111$, find 26×7 or If $255 \div 15 = 17$, find $2550 \div 15$ • doubling/tripling/quadrupling second number of a multiplication fact e.g. If $13 \times 7 = 111$, find 13×21 • doubling/tripling/quadrupling second number of a division fact If $255 \div 15 = 17$, find $255 \div 5$	What's the same and what's different? 4565 ÷ 15 and 456.5 ÷ 1.5 and 45.65 ÷ 0.15 Convince me that 7314×11=80454 and ... that 7.314 x 11 = 80.454
Mental Multiplication and Division • Using the part-part-whole model to partition a calculation and then recombine For example, 36×4 	2. Carry out multiplication and division mentally (with jottings) • 2-digit x 1-digit mentally (partitioning) e.g. 36×4 • simple 3-digit x 1-digit mentally (partitioning) e.g. 236×4 • 2-digit ÷ 1-digit mentally (partitioning) e.g. $76 \div 4$ • simple 3-digit ÷ 1-digit mentally (partitioning) e.g. $116 \div 4$	Show me how you can calculate $116 \div 4$ in your head Convince me that 6.24×8 does not equal (6x8)+(0.2×8)+(0.04×8)=48 + 0.16 + 0.032 = 48.192
Complex Mental Calculations • Using a combination of number lines to represent partitioned jumps when adding and subtracting and a part-part-whole model (above) for multiplying and dividing.	3. Carry out complex mental calculations using all four operations • addition and subtraction only – large numbers • two-step problems e.g. multiplication and addition	Show me two different ways to calculate (1034 + 576) x 5 Convince me of how you could calculate 39 x 801 mentally.

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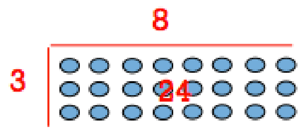
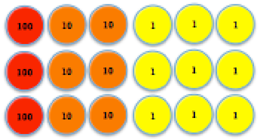
Dividing

- For a calculation p ÷ q, grouping a set of p counters into groups of size q, arranging these groups as an array. For example, for 24 ÷ 3, count out 24 counters and arrange in columns of 3.... then read off the answer of 8 as the number of columns



- Building a number using place value counters and grouping them into groups that are the size of the divisor, before arranging these groups as an array to explore the partitioning approach. For example, 369 ÷ 3



- Repeating the above, but exchanging remaining counters for 10 counters of the next size down before continuing to group. For example, 372 ÷ 3

- Recording these methods to arrive at compact division

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4. Recap: Divide a 3/4-digit number by a 1-digit number using a written method
   - no exchange necessary e.g. 9366 ÷ 3
   - first digit is lower than divisor requiring exchange e.g. 2196 ÷ 3
   - single exchange e.g. 2376 ÷ 3 or 8476 ÷ 4
   - two or more exchanges e.g. 4185 ÷ 5
   - examples with remainders e.g. 4189 ÷ 5

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Show me how you can divide 4305 by 5
- using place value counters
- using a written method

Convince me 1756 ÷ 5 will have a remainder of 1

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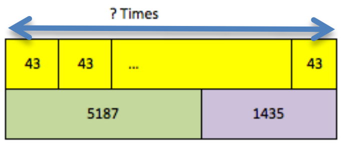
Division by a 2-digit number • Using place value counters to produce the dividend and then grouping column by column into an array with the correct (2-digit) number of rows, exchanging each remaining counter in a column for 10 of the next value down. • Using compacted division as a shorthand for this process	5. Divide a 3-digit number by 2-digit number using a formal written method • known times table, no remainder e.g. $756 \div 12$ • simple times table to derive, no remainder e.g. $966 \div 21$ • larger number requiring derivation of times table, no remainder $986 \div 34$ • examples with remainders e.g. $874 \div 15$	What's the same and what's different? long division and compact division Always, Sometimes, Never? When dividing by a 2-digit number, you have to exchange immediately
	6. Divide a 4-digit number by a 2-digit number using a formal written method • known times table, no remainder e.g. $2856 \div 12$ • simple times table to derive, no remainder e.g. $6594 \div 21$ • larger number requiring derivation of times table, no remainder $3591 \div 57$ • examples with remainders e.g. $4588 \div 16$	Convince me that $6279 \div 23 = 273$ Always, Sometimes, Never? Long division is needed to divide a four digit number by a two digit number Always, Sometimes, Never? Any two numbers can be divided
Remainders • Using place value counters to complete the division practically. Then, when the final remaining units are separated, using bar to represent how much of a full divisor they are. For example, when dividing by 12 and having a remaining 5 ones, this represents  or 5/12 of a whole column	7. Divide a 4-digit number by a 2-digit number using a formal written method , giving the remainder as a fraction • known times table, no remainder e.g. $2857 \div 12$ • simple times table to derive, no remainder e.g. $6597 \div 21$ • larger number requiring derivation of times table, no remainder $3541 \div 57$	Show me how you can divide 4305 by 17 What's the same and what's different? 125/15, 390/4, 1245/9, 216/6 Convince me that if I divide 132 sweets between 5 people, this gives 26 r2 or 26 2/5 each.
Division to give a Decimal Answer • Using place value counters to complete the division practically. Then, when the	8. Divide a 4-digit number by a 2-digit number, giving an answer to up to 2dp, using a formal written method • example resulting in exactly 1dp e.g. $4580 \div 16$	Always, Sometimes, Never? A calculation involving division will have a remainder

final remaining units are separated, exchanging these each for 10 tenths and continuing the division process past the decimal point. Repeat if necessary from tenths to hundredths. • It can be useful to look at examples where a repeat starts to happen e.g. 1000/3	• example resulting in exactly 2dp e.g. $4576 \div 16$ • example resulting in longer decimal (requiring truncation) e.g. $4579 \div 16$	Convince me that $6143 \div 11 = 558.45$ to 2 decimal places.
Division Word Problems • Using the bar model to represent a word or other division problem. For example: 6 people share £1764 equally between them. How much do they each receive? 1764 294 | 294 | 294 | 294 | 294 | 294	9. Recognise and solve a simple division problem, interpreting any remainders in the context as appropriate. • word problem – sharing language e.g. 3282g flour to make 56 cupcakes. How much flour is in each cupcake? • word problem – grouping language e.g. 2543 people go to Wembley. 52 people can fit on each bus. What is the minimum number of buses required? • Remainder problems – an account has £9542 in it. If you spend £86 per day, after how many days will the money run out? • finding unit fractions of an amount e.g. find a twelfth of 5184 • problems with links to factors (and multiples) e.g. show that 14 is a factor of 8834	Convince me that a remainder of 5 can mean different things in different questions Show me all the different words that can signify 'divide' in a problem
Order of Operations • Calculating using scientific calculators and basic (non-scientific) calculators simultaneously and observing the differences in result e.g. for 2 + 5 x 3. • Exploring which calculations this matters for and which it does not. • Using number cards (and bracket) cards and positioning/rearranging to try to reach a given result	10. Calculate using the correct order of operations (no brackets) • examples with one of multiply/divide and one of add/subtract e.g. 4 + 2 x 7 or 12 + 20 ÷ 5 • examples with three operations e.g. 24 ÷ 3 + 5 x 2 • examples with strings of additions/subtractions or multiplications/divisions e.g. 18 – 2 + 3 – 1 or 9 ÷ 3 x 6 ÷ 2 • examples involving strings and other operations e.g. 16 ÷ 2 + 4 - 1	Always, Sometimes, Never? Addition comes before subtraction when calculating Convince me that 3 + 5 x 2 can be done in two orders and give different answers
	11. Calculate using the correct order of operations (including brackets) • examples involving a single bracket e.g. 4 x (2 + 7) ÷ 6 • examples involving more than one bracket e.g. (24 – 4) x (3 + 5)	Always, Sometimes, Never? Multiplication always comes before addition Convince me that 4 x (2 + 7) ÷ 6 = 6
Missing Number Problems • Using a bar model to represent the problem For example: $43 \times ■ = 5187 + 1435$ could be represented as	12. Solve missing number problems involving addition, subtraction, multiplication and division • one operation e.g. $43 \times ■ = 6622$ (missing box either position, answer can come first) • two operations e.g. $43 \times ■ = 5187 + 1435$ (missing box	Always, Sometimes, Never? Division is the inverse of multiplication What's the same and what's different? $a \times ■ = b + c$;

	anywhere) • three or more operations	$\blacksquare \times a = b + c$; $a \times b = \blacksquare + c$; $a \times b = c + \blacksquare$
Solving Problems • Drawing bar models to represent problems involving addition, subtraction, multiplication and division (using a new line to show a new stage of the calculation)	13. Solve complex calculations involving all four operations e.g. $614 + 43 \times 23$ or $454.5 \div 18$	Show me the operations you would carry out to solve this problem Printing charges for a book are 3p per page and 75p for the cover. I paid £4.35 to get this book printed. How many pages are there in the book?

Further Extension	Rich and Sophisticated Tasks
1. A box of labels costs £63. There are 140 sheets in the box. There are 15 labels on each sheet. Sara, Ramesh and Trevor want to calculate the cost of one label, in pence. Ramesh uses the number sentence (6300 ÷ 140) × 15. Sara uses the number sentence 63 ÷ 1·4 ÷ 15. Trevor uses the number sentence (15 × 140) ÷ 6300. Who is using the right number sentence? Explain your choice. 2. Miriam buys 19 tins of soup. All the tins cost the same price. She goes to the shop with just one note, and comes home with the tins and the change in coins. On the way home she drops the change. She looks carefully and she thinks she picks it all up. When she gets home she gives £2·23 change to her mother. Do you think that Miriam picked up all the change that she dropped? Explain your reasoning.	Perform mental calculations, including with mixed operations and large numbers **NRICH:** **Exploring Number Patterns You Make** **** P I** **NRICH:** **Become Maths Detectives** *** P I** Solve problems involving addition, subtraction, multiplication and division **NRICH:** **Always, Sometimes or Never? Number** *** P**

3. Which calculation is the odd one out? ▪ $753 \times 1{\cdot}8$ ▪ $(75{\cdot}3 \times 3) \times 6$ ▪ $753 + 753 \div 5 \times 4$ ▪ $7{\cdot}53 \times 1800$ ▪ $753 \times 2 - 753 \times 0{\cdot}2$ ▪ $750 \times 1{\cdot}8 + 3 \times 1{\cdot}8$ Explain your reasoning.	
Misconceptions	**Teacher Guidance and Notes**
When dividing, children often forget to carry a remainder over as part of the exchange process. This is usually because there is too much information to hold in their head with the nearest multiple and the remainder then not accounted for. This particularly happens at the start of the number where a child may incorrectly 'carry' the divisor across, rather than the first digit of the dividend. Children have difficulty interpreting remainders resulting from a division as fractions, e.g. if the remainder is 3 from a calculation involving the divisor 5, children will write the remainder as 1/3 rather than 3/5 Children will sometimes approach multi operational problems with the correct operations but carry them out in the wrong order. They may also sometimes choose the incorrect operation directly. Children may sometimes solve problems by choosing the least efficient operation e.g. using repeated addition instead of multiplication When solving multi-step problems, some children complete only the first step of a problem and use it as the answer without going on to the next step	• The initial focus of this unit lies in division skills, building on Stage 5 learning to approach division by a 2-digit number. • The pitch is numbers up to 4-digits, but can include examples where the division continues past the decimal point to give a decimal of up to 2dp. • At this level remainders need to be written as fractions of the divisor or rounded appropriately if in context • As always, ensure children carry out the apparatus and written approaches simultaneously during the learning process so that they realise these are the same methods before moving to the solely abstract. • The unit then moves on to looking at the correct order of operations – children will not have seen this before so an exploration of why it is necessary is essential. • Ensure you are clear with children that multiplication and division are of equal precedence (as are addition and subtraction) and hence, when they appear together, should be carried out from left to right once any brackets have been dealt with. • See the videos at the NCETM website for more details of the use of place value counters for division. • Also cross-reference with the AET calculation policy. • Draw attention to language used within addition, subtraction, multiplication and division problems and model this. It is recommended that you teach children the language of the calculation e.g. dividend • Make children aware that some problems involve multiple steps which could mean 2 or more different operations. Use the bar model to represent the problem visually before deciding the operations to carry out and the order of these.

Key Assessment Checklist
1. I can perform mental calculations including mixed operations
2. I can divide a four digit number by a two digit number using a written method of long division
3. I can interpret remainders as whole number remainders, fractions or by rounding
4. I can use written division methods in cases where the answer has up to two decimal places
5. I can recognise and solve simple division problems with numbers up to 4d and 2d.
6. I can solve multi step four operation problems choosing the correct operation and using the most appropriate methods
7. I can calculate in the correct mathematical order, with and without brackets.
8. I can solve problems involving addition, subtraction, multiplication and division

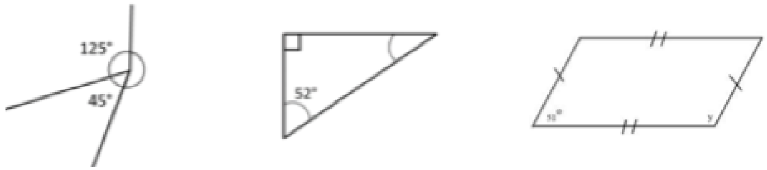
Year 6	Unit 5: Exploring Shape	
12 learning hours	In this unit children and students explore the properties of shapes, both 2D and 3D. At KS1 this is focused on common shape names and basic features of vertices, sides etc. but this then develops to classifying quadrilaterals and triangles in KS2. Alongside this focus children begin to explore angle and turn in KS2 and develop this to more formal angle rules through Stages 5, 6, 7, 8. Older students begin to explore the field of trigonometry, encountering first Pythagoras' Theorem, then RA-triangle trig before finally looking a the sine rule and cosine rule.	
Prior Learning	**Core Learning**	**Learning Leads to...**
➢ know angles are measured in degrees: estimate and compare acute, obtuse and reflex angles ➢ use the properties of rectangles to deduce related facts and find missing lengths and angles ➢ distinguish between regular and irregular polygons based on reasoning about equal sides and angles.	➢ **illustrate and name parts of circles, including radius, diameter and circumference and know that the diameter is twice the radius** ➢ **compare and classify geometric shapes based on their properties and sizes … and find unknown angles in any triangles, quadrilaterals, and regular polygons** ➢ **recognise angles where they meet at a point, are on a straight line, or are vertically opposite, and find missing angles**	➢ identify properties of the faces, surfaces, edges and vertices of: cubes, cuboids, prisms, cylinders, pyramids, cones and spheres ➢ use conventional terms and notations: points, lines, vertices, edges, planes, parallel lines, perpendicular lines, right angles, polygons, regular polygons and polygons with reflection and/or rotation symmetries ➢ use the standard conventions for labelling and referring to the sides and angles of triangles ➢ apply the properties of angles at a point, angles at a point on a straight line, vertically opposite angles ➢ derive and apply the properties and definitions of: special types of quadrilaterals, including square, rectangle, parallelogram, trapezium, kite and rhombus; and triangles and other plane figures using appropriate language

Exemplification

1. Draw diagrams to explain the difference between a radius and a diameter

2.
a) Explain the difference between an equilateral triangle and an isosceles triangle

b) Find all the missing angles in the shapes below:

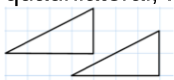


3. Find the value of the missing angle in each diagram

Vocabulary

circle	square
radius	rectangle
diameter	parallelogram
circumference	rhombus
centre	kite
semicircle	delta
degrees	trapezium
right angle	regular
acute	irregular
obtuse	classify
reflex	sort
triangle	compare
quadrilateral	symmetry
pentagon	parallel
hexagon	equal
heptagon	diagonal
octagon	at a point
nonagon	on a straight line
decagon	vertically opposite
isosceles	Venn diagram
right-angled	Carroll Diagram
equilateral	
scalene	

Representation	Fluency	Probing Questions
Circle Parts • Labelling a paper plate with key terms • Using a pen on a fixed piece of string to create a circle • Making circles out of children and finding the radius, diameter etc.	1. Recognise and name parts of a circle • identify from a diagram • sketch given the name • explain differences between parts • use fact that diameter is double the radius	Show me … a radius ... a diameter ... a circumference ... the radius if the diameter is 7cm What’s the same and what’s different? radius, circumference, diameter, chord
Properties of Triangles and Quadrilaterals • measuring sides and angles of cut-out/cardboard 2D shapes to discover equal angles and lengths • folding cut-out/cardboard shapes to identify lines of symmetry (and properties of diagonals)	2. Recognise and describe special types of triangle • describe side lengths, angles, symmetry ○ equilateral ○ isosceles ○ scalene ○ right-angled • compare and contrast shapes	Show me ... a triangle with: i) one acute angle ii) two acute angles iii) one obtuse angle iv) two obtuse angles What’s the same and what’s different?

• using a ten-point circle to make a right-angled triangle, a kite, a rectangle, a trapezium, a shape by jumping round the circle in 2s, 3s, etc. • fitting two right-angled triangles together to make a rectangle, a kite, a parallelogram, a pentagon, a hexagon, an isosceles triangle, another quadrilateral, ...  This activity can also be done with four triangles to get more shape possibilities • Using a geoboard with elastic bands or a square-dotty grid to produce as many different shapes as possible		scalene; right-angled; isosceles; equilateral Always, Sometimes, Never? A right-angled triangle is scalene
	3. Recognise and describe special types of quadrilateral • describe side lengths, side directions, angles, symmetry, diagonals ○ square ○ rectangle ○ parallelogram ○ rhombus ○ kite ○ delta ○ trapezium ○ irregular • compare and contrast shapes	Show me a sketch of a parallelogram with its angles labelled Convince me that a rhombus must be a parallelogram but a parallelogram is not necessarily a rhombus What's the same and what's different? trapezium, parallelogram, kite, square Always, Sometimes, Never? A trapezium is also a parallelogram
Classifying shapes • Sorting cardboard/solid shapes into hoops or boxes using one criterion • Sorting cardboard/solid shapes into a hoop Venn diagram using two criteria • Sorting cardboard/solid shapes into a • Organising a selection of shapes into families • Producing a quadrilateral 'diagnosis chart' using yes/no questions	4. Classify 2D shapes using a given category/criterion • one category/criterion • two categories/criteria - no overlap • two categories/criteria – overlapping (including using a Venn Diagram or Carroll Diagram) • three categories – no overlap • three categories- overlapping • categories to be derived	What's the same and what's different? circle; oval What's the same and what's different? triangle; pentagon; octagon; nonagon What's the same and what's different? regular; irregular
Angle Facts • completing a historical investigation to discover why we use 360 degrees to represent a full turn/circle (and the related facts for a straight line, right-angle etc) • using dynamic geometry packages to see what happens to angles at a point, on a straight line or those that are vertically opposite as the lines involved are moved (e.g. geogebra)	5. Find a missing angle around a point • given one known angle and the unknown • given two known angles and the unknown • given three or more known angles and the unknown • where one of the known angles is a marked right-angle	Convince me that angles around a point add up to 360 degrees Show me a diagram of angles around a point where the missing angle must be 55 degrees.
	6. Find a missing angle on a straight line • given one known angle and the unknown • given two known angles and the unknown • given three or more known angles and the unknown • where one of the known angles is a marked right-angle	Convince me that angles on a straight line add up to 180 degrees Show me a diagram of angles on a straight line where the missing angle must be 25 degrees.

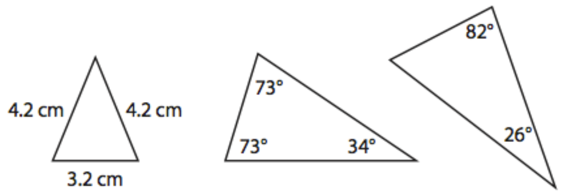
	7. Identify and use vertically opposite angles • find the value of all three remaining angles when given one angle as the intersection of two lines • find the value of all five remaining angles when given one angle as the intersection of three lines	Convince me that vertically opposite angles are equal. Show me a diagram of vertically opposite angles where the missing angle must be 40 degrees.
Angles in Triangles • cutting the corners off a triangle and rearranging to form a straight line •	8. Find missing angles in triangles • given two known angles and the unknown • given two known angles, one of which is a marked right-angle, and the unknown • given one of the equal angles in a marked isosceles triangle • given the unique angle in a marked isosceles triangle • given a marked equilateral triangle • given relative sizes of all three angles (e.g. in a ratio of 3:2:1 or this is angle is double this one, which is triple this one)	Show me the value of the other two angles in this isosceles triangle if this topic one is 40 degrees Always, Sometimes, Never? An isosceles triangle has one angle of 30^{o}. Is this enough information to know the other two angles?
Angles in Quadrilaterals • cutting the corners off a quadrilateral and rearranging to form a full turn	9. Find missing angles in quadrilaterals • irregular quadrilateral, given three known angles and the unknown • irregular quadrilateral, given three known angles some of which are marked right-angles, and the unknown • given one of the angles in a marked parallelogram or rhombus • given one of the angles in a marked symmetrical shape e.g. isosceles trapezium or kite or delta (note: can only find one other for a kite or delta)	Show me a quadrilateral with a missing angle that must be worth 70 degrees Convince me that a trapezium can't have three acute angles Convince me that the angles in a kite/quadrilateral must add up to 360 degrees What's the same and what's different? angles in a triangle, angles round a point, angles on a straight line, angles in a quadrilateral
Angles in Polygons • cutting/dividing regular polygons up into triangles find out their total angle sum (then dividing this by the number of angles to find the value of each)	10. Find the size of an angle in a regular polygon • find sum of angles in a regular polygon • find value of one angle • find missing angles using this	Convince me that the angles in a pentagon add up to 540 degrees Convince me that all the angles in a regular octagon measure 135 degrees. Always, Sometimes, Never? Each angle in a hexagon is 120 degrees.

Finding Missing Angles • Finding any missing angle that is possible on a diagram, before then searching for a route to the desired angle • Creating own angle problems to see how earlier choices of angles force others to be certain values	11. Find missing angles in multi-step problems • shape combined with straight line at one vertex • shape combined with full turn at one vertex • polygon divided up into triangles and quadrilaterals • two triangles joined at a vertex etc. (two quadrilaterals etc) • tessellating pattern	Show me a diagram with a missing angle that must be worth 80 degrees True or False? The order that find missing angles in matters.

Further Extension

1.
Which of these triangles are isosceles?

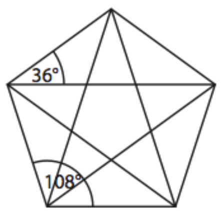
Explain your decisions.



2.
This is a regular pentagon.
Two angles (108° and 36°) are shown.

Which other angles can you work out?

Explain your reasoning.



3.
A triangle has been drawn carefully. You are told that the biggest angle is 20° larger than the second biggest angle and 40° larger than the smallest angle.

Work out how big each angle is.

Rich and Sophisticated Tasks

Compare and classify geometric shapes based on their properties and sizes and find unknown angles in any triangles, quadrilaterals, and regular polygons

STANDARDS UNIT: SS1 Classifying Shapes

NRICH: Where Are They? * P
NRICH: Quadrilaterals *** P I - challenging
NRICH: Round a Hexagon * P
NRICH: Always, Sometines or Never? Shape * P

BOWLAND assessments: Three of a Kind
BOWLAND assessments: Rods and Triangles

4.

Captain Conjecture says, 'The diameter of a circle is twice the length of its radius.'

Do you agree?
Explain your answer.

Captain Conjecture says, 'All circles with a radius of 4 cm have circumferences that are the same length.'

Do you agree?
Explain your answer.

Misconceptions	Teacher Guidance and Notes
There is a lot of vocabulary in this unit, which some children may find challenging as evidenced by interchanging words. However, children may not realise the full implications of the use of a technical word or be imprecise in their use of language or properties. For example, children describe squares as shapes with four equal sides, not considering that this also includes rhombuses. Additionally, children may use everyday language instead of technical e.g. diamond instead of rhombus or kite In the case of circumference, children do not realise that this is just a particular word for perimeter within the context of a circle. Look out for children using a shape's orientation to name it, rather than its intrinsic properties. E.g. a child may say that a square is a 'diamond' or a rhombus when it's base is not horizontal to the bottom of the page. Some children believe that all polygons are regular. Also, children often believe that an irregular polygon is one with all different angles and all sides of different lengths – they do not apply 'irregular' to a polygon with only one angle different to the others or only one side different to the others. When finding angles on straight lines, children may look at the whole line and not just at the point where their angle is involved. Thus they add angles not directly concerned with the point on the line under scrutiny. Children may also apply vertically opposite angle rules when there are three lines crossing and saying that any two of the six angles created must be equal. Children forget to use properties of shapes in angle questions e.g. symmetry will imply two angles of the same value, the definition of a parallelogram is that opposite angles are equal.	• This unit is focused on developing the language and precision of geometry. There is a shift towards being able to reason along a sequence of steps to find a missing piece of information by applying rules of angles and properties of shapes. • Wherever possible, link the language to related words in everyday life (e.g. radius to the radius bone in the arm, circumference to circumnavigate) and/or to their origins (e.g. isosceles means 'same' 'legs' in greek) • Be very precise about your own language e.g. be clear that a square is a special case of a rectangle, that an oblong is a non-square rectangle, that a rhombus is a special case of a parallelogram • It is important to enable children to discover the rules of angles and properties of shapes as much as possible as otherwise this is very memory-dependent and the knowledge may not be retained. • Try to teach children how we notate that two sides or angles are equal on a diagram (as well as parallel sides) • Establish clearly the difference between calculating missing angles using properties of shapes and estimating them from the diagram or measuring them with equipment. • Ensure children realise that sometimes angle questions are multi-step - you have to find one angle to lead you to the one you really want. Use the properties of shapes in these situations to tie the learning together e.g. opposite angles are equal in parallelograms. • A useful starting activity can be to get children to classify ALL the shapes they know! This can be a very revealing activity! • If you are sorting using a Venn or a Caroll diagram, make sure children are labelling these correctly to cover all possibilities.

Key Assessment Checklist

1. I can identify the radius, diameter and circumference of a circle
2. I can describe the relationship between radius and diameter
3. I can calculate missing angles within triangles
4. I can calculate missing angles within quadrilaterals
5. I can calculate missing angles within regular polygons
6. I can calculate missing angles on a straight line and around a point
7. I can calculate missing angles that are vertically opposite each other
8. I can classify shapes using any given criteria
9. I can identify and describe the features of special triangles
10. I can identify and describe the features of special quadrilaterals

Year 6	Unit 5 : Generalising Arithmetic	
14 learning hours	This unit is focused on developing fluency in the manipulation of number. At primary level this is focused on arithmetic itself and the methods for four operations particularly; however, this is naturally generalised to thinking about rules of arithmetic more widely at secondary level i.e. algebra. These aspects have been paired together intentionally to help teachers describe algebra as simply a generalisation of number. It is expected that teachers will go back to arithmetic to help students see where the 'rules' of algebra come from. Note that the greyed out content is covered previously and hence is not required content here unless of concern.	
Prior Learning	**Core Learning**	**Learning Leads to...**
	➢ **perform mental calculations, including with mixed operations and large numbers**	
➢ divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context ➢ multiply and divide whole numbers and those involving decimals by 10, 100 and 1000	➢ **divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context** ➢ **use written division methods in cases where the answer has up to two decimal places**	➢ formal written methods, to integers, decimals and simple fractions (proper and improper), and mixed numbers
	➢ **use their knowledge of the order of operations to carry out calculations involving the four operations** ➢ **solve problems involving addition, subtraction, multiplication and division**	➢ use conventional notation for priority of operations, including brackets ➢ recognise and use relationships between operations, including inverse operations (e.g. cancellation to simplify calculation" ➢ recognise and use relationships between operations, including inverse operations (e.g. cancellation to simplify calculations and expressions

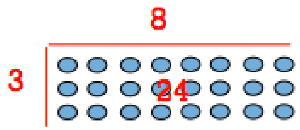
Exemplification	Vocabulary	
1. Calculate mentally a) 70×400 b) 62×4 c) $36\,000 \div 90$ d) $420 \div 3$ e) $72\,000 + 6\,500$ 2. a) Calculate $8211 \div 23$ b) Calculate, leaving any remainder as a fraction	divide dividend divisor remainder round fraction	order of operations precedence addition subtraction multiplication

c) There are 2342 children going to a book festival. They travel on coaches. Each coach can carry 53 people. What is the minimum number of coaches that will be necessary? d) 350 pizzas are shared between 17 people. How many pizzas does each person get? 3. Calculate $7211 \div 32$ to 2 decimal places 4. Calculate a) $25 - 4 \times 2$ b) $(35 + 415) \div 5 + 1$ 5. Find the missing value: $567 \div 3 = \blacksquare \div 34$	exchange decimal places	division brackets balance equal solve

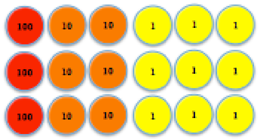
Representation

Dividing

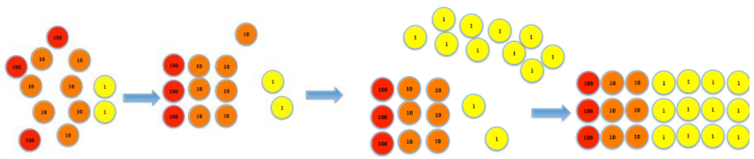
- For a calculation p ÷ q, grouping a set of p counters into groups of size q, arranging these groups as an array. For example, for 24 ÷ 3, count out 24 counters and arrange in columns of 3.... then read off the answer of 8 as the number of columns



- Building a number using place value counters and grouping them into groups that are the size of the divisor, before arranging these groups as an array to explore the partitioning approach. For example, 369 ÷ 3



- Repeating the above, but exchanging remaining counters for 10 counters of the next size down before continuing to group. For example, 372 ÷ 3



Fluency

14. Use a known multiplication/division fact to find other related facts
    - multiple of 10/100/1000 x multiple of 10/100/1000 e.g. 3000×40
    - multiple of 10/100/1000 ÷ multiple of 10/100/1000 e.g. $32000 \div 40$
    - decimal multiplication e.g. 0.3×8
    - decimal division e.g. $7.2 \div 6$
    - doubling/tripling/quadrupling first number of a multiplication/division fact e.g. If $13 \times 7 = 111$, find 26×7 or If $255 \div 15 = 17$, find $2550 \div 15$
    - doubling/tripling/quadrupling second number of a multiplication fact e.g. If $13 \times 7 = 111$, find 13×21
    - doubling/tripling/quadrupling second number of a division fact If $255 \div 15 = 17$, find $255 \div 5$

15. Carry out multiplication and division mentally (with jottings)
    - 2-digit x 1-digit mentally (partitioning) e.g. 36×4
    - simple 3-digit x 1-digit mentally (partitioning) e.g. 236×4
    - 2-digit ÷ 1-digit mentally (partitioning) e.g. $76 \div 4$
    - simple 3-digit ÷ 1-digit mentally (partitioning) e.g. $116 \div 4$

16. Carry out complex mental calculations using all four operations
    - addition and subtraction only – large numbers
    - two-step problems e.g. multiplication and addition

17. Recap: Divide a 3/4-digit number by a 1-digit number using a written method
    - no exchange necessary e.g. $9366 \div 3$
    - first digit is lower than divisor requiring exchange e.g. $2196 \div 3$
    - single exchange e.g. $2376 \div 3$ or $8476 \div 4$
    - two or more exchanges e.g. $4185 \div 5$

- Exploring with larger numbers and decimals also (i.e. exchanging a 1 counter for ten 0.1s)
- Recording these methods to arrive at compact division
- Using the bar model to represent a word or other division problem. For example, 1764 ÷ 6
-

1764					
294	294	294	294	294	294

Order of Operations

- Calculating using scientific calculators and basic (non-scientific) calculators simultaneously and observing the differences in result e.g. for 2 + 5 x 3.
- Exploring which calculations this matters for and which it does not.
- Using number cards (and bracket) cards and positioning/rearranging to try to reach a given result

Solving Problems

- Drawing bar models to represent problems involving addition, subtraction, multiplication and division (using a new line to show a new stage of the calculation)

- examples with remainders e.g. $4189 \div 5$

18. Divide a 3-digit number by 2-digit number using a formal written method
    - known times table, no remainder e.g. $756 \div 12$
    - simple times table to derive, no remainder e.g. $966 \div 21$
    - larger number requiring derivation of times table, no remainder $986 \div 34$
    - examples with remainders e.g. $874 \div 15$

19. Divide a 4-digit number by a 2-digit number using a formal written method
    - known times table, no remainder e.g. $2856 \div 12$
    - simple times table to derive, no remainder e.g. $6594 \div 21$
    - larger number requiring derivation of times table, no remainder 3591 ÷ 57
    - examples with remainders e.g. $4588 \div 16$

20. Divide a 4-digit number by a 2-digit number using a formal written method , giving the remainder as a fraction
    - known times table, no remainder e.g. $2857 \div 12$
    - simple times table to derive, no remainder e.g. $6597 \div 21$
    - larger number requiring derivation of times table, no remainder 3541 ÷ 57

21. Divide a 4-digit number by a 2-digit number, giving an answer to up to 2dp, using a formal written method
    - example resulting in exactly 1dp e.g. $4580 \div 16$
    - example resulting in exactly 2dp e.g. $4576 \div 16$
    - example resulting in longer decimal (requiring truncation) e.g. $4579 \div 16$

22. Recognise and solve a simple division problem, interpreting any remainders in the context as appropriate.
    - word problem – sharing language e.g. 3282g flour to make 56 cupcakes. How much flour is in each cupcake?
    - word problem – grouping language e.g. 2543 people go to Wembley. 52 people can fit on each bus. What is the minimum number of buses required?
    - Remainder problems – an account has £9542 in it. If you spend £86 per day, after how many days will the money run out?
    - finding unit fractions of an amount e.g. find a twelfth of 5184
    - problems with links to factors (and multiples) e.g. show that 14 is a factor

of 8834

23. Calculate using the correct order of operations (no brackets)
    - examples with one of multiply/divide and one of add/subtract e.g. 4 + 2 x 7 or 12 + 20 ÷ 5
    - examples with three operations e.g. 24 ÷ 3 + 5 x 2
    - examples with strings of additions/subtractions or multiplications/divisions e.g. 18 – 2 + 3 – 1 or 9 ÷ 3 x 6 ÷ 2
    - examples involving strings and other operations e.g. 16 ÷ 2 + 4 - 1
24. Calculate using the correct order of operations (including brackets)
    - examples involving a single bracket e.g. 4 x (2 + 7) ÷ 6
    - examples involving more than one bracket e.g. (24 – 4) x (3 + 5)

25. Solve missing number problems involving addition, subtraction, multiplication and division
    - one operation e.g. $43 \times \blacksquare = 6622$ (missing box either position, answer can come first)
    - two operations e.g. $43 \times \blacksquare = 5187 + 1435$ (missing box anywhere)
    - three or more operations

26. Solve complex calculations involving all four operations
    e.g. $614 + 43 \times 23$ or $454.5 \div 18$

Probing Questions

Show me...	Convince me...	What's the same? What's different?	Always, sometimes, never
... the operations you would carry out to solve this problem Printing charges for a book are 3p per page and 75p for the cover. I paid £4.35 to get this book printed. How many pages are there in the book? ... how you can multiply 4305 by 37 using - a grid method - a column method ...how you can divide 4305 by 5 - using place value counters	... that if I divide 132 sweets between 5 people, this gives 26 r2 or 26 2/5 each. ... that 3 + 5 x 2 can be done in two orders and give different answers ... 6.24×8 does not equal (6x8)+(0.2×8)+(0.04×8)=48 + 0.16 + 0.032 = 48.192 ... that 7314×11=80454 and ... that 7.314 x 11 = 80.454	...long division and compact division ... 4565 ÷ 15 and 456.5 ÷ 1.5 and 45.65 ÷ 0.15 125/15, 390/4, 1245/9, 216/6	When dividing by a 2-digit number, you have to exchange immediately Multiplication always comes before addition Long division is needed to divide a four digit number by a two digit number A calculation involving division will have a remainder Division is the inverse of multiplication

- using a written method	... 1756 ÷ 5 will have a remainder of 1 ... that a remainder of 5 can mean different things in different questions		Addition comes before subtraction when calculating

Further Extension	Rich and Sophisticated Tasks
1. A box of labels costs £63. There are 140 sheets in the box. There are 15 labels on each sheet. Sara, Ramesh and Trevor want to calculate the cost of one label, in pence. Ramesh uses the number sentence (6300 ÷ 140) × 15. Sara uses the number sentence 63 ÷ 1·4 ÷ 15. Trevor uses the number sentence (15 × 140) ÷ 6300. Who is using the right number sentence? Explain your choice. 2. Miriam buys 19 tins of soup. All the tins cost the same price. She goes to the shop with just one note, and comes home with the tins and the change in coins. On the way home she drops the change. She looks carefully and she thinks she picks it all up. When she gets home she gives £2·23 change to her mother. Do you think that Miriam picked up all the change that she dropped? Explain your reasoning. 3. Which calculation is the odd one out? ▪ 753 × 1·8 ▪ (75·3 × 3) × 6 ▪ 753 + 753 ÷ 5 × 4 ▪ 7·53 × 1800 ▪ 753 × 2 – 753 × 0·2 ▪ 750 × 1·8 + 3 × 1·8 Explain your reasoning.	Perform mental calculations, including with mixed operations and large numbers **NRICH:** [Exploring Number Patterns You Make](#) **** P I** **NRICH:** [Become Maths Detectives](#) *** P I** Solve problems involving addition, subtraction, multiplication and division **NRICH:** [Always, Sometimes or Never? Number](#) *** P**

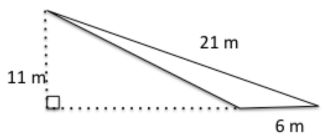
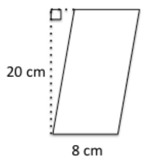
Misconceptions	Teacher Guidance and Notes
When dividing, children often forget to carry a remainder over as part of the exchange process. This is usually because there is too much information to hold in their head with the nearest multiple and the remainder then not accounted for. This particularly happens at the start of the number where a child may incorrectly 'carry' the divisor across, rather than the first digit of the dividend. Children have difficulty interpreting remainders resulting from a division as fractions, e.g. if the remainder is 3 from a calculation involving the divisor 5, children will write the remainder as 1/3 rather than 3/5 Children will sometimes approach multi operational problems with the correct operations but carry them out in the wrong order. They may also sometimes choose the incorrect operation directly. Children may sometimes solve problems by choosing the least efficient operation e.g. using repeated addition instead of multiplication When solving multi-step problems, some children complete only the first step of a problem and use it as the answer without going on to the next step	• The initial focus of this unit lies in division skills, building on Stage 5 learning to approach division by a 2-digit number. • The pitch is numbers up to 4-digits, but can include examples where the division continues past the decimal point to give a decimal of up to 2dp. • At this level remainders need to be written as fractions of the divisor or rounded appropriately if in context • As always, ensure children carry out the apparatus and written approaches simultaneously during the learning process so that they realise these are the same methods before moving to the solely abstract. • The unit then moves on to looking at the correct order of operations – children will not have seen this before so an exploration of why it is necessary is essential. • Ensure you are clear with children that multiplication and division are of equal precedence (as are addition and subtraction) and hence, when they appear together, should be carried out from left to right once any brackets have been dealt with. • See the videos at the NCETM website for more details of the use of place value counters for division. • Also cross-reference with the AET calculation policy. • Draw attention to language used within addition, subtraction, multiplication and division problems and model this. It is recommended that you teach children the language of the calculation e.g. dividend • Make children aware that some problems involve multiple steps which could mean 2 or more different operations. Use the bar model to represent the problem visually before deciding the operations to carry out and the order of these.

Key Assessment Checklist

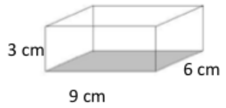
9. I can perform mental calculations including mixed operations
10. I can divide a four digit number by a two digit number using a written method of long division
11. I can interpret remainders as whole number remainders, fractions or by rounding
12. I can use written division methods in cases where the answer has up to two decimal places
13. I can recognise and solve simple division problems with numbers up to 4d and 2d.
14. I can solve multi step four operation problems choosing the correct operation and using the most appropriate methods
15. I can calculate in the correct mathematical order, with and without brackets.
16. I can solve problems involving addition, subtraction, multiplication and division

Year 6	Unit 6 : Reasoning with Measures
10 learning hours	This unit focuses on mensuration and particularly the concepts of perimeter, area and volume. Primary children are also working on money concepts at this stage, while older secondary students develop mensuration into volume and surface area of challenging shapes, applying Pythagoras' Theorem and trigonometry also in combination with these problems. Note the focus on reasoning within this unit: it is common for children to complete routine problems involving mensuration but this unit is about the developing a secure conceptual understanding of these ideas that they can apply to a wide range of problems and contexts. The opportunity to use and build on earlier number work is built into this unit and it is expected that children apply their arithmetic skills, for example, in these problems.

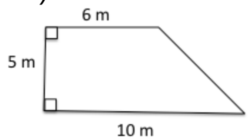
Prior Learning	Core Learning	Learning Leads to...
➢ measure and calculate the perimeter of composite rectilinear shapes in centimetres and metres ➢ calculate and compare the area of rectangles (including squares), and including using standard units, square centimetres (cm^2) and square metres(m^2) and estimate the area of irregular shapes	➢ **recognise that shapes with the same areas can have different perimeters and vice versa** ➢ **calculate the area of parallelograms and triangles**	➢ calculate perimeters of 2D shapes ➢ know the formulae: circumference of a circle = $2\pi r = \pi d$, ➢ know and apply formulae to calculate area of triangles, parallelograms, trapezia ➢ know the formulae: area of a circle = πr^2
➢ estimate volume [for example, using 1 cm^3 blocks to build cuboids (including cubes)] and capacity [for example, using water]	➢ **calculate, estimate and compare volume of cubes and cuboids using standard units, including cubic centimetres (cm^3) and cubic metres (m^3), and extending to other units [for example, mm^3 and km^3]** ➢ **recognise when it is possible to use formulae for area and volume of shapes**	➢ know and apply formulae to calculate volume of cuboids

Exemplification	Vocabulary
1. A rectangle has an area of $14cm^2$. The rectangle has side lengths that are whole numbers of centimetres. a) Sketch a possible rectangle and find its perimeter. b) Sketch another possible rectangle and find its perimeter. 2. Find the area of a)  b) 	perimeter length total sum area square units e.g. mm^2, m^2, cm^2 base width breadth height parallelogram

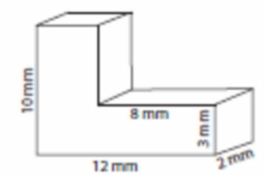
3. Which shape has the larger volume? A cube of side 6cm or this cuboid?



4. a) Find the area of



b) Find the volume of



triangle
compound shape
dissect
formula
geoboard
grid
inverse
volume
cubed units e.g. km^3, m^3, cm^3, mm^3
cube
cuboid
layers
maximum
minimum
multiply
perpendicular
rectilinear

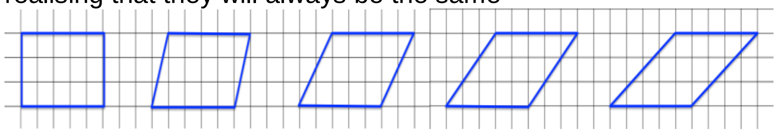
Representation

Perimeter

- Measuring distance around shapes with string, clinometers, metre rules, trundle wheels
- Using geoboards to produce shapes with a given perimeter

Area

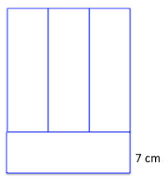
- All: Dissecting rectangles to form parallelograms and dissecting parallelograms to form triangles.
- Rectangles: Exploring shapes drawn on squared paper to develop efficient counting strategies to deduce $A = l \times w$
- Using squares and arranging into arrays (rectangles) to suggest possible rectangles with a given area
- Parallelograms: Exploring the sequence of toppling parallelograms drawn on squared paper, counting the squares to find their areas and realising that they will always be the same



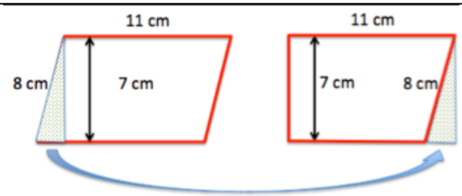
- Cutting the 'slanted' piece of a parallelogram off and repositioning it to form a rectangle to show why $A = length \times perpendicular\ height$

Fluency

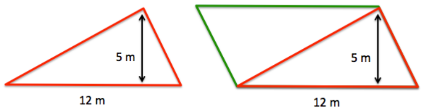
1. Recap: Find the perimeter of a shape
   - shape shown on a grid
   - shape with all lengths marked (triangle, quadrilateral etc)
   - rectangle with length and width marked (whole numbers then decimals/fractions)
   - derive formula for perimeter of a rectangle $P = 2(a + b)$
   - given the area of a rectangle and the length, find the width
   - other shapes with equal lengths marked e.g. isosceles triangle
   - shape with equal lengths implicit e.g. kite, rhombus
   - composite shape made of rectangles
   - composite shape made of rectangles where length of basic rectangle is a multiple of width e.g. this shape is made from four identical rectangles – find its perimeter



2. Recap: find the area of a rectangle
   - rectangle shown on a grid



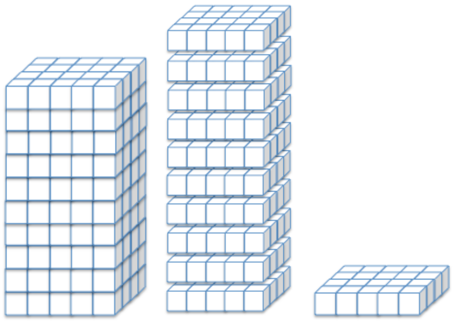
- Triangles: Putting two identical triangles together to form a parallelogram to realise that every triangle is half of a parallelogram. [It is best to avoid starting with a right-angled triangle if possible as this leads straight to a rectangle but acute-angled and obtuse-angled triangles do not].



- Drawing 'the other triangle' to form the parallelogram of which a given triangle is half to deduce the formula
  $A = \frac{1}{2} \times length \times perpendicular\ height$
- Using geoboards to produce shapes with a given area

Volume

- Dissecting a cuboid made from small cubes (e.g. unifix or multilink) into layers to find ways of counting efficiently



x 10

- Building cuboids out of a given number of cubes to find different cuboids of the same volume

  - rectangle with length and width given (whole numbers then decimals/fractions)
  - square
  - derive formula $A = l \times w$
  - given the area and length, find the width
  - find the area of composite shape made of rectangles
3. Suggest rectangles with a given area or perimeter
   - given area, sketch possible rectangles with integer sides and find their perimeters
   - given area, sketch possible rectangles with non-integer sides and find their perimeters
   - ext: given area, find minimum perimeter
   - given perimeter, sketch possible rectangles with integer sides and find their areas
   - given perimeter, sketch possible rectangles with non-integer sides and find their areas
4. Find the area of a parallelogram
   - examples given base and perpendicular height
   - examples given base, perpendicular height and another (extraneous) length
   - examples oriented differently i.e. base not parallel to bottom of page
   - derive formula for area of a parallelogram as $A = length \times perpendicular\ height$
   - given the area of a parallelogram and its length (or height), find the height (or length)
5. Find the area of a triangle
   - examples given base and perpendicular height (acute-angled, then obtuse-angled)
   - right-angled triangles
   - examples given base, perpendicular height and another (extraneous) length
   - example oriented differently i.e. base not parallel to bottom of page
   - derive formula for area of a triangle as $A = \frac{1}{2} \times length \times perpendicular\ height$
   - given the area of a triangle and its length (or height), find the height (or length)
6. Solve problems involving area
   - Find the area of a composite shape – triangle and rectangle e.g. trapezium

- Find the area of a composite shape – parallelogram and rectangle
- Find the area of a composite shape – triangle and parallelogram
- Find the larger area of a selection of shapes/order shapes by area
- Set area of two shapes equal and use this to find a missing length of one of them
- mixed units

7. Find the volume of a cuboid
    - examples where the shape is shown broken into cubes
    - examples where the length, width and height are given
    - cubes
    - examples oriented differently i.e. where base is not parallel to bottom of page
    - examples where area of a face and other length given
    - derive formula for volume of a cuboid $V = l \times w \times h$
    - mixed units

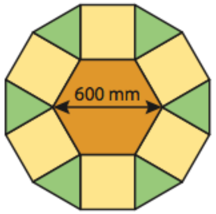
8. Solve reverse problems using the volume of a cuboid
    - given the volume of a cuboid and two lengths, find the other length
    - given the volume of a cuboid and area of a face, find the length
    - given the volume of a cuboid and one length, suggest other possible lengths for the remaining two dimensions (or sketch possible cuboids)
    - given the volume of a cuboid, suggest possible dimensions for the cuboid
9. Find the volume of a composite shape
    - composite shape of two cuboids (e.g. L-shape, T-shape), broken into cubes
    - composite shape of two cuboids, all lengths marked
    - composite shape of two cuboids, some lengths to be deduced

10. Solve other problems involving volume
    - find the larger volume of a selection of shapes/order shapes by volume
    - given an irregular shape made of cubes, say how many more are required to make a cuboid

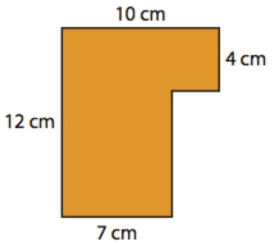
Probing Questions

Show me...	Convince me...	What's the same? What's different?	Always, sometimes, never
... how to find the area of a (specified) shape on a geoboard	... that the formula for the area of a parallelogram gives the same answer	Buying carpet, buying lawn seed, buying skirting board.	Triangles are half a parallelogram.

... how to find the area of a (specified) triangular shape on a geoboard .. the area of this triangle/parallelogram etc ... a triangle with area $12cm^2$ ... a parallelogram with the same area as this rectangle ... the volume of this cuboid ... another cuboid with the same volume	as counting squares. ... that every parallelogram can be rearranged to make a rectangle ... that you only need to know the base and the height of a triangle to find its area ... that any triangle is half a parallelogram ... that you can find the volume of a cuboid if you know its height and the area of its base.	rectangle; parallelogram; right-angled triangle; (general) triangle perimeter; area; volume	If one box fits inside another, the inner box has a smaller volume than the outer box. A box with volume 8cm3 will fit completely inside a box of 9cm3. You find the volume by multiplying all the lengths together.

Further Extension	Rich and Sophisticated Tasks
1. Here is a tiled floor pattern. It is made from equilateral triangles, squares and a regular hexagon. Work out the perimeter of the design. Give your answer in metres.  2.	STANDARDS UNIT: SS2 Understanding Perimeter and Area Recognise shapes with same area can have different perimeters **NRICH: Dicey Perimeter, Dicey Area * Game** Volume of cuboids **NRICH: Next Size Up ** P**

Sami worked out the area of the orange shape as $10 \times 4 + 8 \times 7 = 96\,cm^2$.
Razina worked out the area as $12 \times 7 + 3 \times 4 = 96\,cm^2$.
Lukas worked out the area as $10 \times 10 - 2 \times 2 = 96\,cm^2$.



Are you convinced by Sami, Razina or Lukas's reasoning?

Explain your answer.

3.
Liping says, 'If you draw two rectangles and the second one has a greater perimeter than the first one, then the second one will also have a greater area.'

Do you agree or disagree with her?

Explain your reasoning.

4.
Can you find two or more different cuboids each with a volume of $64\,cm^3$?
What's the same and what's different about your cuboids?

5.
The diameter of a golf ball is 4 cm. I want to make a box which will hold six golf balls.

What size could my box be?
Is there more than one answer?

Misconceptions	Teacher Guidance and Notes
At this stage there may still be some confusion between perimeter and area for children. Try to teach it separately initially to avoid this issue and to bring perimeter into revision activities linked to addition and then area into revision activities linked to multiplication.	• This unit consolidates previous work on perimeter and area and extends beyond rectangles and right-angled triangles to parallelograms and triangles. It also brings in the volume of a cuboid. Although not mentioned specifically, there is an expectation that children will be able to find the area and volume of simple composite (or compound) shapes

An insecure concept of area can cause some children to not realise that dissecting and rearranging a shape preserves the area. Similarly, some children may not be able to explain the formula for area of a rectangle (or other shapes beyond). Often there is confusion between different shape types due to weaker knowledge of the properties of quadrilaterals and triangle. Many children do not realise the importance of perpendicular height for parallelograms and triangles and, even if they do, may find it hard to identify the perpendicular height. They find this particularly hard where 'extra' lengths are given that are not needed. It is common to see some children multiplying any lengths given rather than identifying those they need. Many children lack a sense of the concept of volume as the number of unit cubes needed to fill a solid. There can be difficulty in linking area to volume, specifically difficulty in seeing equivalence of area of base and no. of cubes in bottom layer. Note that this can sometimes be caused by a lack of experience or basic knowledge to support estimates of measures.	by breaking them up into their constituent pieces (links to the statement about recognising when to use a formula) • Earlier work on right-angled triangles as half of rectangle now extends to general triangle. A similar approach to parallelograms can be taken by cutting and rearranging. However, it is recommended that children rearrange triangles into halves of parallelograms (rather than rectangles, which is more difficult). Therefore, triangles should follow parallelograms as they are half a parallelogram. • It is recommended that teachers take an investigative approach to finding area and volume. • As indicated in the calculation units, link area to grid method via (eg) 23cm x 14 cm rectangle on cm squared paper. Dissect into 20x10 + 20x4 + 3x10 + 3x4 rectangles and compare to grid method. Grid thinking can also support fraction work and algebra later on. • It is worth emphasising that the height of a rectangle is a perpendicular height already. • Previous work on volume now extends to developing the formula beyond the approach of counting, however efficiently, in stage 5.

Key Assessment Checklist

1. I can find rectangles with the same perimeter but different areas and vice versa. I can investigate the max area for a given perimeter or min perimeter for a given area.
2. I can find the area of a composite shape by dissecting it into rectangular pieces. I can use this idea to explain the grid method of multiplication in terms of areas.
3. I can use formulas to solve problems involving the perimeter and area of rectilinear shapes, including inverse problems.
4. I can explain the formulas for (i) area of a parallelogram by relating it via cut and paste to the area of a rectangle (ii) area of a triangle as half of a parallelogram
5. I can calculate the area of a parallelogram by using base x perpendicular height and of a general triangle as 1/2 base x perpendicular height. I understand that two triangles with the same base and same perpendicular height have the same area.
6. I can identify composite shapes where area formulas can be applied to different parts and combined, and solve related problems including areas of geoboard shapes.

7. I can explain why the volume of cuboid is l x w x h by referring to rectangular layers of unit cubes. I can calculate simple cases using this formula.
8. I can solve problems in estimating, calculating and comparing volumes of cubes and cuboids, including inverse problems using mm^3, cm^3, m^3 and km^3 as appropriate.

Year 6	**Unit 7 : Discovering Equivalence**	
8 learning hours	This unit explores the concepts of fractions, decimals and percentages as ways of representing non-whole quantities and proportions. For the youngest children, the work is focused on fractions and developing security in recognising and naming them. At KS2 this then builds to looking at families of fractions and decimals and percentages. At secondary level this is extended to more complex % work and equivalence with recurring decimals and surds.	
Prior Learning	**Core Learning**	**Learning Leads to...**
➢ identify, name and write equivalent fractions of a given fraction, represented visually, including tenths and hundredths	➢ **use common factors to simplify fractions; use common multiples to express fractions in the same denomination**	
➢ compare and order fractions whose denominators are all multiples of the same number	➢ **compare and order fractions, including fractions > 1**	➢ order positive and negative integers, decimals and fractions
➢ read and write decimal numbers as fractions [for example, 0.71 = 71/100]. recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents	➢ **associate a fraction with division and calculate decimal fraction equivalents [for example, 0.375] for a simple fraction [for example, 3/8]**	
➢ solve problems which require knowing percentage and decimal equivalents of 1/2, 1/4, 1/5, 2/5, 4/5 and those fractions with a denominator of a multiple of 10 or 25.	➢ **recall and use equivalences between simple fractions, decimals and percentages, including in different contexts**	➢ define percentage as 'number of parts per hundred' ➢ interpret percentages and percentage changes as a fraction or a decimal, and interpret these multiplicatively
➢ recognise the per cent symbol (%) and understand that per cent relates to 'number of parts per hundred' ➢ write percentages as a fraction with denominator 100, and as a decimal	➢ **solve problems involving the calculation of percentages [for example, of measures, and such as 15% of 360] and the use of percentages for comparison**	➢ express one quantity as a percentage of another ➢ compare two quantities using percentages ➢ solve problems involving percentage change, including percentage increase/decrease ➢ interpret fractions and percentages as operators

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Exemplification

1. a) Simplify (i) $\frac{16}{20}$ (ii) $\frac{5}{35}$

   b) Find the value of ■ to produce equivalent fractions (i) $\frac{2}{3} = \frac{■}{18}$ (ii) $\frac{7}{9} = \frac{35}{■}$

2. a) Place these fractions in order from smallest to largest:

$$\frac{3}{4} \qquad \frac{3}{5} \qquad \frac{13}{20} \qquad \frac{7}{10} \qquad \frac{4}{5}$$

   b) Which is greater? $1\frac{1}{3}$ or $\frac{5}{4}$?

3. a) Three cakes are shared between 15 people equally. What proportion of a cake does each person receive?

   b) Convert $\frac{5}{8}$ to a decimal using division

4. $\frac{4}{5}$ of Class A completed their Year 6 Challenge. 82% of Class B completed the challenge.
Which class had a higher proportion of pupils that completed the challenge?

5. Calculate a) 5% of 240 b) 85% of 180cm

Vocabulary

fraction part whole denominator numerator equivalent simplify factor multiple common compare greater than less than order	improper fraction mixed number decimal percentage % proportion tenths hundredths

Representation

Fractions

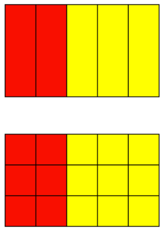
- Folding (and colouring) paper circles to represent a unit (and then non-unit) fraction to compare two or more fractions (and hence order them).
- Similarly using these or pre-made versions (e.g. magnetic pieces) to discover equivalent fractions
- Folding (and colouring) paper strips to represent a unit (and then non-unit) fractions to compare two or more fractions (and hence order them)
- Producing own fraction wall or fraction-fan to help identify equivalent

  fractions
- Positioning fractions on a number line (washing line) including beyond 1.
- Folding paper strips vertically (rectangles) to represent a fraction and then folding horizontally to discover equivalent fractions and the proportional link between numerators and denominators

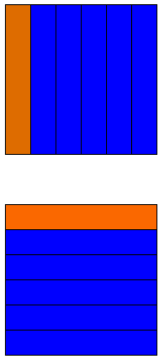
Fluency

1. Simplify a fraction
   - those requiring division by 2 e.g. $\frac{14}{20}$ or $\frac{16}{28}$
   - those requiring division by 5 e.g. $\frac{15}{20}$
   - those requiring division by 3, 7, and other primes e.g. $\frac{9}{21}$
   - efficiently, using higher common factors e.g. $\frac{16}{24}$
   - improper fractions and mixed numbers e.g. $\frac{18}{4}$
2. Find equivalent fractions
   - unit fractions, find any equivalent or a list of equivalent fractions
   - non-unit fractions, find any equivalent or a list of equivalent fractions
   - given denominator, find numerator of equivalent fraction e.g. $\frac{1}{7} = \frac{■}{28}$
   - given numerator, find denominator of equivalent fraction e.g. $\frac{2}{11} = \frac{12}{■}$
3. Compare and order fractions
   - compare two fractions, same denominator
   - compare two fractions, related denominators (i.e. one a multiple of the other)
   - compare two fractions, different denominators

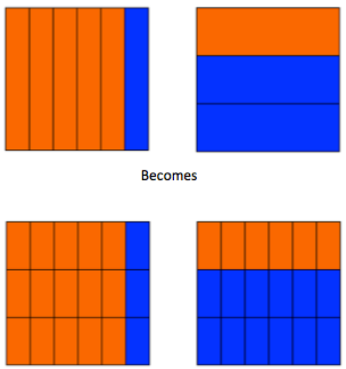
For example, for $\frac{2}{5}$ is equivalent to $\frac{6}{15}$



- Representing fractions using the bar model (vertically and horizontally) e.g. $1/6$



- Comparing two fractions with different denominators using an array
  For example, $\frac{5}{6}$ and $\frac{1}{3}$



$\frac{15}{18}$ and $\frac{6}{18}$

- Using Fractions ITP (Nat Strat) http://www.taw.org.uk/lic/itp/fractions.html

- order three or more fractions
- compare two mixed numbers or two improper fractions
- compare a mixed number to an improper fraction
- order mixed numbers and improper fractions

4. Represent divisions using fractions
   - write a sharing or grouping problem as a fraction
   - recognise that such a fraction is the answer in its own right (although thi can be calculated as a decimal if desired)
   - compare results of such divisions by comparing fractions (e.g. is it better to part of a group of 7 people sharing 4 pizzas or 5 people sharing 3 pizzas)
5. Convert fractions to decimals
   - denominator 2 e.g. $\frac{5}{2}$
   - denominator 10 e.g. $\frac{3}{10}$
   - denominator 100 e.g. $\frac{17}{100}$
   - denominator 5 e.g. $\frac{3}{5}$
   - denominator 4 e.g. $\frac{7}{4}$
   - denominator 8 e.g. $\frac{3}{8}$
   - denominator 20 e.g. $\frac{7}{20}$
6. Convert decimals to fractions
   - tenths, no simplifying e.g. 0.3
   - tenths, simplifying, e.g. 0.4
   - hundredths only, no simplifying e.g. 0.07
   - hundredths only, simplifying e.g. 0.08
   - tenths and hundredths, no simplifying e.g. 0.27
   - tenths and hundredths, simplifying e.g. 0.32
7. Use fraction, decimal and percentage equivalence
   - convert between tenths and percent e.g. 0.3
   - convert between hundredths and percent e.g. 0.07 or 0.17
   - compare a decimal and a percentage to say which is greater
   - compare a fraction and a decimal to say which is greater
   - compare a fraction and a percentage to say which is greater
   - recall equivalences for halves, tenths, quarters, fifths, hundredths, thirds, sixths and eighths.
   - order fractions, decimals and percentages
8. Calculate a percentage of an amount (number or measure)
   - 50%

Fractions, Decimals and Percentages

- Representing a given fraction is as many different forms as possible e.g. three quarters
- Using the bar model (or paper strips) to represent percentages by making the whole worth 100%. Hence finding equivalent percentages as fractions of 100.
- Using place value counters to help complete fraction division to find equivalent decimals e.g. $1/8$ as $1 \div 8$ using bus stop method, exchanging the 1 for 10 tenths and so on.

- 10%, number ending in 0
- 10%, number not ending in 0
- multiples of 10% e.g. 20%, 30%, 70%
- 5%
- 1% (from 10% ending in 0)
- 1% (from 10% not ending in 0)
- 1% directly from 100% ending in 00
- 1% directly from 100% not ending in 00
- multiples of 1% e.g. 3%
- combinations e.g. 15% or 31%
- compare percentages e.g. which is better? 50% of 20 or 20% of 50?
- compare fraction of an amount with a percentage of an amount e.g. which is higher? 3/5 of 75 or 40% of 120

Probing Questions

Show me...	Convince me...	What's the same? What's different?	Always, sometimes, never
... a fraction that cannot be simplified ... three ways that you could simplify 18/30 ... three fractions that are equivalent to 2/7 ... four possible common denominators for 3/5 and 7/20 ... the equivalent decimal for 1/4, 7/20, 1/8, 4/25	... how to simplify a fraction ... how to order a set of fractions such as 2/3, 1/4, 5/6, 1/2 ... what is wrong with 2/3 = 1/1.5 ... that 8/5 > 3/2 ... that there are an infinite number of fractions equivalent to 2/3	14/21, 8/12, 20/30 $1\frac{2}{5}, 1.4, \frac{10}{8}, 1\frac{1}{4}, \frac{7}{5}, \frac{21}{15}, 1.25, \frac{5}{4}$ 30%, 3/10, 0.3, 0.03, 3/100, 3% 1/5, 5%,1/20, 20% 20% of 25 and 25% of 20	... you can have more than 100% ... to cancel a fraction, you halve the numerator and denominator until you can't do it any more ... you can always simplify a fraction ... to find x% of a number, just divide it by x ... you can do long division when the dividend is less than the divisor

Further Extension

1.
Only a fraction of each whole rod is shown. Using the given information, identify which whole rod is longer.

Rich and Sophisticated Tasks

Recall and use equivalences between simple fractions, decimals and percentages, including in different contexts
NRICH: Doughnut Percents * P

Solve problems involving the calculation of percentages [for example, of measures, and such as 15% of 360] and the use of percentages for comparison
NRICH: Would you Rather? * P

Explain your reasoning.

2.
Which is the odd one out?
$\frac{2}{5}$, 0·4, $\frac{4}{10}$, $\frac{3}{6}$, $\frac{6}{15}$

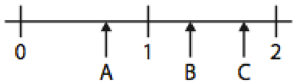
Explain your choice.

Put the following numbers into groups:
$\frac{3}{4}$, $\frac{3}{2}$, 0·5, 1·25, $\frac{3}{8}$, 0·125.

Explain your choices.

3.
Put the following numbers on a number line:
$\frac{3}{4}$, $\frac{3}{2}$, 0·5, 1·25, 3 ÷ 8, 0·125

4.
Suggest a fraction that could be at point A, a decimal that could be at point B and an improper fraction that could be at point C on this number line.



5.
Last month Kira saved $\frac{3}{5}$ of her £10 pocket money. She also saved 15% of her £20 birthday money.

How much did she save altogether?

Misconceptions	Teacher Guidance and Notes
Even at this stage, some children may still not appreciate that equivalent fractions must be an equivalent proportion of a whole. They may still fail to ensure that each part of the whole is of equal size. Children may confuse the definitions of factors and multiples and hence muddle the processes of simplifying and finding fractions with a common denominator. This work will be greatly hindered by children not knowing their times tables securely as this may lead to errors as well as unnoticed opportunities to simplify. Some children believe that a fraction is simplified by repeatedly dividing by 2 and do not realise that division by any and all common factors is expected. Similarly, they may cancel by simply removing a matching digit, saying that, for example, 17/27 is equivalent to ½ by removing the 7 digits in the numerator and denominator. When finding equivalent fractions, children may forget to adjust the numerator as well as the denominator, or they may multiply it by a different value, thus losing the value of the fraction. They may also try to think additively rather than multiplicatively, for instance saying that 3/5 is equivalent to 5/7 because they have added two to both the numerator and denominator. Some children will still try to make comparisons of fractions and order fractions by inspection of the numerators and/or denominators rather than seeking equivalent fractions with matching denominators to facilitate the comparison. Children do not always realise that a percentage is simply a different way of describing the proportion of the whole. When finding percentages of amounts, children sometimes revert to thinking that (because one can find 10% by dividing by 10) one can find, for example, 20% by dividing by 20. There is often an interchange of fifths and twentieths i.e. children may think that 1/5 is 5% instinctively. When finding 10%, children may make errors in their division by 10, for example, by thinking that dividing by ten means removing the zero rather than moving all the digits down one place. When finding decimal equivalents to more complex problems, the issues and misconceptions of division may arise. Commonly children often struggle to conceive of how we might divide a smaller number by a larger and thus may put the dividend and the divisor in the wrong places within the formal written method.	• This unit is all about securing children's understanding of and flexibility with representations of fractions (decimals and percentages). • In the next unit, children will need to conduct arithmetic with fractions but for now they are simply representing, manipulating and ordering them. • Therefore, spend time ensuring that all possible representations are fully understood at this point and that, even when we use improper fractions or those with more unusual denominators, children have strong skills in representing these visually (e.g. using a number line, proportion of a shape, proportion of a set of objects, proportion of a bar) as well as in decimal and percentage form. • Some of these fraction/decimal/% equivalents simply need to be learned to be recalled - make sure you explore the concepts first but then insist on rapid recall. • You will need to include regular rehearsal of times tables and corresponding division facts in order to maintain the recall of knowledge that supports the processes of simplifying and finding equivalent fractions with a common denominator. • When children struggle with a representation, go back to first principles to unpick the issue and build it back up - make sure you don't rely on a single representation to work with fractions here - you need to use a range to build a confident learner. It is better to spend time ensuring children can recognise and draw representations in all formats than move straight to the number work. At all costs, refrain from instructing children to mechanically multiply (or divide) the top and bottom numbers by the same amount as this type of algorithmic approach will not generate the conceptual understanding you will need in the next unit and beyond. • When working with decimals and percentages, spend time making sure children are able to multiply and divide by 10, 100, 1000 by moving digits using the unitisation argument (this will be critical at secondary school later as they apply this logic to algebra, surds and much more!). • It is key that children have efficient methods for division to enable them to divide the numerator by the denominator so revisit this if necessary beforehand.

Key Assessment Checklist
1. I can find common factors of two numbers and use these to simplify fractions
2. I can find common multiples of two numbers and use these to find common denominators
3. I can compare and order fractions by finding a common denominator
4. I can compare and order improper fractions (and mixed numbers) by finding common denominators and converting
5. I can find decimals that are equivalent to fractions by dividing the numerator by the denominator (e.g. 3/8 = 3 divided by 8 = 0.375)
6. I can recall and use equivalent fractions, decimals and percentages to solve problems
7. I can calculate percentages of amounts by finding 10% and then adjusting (e.g. halving that to find 5%). I can use this to solve problems.
8. I can solve percentage comparison problems

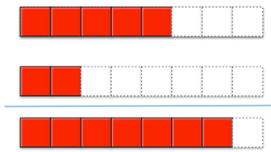
Year 6	Unit 8 : Reasoning with Fractions
8 learning hours	This unit progresses from the development of the understanding of non-whole items at the lowest end to flexibility and fluency with calculations involving fractions for older primary students. This knowledge is then applied within the secondary curriculum to the topic of probability, thus providing a clear context in which the skills of adding and multiplying fractions particularly are needed. It is critical that pupils develop confidence and security in understanding and manipulating fractions as well as flexibility in representing a number as a fraction or as a decimal, percentage, diagram etc. Note that once fraction calculations are mastered here, they should be used in following units as examples just as other numbers are in order to keep the skills fresh.

Prior Learning	Core Learning	Learning Leads to...
➢ add and subtract fractions with the same denominator and denominators that are multiples of the same number ➢ multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams	➢ **add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions** ➢ **multiply simple pairs of proper fractions, writing the answer in its simplest form [for example, 1/4 × 1/2 = 1/8]** ➢ **divide proper fractions by whole numbers [for example, 1/3 ÷ 2 = 1/6]**	➢ express one quantity as a fraction of another, where the fraction is less than 1 or greater than 1 ➢ record describe and analyse the frequency of outcomes of probability experiments using tables and frequency trees

Exemplification	Vocabulary	
1. Calculate a) $\frac{7}{9}-\frac{2}{5}$ b) $1\frac{3}{4}+\frac{5}{7}$ 2. Calculate, writing your answer in its simplest form a) $\frac{7}{10}\times\frac{1}{3}$ b) $\frac{8}{15}\times\frac{9}{10}$ 3. Calculate a) $\frac{8}{11}\div 4$ b) $\frac{2}{5}\div 3$	fraction proper fraction improper fraction mixed number equivalent reduced to cancel numerator denominator simplify lowest terms	common factors proportion sum difference product quotient dividend divisor

Representation	Fluency
Adding and Subtracting Fractions • Using the bar model to add and subtract fractions with the same denominator	1. Recap: convert between mixed numbers and proper fractions • improper fraction to mixed number (<2) e.g. 8/5 • improper fraction to mixed number (exact integer) e.g. 24/6 • improper fraction to mixed number (>2) e.g. 11/4 • mixed number to improper fraction (<2) e.g. $1\frac{2}{3}$ • mixed number to improper fraction (exact integer) e.g. 3 (to fifths) • mixed number to improper fraction (>2) e.g. $2\frac{3}{7}$

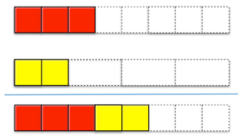
$$\frac{5}{8}+\frac{2}{8}$$



or $7/8$ in total

- Using the bar model to add and subtract fractions with one denominator that is a multiple of the other (by breaking the fraction with the smaller denominator up into the larger denominator-sized pieces)

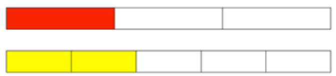
$$\frac{3}{8}+\frac{1}{4}$$



or $\frac{5}{8}$ in total

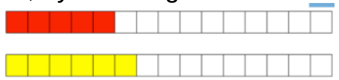
- Using the bar model to add and subtract fractions with different denominators

$$\frac{1}{3}+\frac{2}{5}$$



In total:

Or, by breaking into fifteenths:

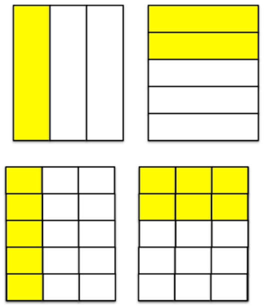


2. Recap: add fractions with the same denominator and denominators that are multiples of the same number
   - add two fractions with same denominator (answer a proper fraction)
   - add two fractions with same denominator (answer an improper fraction)
   - add two fractions with denominators that are multiples of each other (answer a proper fraction)
   - add two fractions with denominators that are multiples of each other (answer an improper fraction)
   - add two fractions with denominators that are multiples of each other (answer requiring simplification)
3. Add two proper fractions with different denominators
   - add two unit fractions
   - add two fractions with different denominators (answer a proper fraction)
   - add two fractions with different denominators (answer an improper fraction)
   - add two fractions with different denominators (answer requiring simplification)
4. subtract two proper fractions
   - subtract two fractions with the same denominator
   - subtract two fractions with denominators that are multiples of each other
   - subtract two fractions with different denominators
5. Add and subtract a mixed number and a fraction
   - add, not crossing the next integer
   - add, then crossing the next integer
   - subtract, not crossing next integer
   - subtract, crossing next integer but same denominators
   - subtract, crossing next integer but different denominators
6. Add and subtract two mixed numbers
   - add, not crossing next integer
   - add, crossing next integer
   - subtract, not crossing next integer
   - subtract, crossing next integer
7. Multiply two proper fractions
   - two unit fractions e.g. $\frac{1}{3}\times\frac{1}{7}$
   - one unit fraction, one proper fraction e.g. $\frac{1}{5}\times\frac{2}{3}$
   - one unit fraction, one proper fraction (requiring simplification or cancellation) e.g. $\frac{1}{3}\times\frac{9}{11}$
   - two non-unit fractions e.g. $\frac{2}{3}\times\frac{5}{7}$

which is $\frac{11}{15}$ altogether

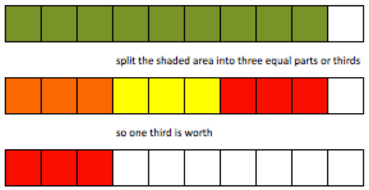
- Using an array model to add and subtract fractions (with different denominators). This can be achieved visually or by folding a rectangle of paper horizontally and vertically to be more concrete

  $\frac{1}{3}+\frac{2}{5}$

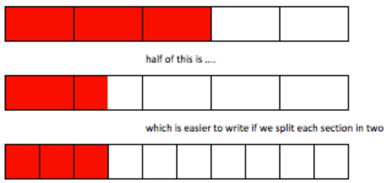


Multiplying Fractions

- Using the bar model to produce repeated addition of a fraction or a fraction of another fraction
  For example, $\frac{9}{10}\times\frac{1}{3}$ is the same as asking what is one third of nine tenths



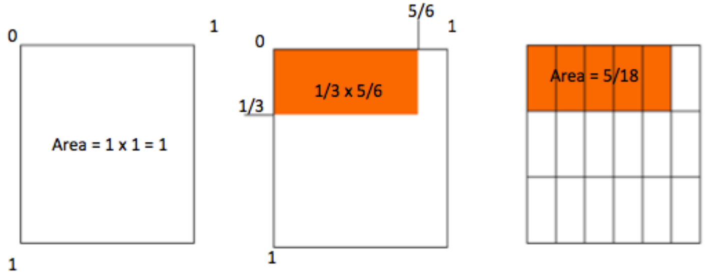
  Alternatively, $\frac{3}{5}\times\frac{1}{2}$ the same as asking what is one half of three fifths



- Using the area model to compare fractions to a whole

- two non-unit fractions (requiring simplification or cancellation) e.g. $\frac{2}{3}\times\frac{6}{7}$

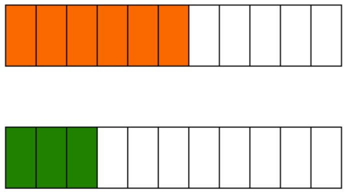
8. Divide a proper fraction by a whole number
- unit fraction divided by 2 e.g. $\frac{1}{5}\div 2$
- unit fraction divided by another single-digit number e.g. $\frac{1}{3}\div 5$
- unit fraction divided by a whole number e.g. $\frac{1}{5}\div 11$
- non-unit fraction divided by 2, no simplification needed because numerator a multiple of 2 e.g. $\frac{6}{11}\div 2$
- non-unit fraction divided by 2, simplification needed e.g. $\frac{3}{5}\div 2$
- non-unit fraction divided by a single-digit number, no simplification needed because numerator a multiple of divisor e.g. e.g. $\frac{8}{11}\div 4$
- non-unit fraction divided by a single-digit number, simplification needed e.g. e.g. $\frac{8}{9}\div 3$
- non-unit fraction divided by a whole number, with or without simplification e.g. e.g. $\frac{3}{10}\div 12$



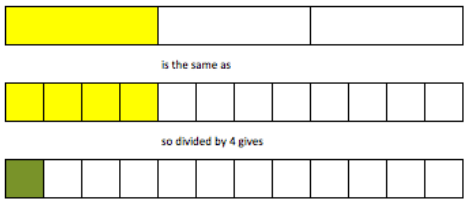
Dividing Fractions

- Using the bar model to divide a fraction by a whole number by sharing and revalue the result compared to the whole

  For example, $\frac{6}{11} \div 2 = \frac{3}{11}$



  Alternatively, $\frac{1}{3} \div 4 = \frac{1}{12}$



Probing Questions			
Show me...	**Convince me...**	**What's the same? What's different?**	**Always, sometimes, never**
…. how you can represent 11/4-1/2 using equivalent fractions ... two fractions with a sum of 7/8 ... two fractions with a difference of 1/5 … 3/4 x 3/4 in its simplest form ... two fractions with a product of 12/21 ... the fraction that gives 1/12 when divided by 3	….that 1 1/4 - 1/2 = ¾ ….that you can add fractions with different denominators ... that when you calculate ¼ x ¼, you get a smaller answer	2 1/3, 1 1/2, 2/3, 3 2/3 2 7/8, 6 2/3, 10 3/4, 33 1/3 Double one third of 15 One third of 30 2×5 $15 \times 2 \div 3$ $15 \div 3 \times 2$ $15 \times \frac{2}{3}$	Multiplying two fractions always results in a smaller number When I divide a fraction by a whole number the fraction always gets smaller. Dividing a whole number by a fraction results in a fraction To find a fraction of an amount, multiply the amount by the numerator of the fraction and then divide by the denominator. The sum of two fractions is greater than their product When you divide a fraction by a whole number, the result (the quotient) is smaller than the starting number (the dividend)

Further Extension	Rich and Sophisticated Tasks
1. Altogether on Monday and Tuesday I ran $3\frac{1}{2}$ km. On neither day did I run a whole number of km. Suggest how far I ran on Monday and how far on Tuesday. On Wednesday I ran some km and my sister ran $1\frac{1}{6}$ km further than I did. Altogether we ran $4\frac{1}{2}$ km. How far did I run on Wednesday? 2.	Calculate with fractions **NRICH: Fractions Jigsaw** **NRICH: Peaches Today, Peaches Tomorrow** **NRICH: Andy's Marbles** Solve problems involving the relative sizes of two quantities where missing values can be found by using integer multiplication and division facts **NRICH: Fraction Fascination *** P**

Roland cuts a sandwich into two pieces. First, Roland gives one piece to Ayat and the other piece to Claire. Then Claire gives Ayat half of her piece. Now Ayat has $\frac{7}{8}$ of the original sandwich.

Did Roland cut the sandwich into two equal pieces? If not, how did he cut the sandwich?

3.

In each number sentence, replace the boxes with different whole numbers less than 20 so that the number sentence is true.

$\frac{1}{\square} \times \frac{3}{\square} = \frac{\square}{\square}$

$\frac{\square}{\square} \times \frac{\square}{\square} = \frac{8}{15}$

$\frac{2}{\square} \times \frac{5}{\square} < \frac{10}{\square}$

$\frac{\square}{\square} \div 3 = \frac{1}{\square}$

$\frac{\square}{\square} \div 3 > \frac{1}{4}$

Misconceptions

When adding or subtracting fractions children may add (or subtract) both the numerators and denominators. This is because they do not recognise that the denominator indicates the number of parts of the whole and so treat the fractions as 4 'whole numbers' to be added together.

When working with mixed numbers, children may only deal with the fractional part, for example obtaining $3\frac{1}{3} - 2\frac{1}{2} = 1\frac{-1}{6}$

Errors in simplification may still persist, for example, cancelling by removing matching digits or thinking additively rather than multiplicatively

When multiplying, some children may apply the process of finding common denominators unnecessarily.
They may also not notice when they can cancel prior to multiplication and hence end up with large numbers as the numerator and denominator that require significant simplification.

When dividing a fraction by an integer, children may create a fraction with a decimal numerator.

Teacher Guidance and Notes

- This unit applies the concepts of fractions and their equivalents to calculation
- The pitch of this unit includes the addition and subtraction of all fractions and mixed numbers. However, multiplication is only expected of simple proper fractions and division of proper fractions is only by an integer
- Children have worked with addition and subtraction of proper fractions previously and hence should be building on prior knowledge here
- It is advised that you recommend to children that they convert mixed numbers to improper fractions when calculating with them to avoid errors
- It is important that once the concepts and processes have been established, children have the opportunity to apply these calculations to real life contexts, particularly linking to measures. It is advised that these skills be maintained in calculations in forthcoming units.

Alternatively, they may divide both numerator and denominator by the divisor incorrectly. Children may not use their mental skills when appropriate e.g. for dividing 16/19 by 2 (to get 8/19)	

Key Assessment Checklist

1. I can say which is the bigger of two fractions with different denominators and explain why using representations
2. I can add and subtract fractions with different denominators using the concept of equivalent fractions.
3. I can add and subtract mixed numbers and fractions with different denominators using the concept of equivalent fractions.
4. I can solve problems involving adding different size fractions by using visual representations of them.
5. I can multiply simple pairs of proper fractions, then write the answer in it simplest form.
6. I can show my understanding of the effect of multiplying pairs of fractions.
7. I can divide proper fractions by whole numbers and describe how I do this.
8. I can show my understanding of the effect of dividing a whole number by a fraction.

Year 6	Unit 9 : Solving Number Problems	
6 learning hours	This unit continues pupils' earlier study of arithmetic (and algebra for secondary students). At Key Stage 1 children are working on multiplication (and division in Stage 2) as a way to represented repeated addition and scaling (and repeated subtraction – grouping - and sharing) At Key Stage 2 children are developing skills in applying their arithmetic to more complex problems. At secondary level and in Stage 6, students begin to find unknown values by applying inverse operations. Equations of all types including quadratic and simultaneous are covered in later stages.	
Prior Learning	**Core Learning**	**Learning Leads to...**
➢ multiply and divide whole numbers and those involving decimals by 10, 100 and 1000 ➢ multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers ➢ divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context ➢ solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes ➢ solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign ➢ solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates ➢ solve problems involving number up to three decimal places	➢ **multiply and divide numbers by 10, 100 and 1000 giving answers up to three decimal places** ➢ **perform mental calculations, including with mixed operations and large numbers** ➢ **multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication** ➢ **divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context** ➢ **multiply one-digit numbers with up to two decimal places by whole numbers** ➢ **use written division methods in cases where the answer has up to two decimal places** ➢ **solve problems involving addition, subtraction, multiplication and division** ➢ **use their knowledge of the order of operations to carry out calculations involving the four operations** ➢ **solve problems which require answers to be rounded to specified degrees of accuracy** ➢ **express missing number problems algebraically** ➢ **find pairs of numbers that satisfy an equation with two unknowns** ➢ **enumerate possibilities of combinations of two variables**	➢ solve linear equations in one unknown algebraically ➢ use the symbols =, ≠, <, >, ≤, ≥

Exemplification	Vocabulary
1. A chicken farmer has 387 eggs that his hens laid this morning. She packs the eggs in dozens. How many egg boxes will be needed? 2. Tom's height is x cm. Tim is 7 cm taller than Tom. a) Write an expression for Tim's height Tom's father is three times as tall as Tom. b) Write an expression for Tom's father's height. 3. x and y are whole numbers. $x + y = 17$ Find a pair of numbers, x and y, that satisfy the equation. 4. x and y are variables $x - y = 11$ a) Find three possible pairs of values that satisfy the equation. b) Josh says that $x = 8$ and $y = 3$ is a possible solution. Do you agree with Josh? Explain your answer.	multiply divide place value decimal places rounding remainder problem solve context missing number order of operations value equation expression algebra variable unknown solution satisfy pairs of values

Representation	Fluency
Solve Problems involving Rounding and Accuracy • Represent word problems using the bar model to help children decide the correct calculation. • Representing a problem with objects e.g. egg boxes and eggs to help children understand the context **Algebra** • Using a empty box/question mark/shape to represent an unknown number (different shapes if more than one unknown). • Table of unknown with totals – problem – find • Building an expression or equation with algebra tiles and then turning them over to write on the reverse to substitute values in. • Matchbox algebra - find	1. Recap: multiply and divide a whole number by a 1 or 2 digit number ○ whole number multiplied by 1-digit number ○ whole number multiplied by 2-digit number ○ whole number divided by 1-digit number ○ whole number divided by 2-digit number ○ find missing numbers in formal calculations 2. Recap: round a number to a given degree of accuracy ○ Nearest whole number ○ 1 decimal place ○ 2 decimal places ○ Nearest 10, 100, 1000, … 3. Solve problems involving 4 operations and some element of rounding ○ One-step problem where trigger word for operation is clear and degree of accuracy is specified ○ One-step problem where trigger word is subtle or missing for the operation but degree of accuracy is specified

	○ One-step problem where the degree of accuracy must be decided by the pupil ○ Two-step problem with specified degree of accuracy ○ Two-step problem where the degree of accuracy must be decided by the pupil ○ Reverse problem where solution has been found and pupil needs to identify the (set of) possible starting numbers that could have led to that solution 4. Write expressions to represent quantities using a given letter(s) ○ addition e.g. one more than x as $x + 1$ ○ general addition of two or more letters e.g. $x + y$ ○ subtraction from x e.g. two less than x as $x - 2$ ○ subtraction of x e.g. x less than 7 as $7 - x$ ○ general subtraction e.g. $x - y$ ○ multiplication of x e.g. 3 times as large as x as $3 \times x$ or $3x$ ○ division of x e.g. a quarter of the size of x as $x \div 4$ ○ combinations of the above e.g. $2x + 1$ 5. Write missing number problems algebraically and solve them informally: ○ solve a missing number problem e.g. $14 + \blacksquare = 72 \div 3$ ○ write and solve an addition statement e.g. when I add 3 to x I get 11 as $x + 3 = 11$ ○ write and solve a subtraction statement e.g. when I subtract x from 20 I get 14 as $20 - x = 14$ ○ write and solve a multiplication statement e.g. when I multiply x by 4 I get 18 as $4 \times x = 18$ ○ write and solve a division statement e.g. when I divide x by 3 I get 18 as $x \div 3 = 18$ 6. Find values that satisfy an equation e.g. $x + y = 10$ ○ suggest one pair of (whole number) values that make an equation true e.g. $x = 3$ and $y = 7$ ○ find all pairs of positive whole numbers that make a statement true, producing a systematic list or table ○ find negative numbers that satisfy an equation e.g. $x = -2$ and $y = 12$ ○ find decimals or fractions that satisfy an equation e.g. $x = 3.5$ and $y = 6.5$

Probing Questions

Show me...	Convince me...	What's the same? What's different?	Always, sometimes, never
... how you would solve this problem: Milly is saving £2.75 a week to buy a pair of jeans. The jeans cost £37. For how many weeks does she need to save? ... how you would solve this problem: In Sports 4 U, there are 18 large boxes each containing 136 footballs. How many footballs are there altogether?	… that $134 \div 7$ and $130 \div 7$ have the same answer to the nearest whole numbe... that there are an infinite number of solutions to $x + y = 12$. ... why 6×100 and 60×10 give the same answer ... what 0.6 would mean on a calculator display if the units were pounds, metres, hours, cars	$x + y = 10$ $x + y = 7$ $2x + 2y = 20$ $3 + x = 10$ $2x + y = 17$ $4x = 20$ $3x + 2y = 29$ $8x + 4y = 68$	There are an infinite number of possible values for: $x + y = 11$ $x + 5 = 20$ $2x + y = 10$ Algebra always uses x and y. x and y are whole numbers.

Further Extension

1. Sam's grandmother has an old recipe for cherry buns.

To make them, she weighs two eggs. Then she takes the same weight in flour, and in sugar and in butter. She mixes all this together and then she adds half the weight of the 2 eggs in chopped glace cherries. She has enough mixture to put 45 grams in each of 12 paper cake cases.
What was the weight of one egg?

2. Cassandra, David and Lachlan are brothers and sister.
They range in age between 1 year and 14 years.
Can you figure out their exact ages from the clues?

Lachlan's age and David's age added together is 10 years.
Cassandra's age and Lachlan's age added together is 15 years.
David's age and Cassandra's age added together is 19 years.

Rich and Sophisticated Tasks

NRICH: Plenty of Pens * P
NRICH: Two and Two *** P I
NRICH: Carrying Cards

1. I am going to buy some 16p paper and some 13p envelopes. I want to spend exactly 90p. Write this as a symbol sentence and find whole number values that satisfy your sentence. Now tell me how many pieces of paper I should buy.

I want to spend exactly £2.25. Write this as a symbol sentence and find whole number values that satisfy your sentence. Now tell me how many envelopes I should buy.

I want to spend exactly £3.61. Write this as a symbol number sentence. Can you convince yourself that you can’t find whole number values that satisfy your symbol sentence? Explain your reasoning.

<table>
<tr><td>3. I am going to buy some 8p stickers and some 15p chews.

I want to spend exactly 93p. Write this as a symbol sentence and find whole number values that satisfy your sentence. Now tell me how many of each I should buy.

I want to spend exactly £1·93. Write this as a symbol sentence and find whole number values that satisfy your sentence. Now tell me how many of each I could buy.</td><td></td></tr>
<tr><th>Misconceptions</th><th>Teacher Guidance and Notes</th></tr>
<tr><td>See stage 4 and 5 re issues with multiplying and dividing by powers of 10.

When rounding some children round from the end of the number in a 'chain reaction' rather than looking at the next digit to decide whether to keep the stem the same or round the last digit up.

When interpreting remainders, children sometimes see these as literal numbers and cannot say what they mean in the context of the problem e.g. money or time. They may struggle to give the remainder as a fraction.

Children find the order of operations rules non-intuitive sometimes because they are used to reading from left to right. They do not therefore always carry out multiplication and division before addition and subtraction when a calculation is presented e.g. 2 + 3 x 6 = 20 but many children will say 12 because they have added first.

With algebra, children often don't see it is EXACTLY the same as number but with some numbers unknown. They may get confused with how to read terms such as 3a as a multiplication. For example, if a = 3 they would say that 3a is 33 rather than 3 lots of 3.</td><td>See Stage 4 and 5 for guidance on teaching mult and div by powers of 10 conceptually to avoid incorrect use of the 'short cut'.
This unit is about solving problems so ensure that there is considerable exposure to applying the calculation methods here. (the greyed out parts above are only to show you the theory that this application depends upon in case you need to go back to it).
Try to use a range of contexts including measures, money, time and the full range of numbers.
When working with order of operations, you may need to explore what happens when people calculate in different orders to get children to see WHY we need a convention about the order. These rules apply to number and algebra settings so are worth spending time on - look out for children reading from left to right!
Spend time developing the concept of the variable number e.g. in x + y = 10, there are lots of numbers that x and y could be. However, once you pick one for x, you have essentially narrowed down y to just one possibility.
If this is hard for children, try using ? symbols or empty boxes first as missing number problems before moving to letters so that they see that this is just a more 'grown up' way of doing the same thing. Make sure they still calculate in the correct order e.g. 3 + 5 x ? = 48</td></tr>
</table>

Key Assessment Checklist
1. I can use efficient written methods to solve addition, subtraction, multiplication and division problems
2. I can round my answer to a problem as appropriate for the context.
3. I can express remainders as whole numbers, fractions and rounded numbers
4. I can express missing number problems using numbers and symbols.
5. I can find pairs of numbers that satisfy an equation
6. I can solve an algebraic problem by listing possible numbers that satisfy it.

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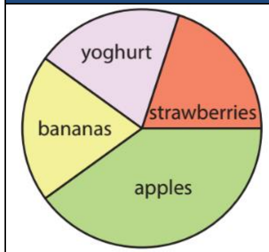
Year 6	Unit 10 : Investigating Statistics	
8 learning hours	In this unit children and students explore the collection, representation, analysis and interpretation of data. It covers a range of calculations of central tendency and spread as well as multiple charts and graphs to represent data. As it is the only unit directly exploring statistics, it is critical that children have time to explore the handling data cycle here and to focus sufficient time on interpreting their results.	
Prior Learning	**Core Learning**	**Learning Leads to...**
➢ solve comparison, sum and difference problems using information presented in a line graph complete, read and interpret information in tables, including timetables	➢ **interpret and construct pie charts and line graphs and use these to solve problems** ➢ **calculate and interpret the mean as an average**	➢ interpret and construct tables, charts and diagrams, including frequency tables, bar charts, pie charts and pictograms for categorical data, vertical line charts for ungrouped discrete numerical data and know their appropriate use ➢ interpret, analyse and compare the distributions of data sets from univariate empirical distributions through appropriate measures of central tendency (median, mean, mode and modal class) and spread (range)

Exemplification



1. The pie chart represents the proportions of the four ingredients in a smoothie drink.

The sector representing the amount of strawberries takes up 22% of the pie chart.
The sector representing the amount of apple is twice as big as the sector representing the amount of strawberries.
The sectors representing the amount of yoghurt and the amount of banana are identical.

Calculate the percentage of bananas needed to make a smoothie drink.

Vocabulary

pie chart	line graph
proportion	axes
distribution	scale
scaled up/scaled down	prediction
compare	trend
	mean
	average

What percentage of bananas would be needed to make two smoothie drinks?
Explain your reasoning.

2. Ten pupils take part in some races on Sports Day, and the following times are recorded.

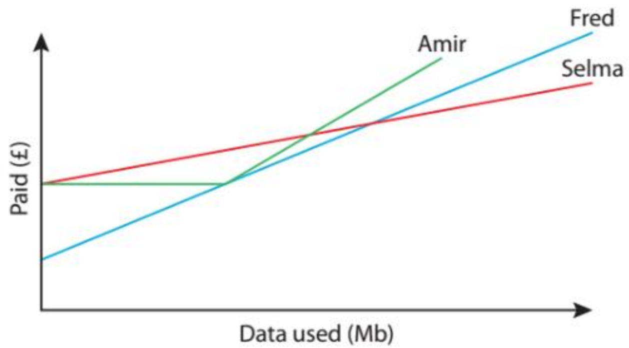
Time to run 100 m (seconds): 23, 21, 21, 20, 21, 22, 24, 23, 22, 20.
Time to run 100 m holding an egg and spoon (seconds): 45, 47, 49, 43, 44, 46, 78, 46, 44, 48.
Time to run 100 m in a three-legged race (seconds): 50, 83, 79, 48, 53, 52, 85, 81, 49, 84.

Calculate the mean average of the times recorded in each race.
For each race, do you think that the mean average of the times would give a useful summary of the ten individual times?
Explain your decision.

3. Three mobile phone companies each have different monthly pay-as-you-go contracts.

Phil's Phones: £5 fee every month and 2p for each Mb of data you use.
Manish's Mobiles: £7 fee every month and 1p for each Mb of data you use.
Harry's Handsets: £7 fee every month and 200Mb of free data, then 3p for each Mb of data after that.

Amir, Selma and Fred have mobile phones and they have recorded for one month how much data they have used (in Mb) and how much they have paid (in £). They have represented their data on this graph.



With which company do you think Amir has his contract?
With which company do you think Selma has her contract?
With which company do you think Fred has his contract?
Explain each of your choices.

<table>
<tr><th>Representation</th><th>Fluency</th></tr>
<tr><td>
Pie Charts

Start students off with simple data that can be easily split into different fractions
Get students to build their pie charts initially by shading in circles split into 12 equal parts (lots of factors)
When moving on to converting to degrees, some students might find it easy to divide frequency by total, then multiply by 360
Other students might find it easier to scale up or down proportionally to 360 (e.g. if there is a total of 60, multiply the frequencies by 6 to find angles)
This method will help students identify that they are working out how many degrees each individual represents
Students might find it easier using templates with pre-drawn centre lines
Spend some time before interpreting pie charts getting students to identify how many degrees each individual frequency would represent by dividing 360 by the total. Guide students by asking them questions like ‘Given that one person is represented by 6^o, how many people are represented by a sector of 30^o?’

Line Graph

Begin by getting students to interpret simple line graphs using questions such as ‘At what point are the values the same?’ or ‘Which one is increasing faster?’
Before interpreting phone contracts etc. get students to create line graphs from data tables
Build the concept of creating line graphs directly from key information by getting students to create data tables with small increments and see that it is a long process

Guide students to see that you can interpret key information and use that to draw your line graph
Look for constant values up to a certain point (these will be horizontal lines)
Encourage students to find gradients (phrase it as for every 10 minutes it costs £5 etc. and then use a ruler to extend lines)

Mean

Explore benefits/drawbacks of mean compared to median and mode
A good visual representation is to get a few students to the front of the class for a challenge where you know one student will greatly

</td><td>
1. Pie Charts

Interpret a simple pie chart split into easy fractions (½ and ¼)
Identify missing percentages/fractions of a pie chart
Use a pie chart with fractions to find amounts given a total
Use a pie chart with percentages to find amounts given a total
Find simple fractions to construct a pie chart with simple data
Convert simple fractions to angles out of 360^o in order to create a pie chart
Convert more complex fractions and percentages into angles out of 360^o in order to create a pie chart
Accurately use a protractor to create a pie chart
Measure an angle on a pie chart
Use the angle on a pie chart and given total to find a frequency

2. Line graph

Interpret simple line graph and explain in own words
Identify horizontal line means no change
Identify gradient is the rate of change
Identify where lines meet the values are equal
Identify constant values are where the line starts on y axis (in phone contracts etc.)

3. Mean

Explain mean is total/sum divided by count of data
Calculate mean for small set of integers with integer outcome
Calculate mean for small set of integers with non-integer outcome
Calculate mean for non-integer data set
Compare data using the mean
Find values with a given mean and range
(EXT): Find missing values given the mean of a data set

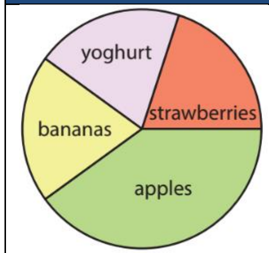
</td></tr>
</table>

outperform the others (e.g. juggling, keepy-ups, hula hoop spins)
  - Record the data and find median and mode if applicable
  - Discuss if it's fair that the good students result does not affect the class average
  - Draw out that mean includes all results including outliers
- Discuss with students what should be done with discrete integer data that has a non-integer mean - identify that mean is just a representation of the data
- Ask students to compare two sets of data just by looking at the numbers - lead them to use the mean to make sense and comparisons

Probing Questions

Show me...	Convince me...	What's the same? What's different?	Always, sometimes, never
... that if the mean height of a class is 150cm. What does this tell you about the tallest and shortest pupil? ... five numbers that have a mean of 6 and a range of 8.	...pie charts are easier to draw when the frequencies add up to a factor of 360 ... that pie charts can be misleading	bar chart, line graph, frequency diagram, pictogram, pie chart average, mean 1, 3, 3, 5 and 1, 2, 4 ,5 as data sets	... You can read the frequency from a pie chart ... You can read the proportion from a pie chart ... You can read the frequency from a bar graph .... If the section is the same size on two pie charts then the section represents the same frequency ... In order to interpret and compare two pie charts, you have to measure the angles on the pie charts. ... that if the mean height of a class is 150cm. What does this tell you about the tallest and shortest pupil? ... five numbers that have a mean of 6 and a range of 8.

Further Extension



Rich and Sophisticated Tasks

Pie charts and line graphs
NRICH: Match the Matches ** P

Mean as an average
NRICH: Birdwatch * INV
NRICH: Probably … * P
NRICH: Odds or Sixes? * GAME
NRICH: Same or Different? ** GAME

1. The pie chart represents the proportions of the four ingredients in a smoothie drink.

The sector representing the amount of strawberries takes up 22% of the pie chart.
The sector representing the amount of apple is twice as big as the sector representing the amount of strawberries.
The sectors representing the amount of yoghurt and the amount of banana are identical.

Estimate the angle of the sector representing the amount of banana.

Explain your reasoning.

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2. Three teams are taking part in the heats of a 4 × 100 m relay race competition on Sports Day. If the mean average time of the four runners in a team is less than 30 seconds, the team will be selected for the finals.

At the start of the last leg of the relay race, the times (in seconds) of each teams' first three runners are:

Team Peacock: 27, 29, 31

Team Farah: 45, 43, 37

Team Ennis: 29, 30, 25

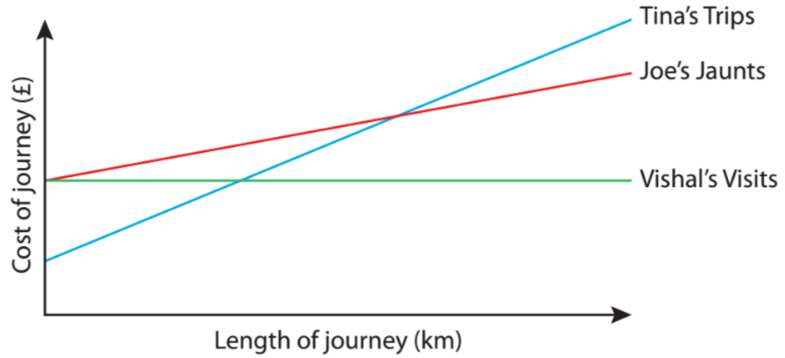
Which of the teams have the best chance of being selected?

Explain your reasoning.

---

3. Three taxi companies each work out the cost of a journey in different ways. I have taken lots of journeys with each of the companies, and have recorded each time how long the journey was (in km) and the cost of the journey (in £). I have represented these data on this graph.

NRICH: Tricky Track ** GAME
NRICH: Winning the Lottery ** P



What's the same and what's different about the ways in which the three companies work out the cost of a journey?

Which might you choose if you wanted to book a taxi to make a journey?

Explain your reasoning.

Misconceptions

Children sometimes struggle to use the fractional, proportional reasoning necessary for both constructing and interpreting pie charts. They need to link slices of the chart to fractions e.g. 1/6, 1/5.

Children do not realise that you cannot tell the size of the data set from a pie chart, only the distribution. Therefore, they believe that two pie charts are directly comparable, even when one represents many more pieces of data than the other.

Children sometimes select a scale for a line graph that compresses the graph so much that no discernible trend can be seen or one that does not fit all the points on.

Children make calculation errors with the mean and do not check to see if their answer is sensible and lies within the range of the data.

Teacher Guidance and Notes

- To teach pie charts, begin with examples using simple fractions. e.g. quarters, thirds, fifths, sixths, etc.
- Then move to a situation where you scale up (or down) your frequencies so that the total is 360 (degrees). E.g. if the frequencies sum to 72, you need to scale up by a factor of 5 to reach a total of 360 degrees. Focusing on the proportional reasoning approach here will be more memorable and connected to other mathematics than trying to learn a formula.
- Students struggle to construct angles correctly when using a protractor, often reading from the wrong side of the protractor (ie not starting from 0) or failing to align it correctly with the centre of the circle. Having pre-printed circles with their centres marked can speed this process up considerably.
- Similarly, when analysing pie charts, make sure children can measure angles correctly (encouraging them to re-orient the paper if necessary) before calculating what fraction or proportion of the whole they have and applying that to the total frequency.
- When constructing line graphs, pay particular attention to the selection of the scale for the axes. Don't forget, the scale doesn't have to start at 0 - but if this is chosen then it should be marked clearly. It can be good to extend children by drawing two line graphs on the same axes and

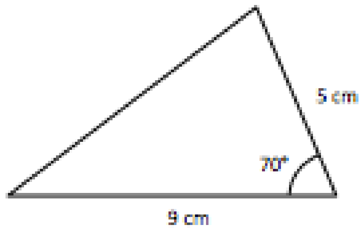
	then comparing them and their trends. • The mean is the last of the famous three averages to be met and is often thought of as the hardest by children - sell the attributes of the mean to children as factoring in ALL of the pieces of the data. It is worth investigating what happens if you have a large outlier - ie what is the impact on the mean (and compare this to the mode or the median). Also make time to go backwards and to find, for example, three numbers with a mean of 5 and a range of 3. • Make sure children understand what having a non-integer answer means in situations where the data is discrete. Encourage them to check whether their answer is sensible i.e. falls within the range of the data.

Key Assessment Checklist

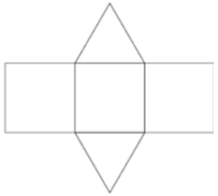
1. I can construct a simple pie chart for a data set using simple proportions of a whole.
2. I can construct a standard pie chart for any data set by scaling up to 360 degrees.
3. I can explain what a pie chart tells me about the distribution of data and compare two pie charts in this way; I understand that the pie charts may represent very different amounts of data.
4. I can construct a line graph for a data set, including drawing and selecting appropriate axes and scales.
5. I can analyse a line graph to describe the overall trend of the data and to make predictions as appropriate.
6. I can calculate the mean of a set of a data; I can say how this is different to finding the mode or the median.
7. I can give a set of data with a given mean
8. I can explain what the mean tells me in the context of the original data set and compare two data sets using means.

Year 6	Unit 11 : Visualising Shape
8 learning hours	In this unit children focus on exploring shapes practically and visually. There is an emphasis on sketching, constructing and modelling to gain a deeper understanding of the properties of shapes. It is therefore necessary to secure the practical skills at the same time as using them to explore the shapes in questions. At secondary level students are developing their skills in construction and the language/notation of shape up to the understanding, use and proof of circle theorems.

Prior Learning	Core Learning	Learning Leads to...
➢ draw given angles, and measure them in degrees (o) ➢ identify 3-D shapes, including cubes and other cuboids, from 2-D representations	➢ **draw 2D shapes using given dimensions and angles** ➢ **recognise, describe and build simple 3-D shapes, including making nets**	➢ draw diagrams from written description ➢ measure line segments and angles in geometric figures ➢ use conventional terms and notations: points, lines, vertices, edges, planes, parallel lines, perpendicular lines, right angles, polygons, regular polygons and polygons with reflection and/or rotation symmetries ➢ use the standard conventions for labelling and referring to the sides and angles of triangles ➢ identify and apply circle definitions and properties, including: centre, radius, chord, diameter, circumference

Exemplification	Vocabulary	
1. Here is a sketch of a triangle.  Draw an accurate full-size diagram of the triangle	angles degrees measure construct draw accurately sketch visualise net 2-D 3-D	hexagon octagon cube cuboid prism pyramid square based corners sides faces edges

2.
a) This is a net of a 3D shape.



What is the shape?

b) Draw an accurate net for a cuboid of length 5cm, width 3cm and height 2cm.
You do not need to include tabs.

protractor
angle measurer
regular
describe
properties
lines
net
circle
square
triangle
rectangle
rectangular
pentagon

vertices
base
draw
straight
shapes
length
width
height
depth

Representation

Drawing 2D Shapes

- Exploring using rulers and protractors to produce accurate drawings
- Investigating whether there are multiple shapes with the same criteria (e.g. a triangle with a side of length 7cm, one of length 5cm and an angle of $40°$)
- Ext: using compasses to draw triangles where all three sides are given (and no angles)

Nets of 3D shapes

- Using Polydron or equivalent to produce a 3d shape by connecting the different faces (and then unfolding- disconnecting to flatten out)

- Exploring packaging and unfolding it to produce nets (e.g. cereal boxes, Toblerone boxes, ...)
- Making nets of shapes from templates (link to example of printable templates)
- Making own nets of shapes without a template and testing them by cutting them out to produce the shapes

Fluency

1. Use a protractor to draw angles accurately
   - acute
   - obtuse
   - reflex
   - measure angles accurately to the nearest degree
2. Use a ruler to draw lines to the nearest millimetre
   - nearest centimetre
   - nearest millimetre
   - measure line accurately to the nearest millimetre
3. Construct rectangles accurately
   - using squared paper as a guide, to nearest centimetre
   - using squared paper as a guide, to nearest millimetre
   - on plain paper, using set square or protractor to ensure a right angle
4. Construct triangles accurately
   - all angles and lengths given
   - some angles and lengths given (unique solution)
   - recognise that there are lots of triangles with the same angles so a length is needed
   - ext: triangle where all three sides are given using compasses
5. Construct other quadrilaterals accurately
   - parallelograms and rhombuses (using equal opposite sides/angles property)
   - kite (using equal adjacent sides and one pair of equal opposite angles property)
   - trapezium

	• examples where an angle needs to be deduced first using 360° 6. Recap: name and describe the properties of 3D shapes • cubes and cuboids • prisms (with different cross-sections) • pyramids (with different bases) • spheres, cones and cylinders 7. Recognise a 3D shape from its net • cubes and cuboids • prisms (with different cross-sections) • pyramids (with different bases) • cones and cylinders • sphere – recognise the difficulty of producing such a net • say if a net works or does not work and explain why • use a net to visualise which edges will meet when folded 8. Sketch the net of a 3D shape • cubes and cuboids • find all the nets for a cube • prisms (with different cross-sections) • pyramids (with different bases) • cones and cylinders • sphere – recognise the difficulty of producing such a net 9. Draw accurate nets for common 3D shapes • cubes • cuboids • triangular prisms • square based pyramid • tetrahedron

Probing Questions

Show me...	Convince me...	What's the same? What's different?	Always, sometimes, never
... a square with 6 cm sides ... a rectangle with 2 sides of 12 cm and 2 sides of 4.5 cm ... a regular hexagon	... that the corners of a pentagon cannot all be 90 degrees ... that a triangle cannot have 3 obtuse angles ... that cylinder has a circular face	face, edge, vertex, corner cone, square based pyramid, cylinder circle, oval, sphere, cylinder net; 3D shape	cubes have 6 square faces A cuboid can have exactly 5 square faces There are 11 different nets for a cube

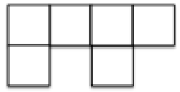
<table>
<tr>
<td>... the net of a cuboid

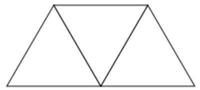
... a 2D shape with a straight side and a curved side.

... a 2D shape with 5 corners

.. a net of a cube ... and another

... a net that won't fold up to make a cube

... the net of a pentagonal prism</td>
<td>
... that this is not the net of a cube


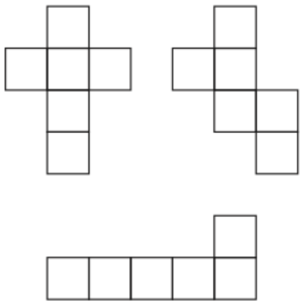
... that there are at least three ways to complete this net of a tetrahedron
</td>
<td>sketch; construct; draw accurately

net of a cube; net of a cuboid

net of a cube; net of a tetrahedron</td>
<td>Any four connected identical equilateral triangles will produce the net of a tetrahedron</td>
</tr>
<tr>
<th colspan="2">Further Extension</th>
<th colspan="2">Rich and Sophisticated Tasks</th>
</tr>
<tr>
<td colspan="2">1.
Accurately draw two right-angled triangles with sides of different lengths.

Compare them and describe what's the same and what's different about them.

2.
Which of these could be the net of a cube?

Explain your choices.


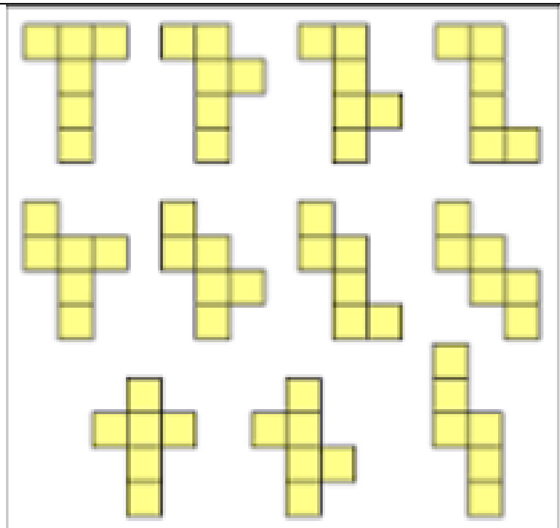
3.
Pascal says that any net made with six squares can be folded to make a cube.

Do you agree with him?

Explain your reasoning.</td>
<td colspan="2">Draw 2-D shapes using given dimensions and angles
NRICH: <u>Making Spirals</u> *** P
NRICH: <u>Shape Draw</u> * P
NRICH: <u>Baravelle</u> * P

Recognise, describe and build simple 3-D shapes, including making nets
NRICH: <u>Cut Nets</u> ** P
NRICH: <u>Making Cuboids</u> ** P I

Exploring/discovering the 11 nets of a cube</td>
</tr>
</table>



Misconceptions	Teacher Guidance and Notes
Children may struggle to differentiate between a sketch and a construction. When drawing 2D shapes to scale (given the side measurements and angles needed) they may find it hard to be sufficiently accurate (particularly with a protractor). When using a protractor some children may fail to identify 0 and thus measure the 'wrong way' round the protractor. For example, they measure an obtuse angle as 50° instead of 130°. They may also fail to centre the protractor correctly on 0 or allow it to move during the measurement. Similarly, children may not correctly align the 0 on a ruler with the start of a line. For some children there will be difficulty visualising where nets will fold up and construct the shape (especially when deciding if a net will make a cube or not). Similarly, they may struggle to link description to a visual image e.g. it has 5 faces and two of them are triangles may be hard to turn into an image. When given a net of a 3D shape some children think that the number of vertices of the 3D shape is found by counting the number of 'corners' on the net.	• Children have worked on the properties of both 2D and 3D shapes in the earlier unit, Exploring Shape. • The focus of this unit, therefore, is on the use of apparatus to construct accurately and on the visualisation skills linking nets to 3D shapes. Therefore, it is strongly advised that there is a high practical nature to this unit throughout. • For construction, give children the opportunity to use angle measurers as well as protractors - they are easier to use for most children. Having worked solutions as overlays is a good way for them to check their work and spot what may have gone wrong more easily. Check that children are secure drawing a single angle first before completing a whole shape. • It is important to enable children to deconstruct 3-D shapes to look at the nets they consist of. They may need to fold up repeatedly and view from the sides and above to really see the individual faces and how they connect.

Key Assessment Checklist

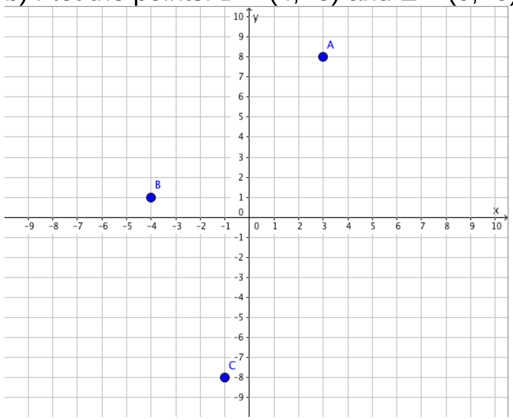
1. I can draw an angle or length accurately
2. I can draw a rectangle with given dimensions accurately
3. I can draw a triangle with given dimensions accurately
4. I can draw a range of 2D shapes when given the required lengths and angles.
5. I can identify the nets of 3D shapes
6. I can sketch 3D shapes from their nets
7. I can construct or complete the nets of simple 3D shapes (accurately where required)
8. I can describe the properties of 3D shapes (using models I have made)

Year 6	Unit 12: Exploring Change
4 learning hours	For primary pupils this unit focuses on the measures elements of time and co-ordinates. There is a progression from sequencing and ordering through telling the time formally to solving problems involving time. The co-ordinate work flows in the secondary students' learning focused on the relationships between co-ordinates. Key objectives include the use of y=mx+c for straight lines, the use of functions and the graphing of more complex functions.

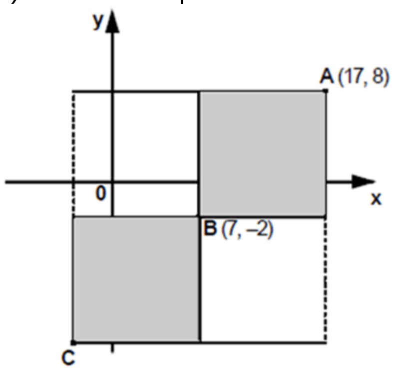
Prior Learning	Core Learning	Learning Leads to...
➢ describe positions on a 2-D grid as coordinates in the first quadrant	➢ **describe positions on the full coordinate grid (all four quadrants)**	➢ work with coordinates in all four quadrants

Exemplification

1.
a) Write the coordinates of points A, B and C
b) Plot the points: D = (4, -3) and E = (0, -9)



c) These two squares are the same size. What are the coordinates of the point C?



Vocabulary

coordinate
(x, y)
x coordinate; y coordinate
quadrant
negative
axis
x-axis
y-axis
first quadrant
2nd quadrant
3rd quadrant
4th quadrant
origin
horizontal
vertical
plot
construct
coordinate grid

Representation	Fluency
Coordinates • Playing coordinate bingo: Each player draws three lines on their axes: one horizontal, one vertical and one at an angle. Players take it in turns to choose a card at random from each set to generate a coordinate. For example, if the cards selected were a 2 and then a 3, the coordinate would be (2, 3). Players should mark off the coordinates on their grid as they occur. A player wins when they get 3 coordinates on any of their lines. • Using large grids on the playground with children as points. Children can show the journey from the origin each time to reinforce the process. • Drawing dot-to-dot pictures using coordinates – and then producing own sequences of coordinates for others to turn into a picture • Playing "Battleships" in pairs using a first-quadrant or a four-quadrant grid • Plotting three coordinates and then finding the fourth to produce a given type of quadrilateral • Coordinates ITP	1. Recap: plot and read coordinates in the first quadrant • recognise and name the x-axis and y-axis • know that a coordinate (x, y) is found by reading the first number across on the x-axis and the second number up or down on the y-axis • plot a coordinate in the first quadrant • plot a coordinate on one of the axes • plot the point (0,0); know that this is the 'origin' • write a coordinate using brackets and a comma 2. Plot and read coordinates in all four quadrants • plot/read a first quadrant coordinate • plot/read a third quadrant coordinate (double negative) • plot/read a second or fourth quadrant coordinate (single negative) • plot/read a coordinate on the axes • recognise and name the four quadrants 3. Construct a coordinate grid on squared paper (of given size) 4. Solve problems involving coordinates • find the final coordinate given the others and the shape they form • plot coordinates in sequence and connect them to form an image; continue the sequence of coordinates • find a coordinate a given number of squares to the left/right and above/below from another given coordinate (without the grid to count across)

Probing Questions

Show me...	Convince me...	What's the same? What's different?	Always, sometimes, never
... a coordinate in the 1st quadrant, in the 2nd, in the 3rd, in the 4th ... a coordinate that is horizontal to the coordinate (3, -2)... and another ... and another ... a coordinate that is vertical to (-2, -1) ... and another ... and another ... a co-ordinate that lies on this line	... that the distance from -2 to 2 is four ... that (-2, 2) is different to (2, -2) ... that (0,0) is a co-ordinate ... that (-4, 5) is in the second quadrant	(0,2), (2,0), (-2,0), (0,-2) the origin, the x-axis, the y-axis (4, -3) and (-3, 4)	... the largest number appears first when writing coordinate pairs in brackets ... if you reverse co-ordinates, you get a different point ... the origin isn't in a quadrant

... four co-ordinates that will form a square/rectangle etc.			

Further Extension	Rich and Sophisticated Tasks
1. A square has two vertices at (0,0) and (3,3). Work out and explain the coordinates where the other two vertices could be. A square has two vertices at (–3,0) and (3,0). Work out and explain the coordinates where the other two vertices could be. 2. An isosceles triangle has two vertices at (–3,2) and (3,2). Explore where the third vertex could be. 3. How many different coordinates can you form from the numbers 4 and -3? How do you know you have got them all? What shape will they form when you plot them? Is this always true? 4. These coordinates form a pattern when connected in order. (-2, 0), (-2, 1), (0, 1), (0, -2), (-4, -2), (-4, 3) (2, 3), (2, -4). Suggest the next three coordinates that should be used to continue the pattern. Hint: Plot the coordinates in order, joining them up as you go.	Describe positions on co-ordinate grid **NRICH: Cops and Robbers * GAME** **NRICH: Eight Hidden Squares ** P** **NRICH: Coordinate Tan ** P** **NRICH: Ten Hidden Squares *** P**

Misconceptions	Teacher Guidance and Notes
Children may confuse the axes and/or the quadrants. Some children may reverse coordinates within brackets. Issues with negative numbers may present themselves here - if children are insecure with negatives on a number line this can lead to issues when plotting negative co-ordinates. - grid reference. Some children are unsure of the origin and its significance - they may 'miss out' the origin and possibly the axes when plotting When constructing axes, children do not always ensure equal divisions between values (particularly between 0 and 1/-1).	• Children first encountered (first quadrant) coordinates in Stage 4 but this is the first time they have seen them used with negative numbers. • Therefore, it is recommended that you spend some time (perhaps in starters) reinforcing knowledge of negative numbers and particularly positioning on a number line. • The national curriculum objective focuses on plotting and reading coordinates; however, it is also expected that children use these skills to solve geometric problems.

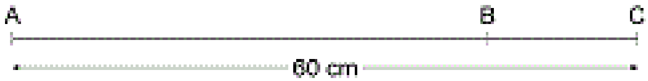
Key Assessment Checklist
1. I can plot points in all four quadrants. 2. I can find and record co-ordinates using brackets and a comma to demarcate them. 3. I can construct a coordinate grid 4. I can solve simple problems involving co-ordinates in all four quadrants.

Year 6	Unit 13: Proportional Reasoning
8 learning hours	In this unit pupils explore proportional relationships, from the operations of multiplication and division on to the concepts of ratio, similarity, direct and inverse proportion. For primary pupils in Stages 1-3, this is focused on developing skills of division. Stages 4 and 5 revisit the whole of calculation to broaden to all four operations in a range of contexts and combination problems; the emphasis here is really on representing and then solving a problem using their calculation skills, not just calculating alone. In Stage 6 the real underpinning concepts of proportion and ratio develop. Secondary pupils begin to formalise their thinking about proportion by finding and applying scale factors, dividing quantities in a given ratio and fully investigating quantities in direct or inverse proportion, including graphically.

Prior Learning	Core Learning	Learning Leads to...
➢ solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes ➢ solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign ➢ solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates of change.	➢ **use all four operations to solve problems involving measure [for example, length, mass, volume, money] using decimal notation, including scaling** ➢ **solve problems involving the relative sizes of two quantities where missing values can be found by using integer multiplication and division facts** ➢ **solve problems involving similar shapes where the scale factor is known or can be found** ➢ **solve problems involving unequal sharing and grouping using knowledge of fractions and multiples**	➢ change freely between related standard units (e.g. time, length, area, mass, volume/capacity) in numerical contexts ➢ Express one quantity as a fraction of another, where the fraction is less than 1 or greater than 1. ➢ use ratio notation, including reduction to simplest form ➢ divide a given quantity into two parts in a given part:part or part:whole ratio

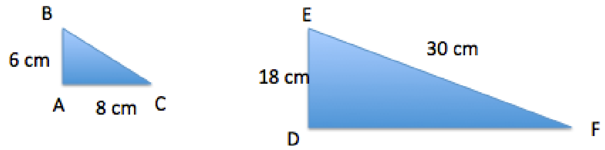
Exemplification	Vocabulary
1. a) Ben wants to cut 3 lengths of rope measuring 1.84m from a 5m rope. Does he have enough rope? Explain how you know. b) There are approximately 2.2 pounds in every kilogram. Ellie weighs 60kg. How much is this in pounds? 2. a) Amy and Beth are sharing some money. Amy receives four times as much as Beth. If Amy receives £72, how much money are they sharing? b) A recipe for shortbread has these ingredients: 300g plain flour, 250g butter, 175 g sugar. Tom makes shortbread using 900g flour. How much butter and sugar will he need? 3. a) 3 apples cost 45p. How much do five apples cost?	multiply divide product quotient scaling proportion scaled up/down scale factor similar ratio per … times as … multiplier, divisor times bigger/larger/longer

b) The distance from A to B is four times as far as the distance from B to C.
The distance from A to C is 60 cm.



Calculate the distance from A to B.

c) These two triangles are similar.



Calculate (i) the length of DE and (ii) the length of BC.

4.

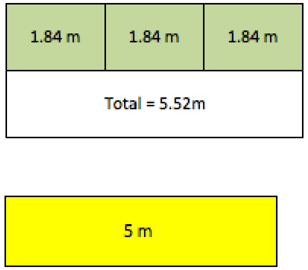
a) Calculate $\frac{5}{6}$ of 120g

b) What fraction is 700ml of 400ml?

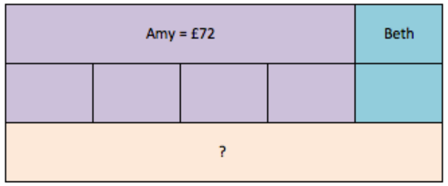
quantity
share
multiples
part(s)
whole
enlargement

Representation	Fluency
Scaling and Ratio • Exploring photographs and pictures that are enlargements of a real object to find the scale factor and corresponding lengths. For example, children could look at photos of themselves or famous landmarks. • Using counters or other objects to produce sets of objects with a given ratio or proportion. For example, a set of counters where the ratio of yellow to red is 2:3; or a set of counters where the proportion of yellow is ¼; or a set of counters where the proportion of blue is twice the proportion of red. • Exploring maps as a tool for finding scale factors and calculating distances in real life using the map • Mixing drinks e.g. squash and water (or paint!) in different ratios to produce different flavours (or colours). Children could look at making different shades of orange using red and yellow paint, for example. **Representing Problems** • Using the bar model to represent complex problems visually	1. Solve problems involving the four operations in measures contexts • Addition problems (e.g. adding decimal weights) • Subtraction problems (e.g. finding difference between two times or capacities) • Multiplication problems (e.g. finding total mass given mass of single object) • Division problems (e.g. finding amount per cup when bottle of drink shared between 8 cups). • Combinations of the operations above (e.g. leftover length given details of lengths so far) 2. Solve problems involving unequal division of a quantity (i.e. fractions) • Find a unit fraction of a quantity • Find a non-unit fraction of a quantity • Solve a word problem involving non-unit fractions of quantities e.g. Amy has 3m of ribbon. She gives 3/5 of it to Ben and 1/ 10 to Charlie. How much does Amy have left?

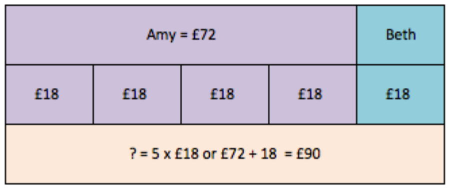
Example 1: *Ben wants to cut 3 lengths of rope measuring 1.84m from a 5m rope. Does he have enough rope? Explain how you know.*
can be represented as



Example 2: *Amy and Beth are sharing some money. Amy receives four times as much as Beth. If Amy receives £72, how much money are they sharing?*
This problem can be represented using this diagram:



which can then be completed to solve the problem:



Example 3: *The distance from A to B is four times as far as the distance from B to C. The distance from A to C is 60 cm.*

Calculate the distance from A to B.

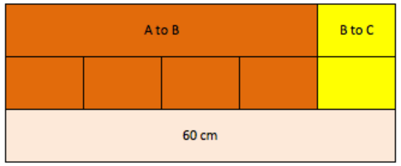
This problem can be represented as:

- Write a quantity as a (proper) fraction of another e.g. 300ml as a fraction of 400ml
- Write a quantity as an improper fraction of another e.g. 500g as a fraction of 400g.

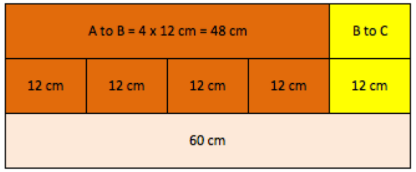
3. Solve simple scaling problems
   - Given a scale factor, use it to find a larger amount. E.g. Hannah has twice as many sweets as Georgia. If Georgia has 21 sweets, how many does Hannah have?
   - Given a scale factor, use it to find a smaller amount e.g. Tom is 3 times as old as Sam. If Tom is 27, how old is Sam?
   - Find the scale factor between two quantities in proportion e.g. Courtney is using a recipe for scones that needs 220g flour. Courtney is using 660g of flour. How much has Courtney scaled the recipe up by?
   - Apply the scale factor to solve problems. E.g. the same recipe needs 150g sugar. How much sugar will Courtney need?
   - Solve problems involving different units *A tennis court is 7m wide and 24m long. A scale plan of it is drawn with a width of 3.5cm. What is its length?*

4. Solve problems involving similar shapes
   - Know that similar shapes have sides in proportion
   - Given the scale factor, find a larger missing side (by multiplying)
   - Given the scale factor, find a smaller missing side (by dividing)
   - Enlarge a shape given a scale factor and the original
   - Find the scale factor of two similar shapes
   - Say whether two shapes are similar given all the sides
   - Know that scale drawings and the original shapes are similar
   - Ext: solve problems where the scale factor is not a whole number e.g. 2.5

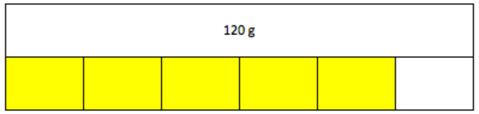
5. State a ratio or proportion
   - Given a set of objects, state the ratio of two sets of objects within it e.g. red to yellow sweets in a packet or boys to girls in a class.
   - State the proportion of a set of objects with a given property e.g. aces in a pack of cards or red sweets in a packet
   - Make a set of objects with a given ratio and/or proportion of specific features
   - Colour (divide) a simple shape in a given ratio e.g. rectangle, circle
   - Colour a given proportion of a simple shape



which can then be completed to solve the problem:



Example 4: *Calculate* $\frac{5}{6}$ *of 120g*



- Using scaling 2x2 grids to link corresponding measurements together For example: *A tennis court is 7m wide and 24m long. A scale plan of it is drawn with a width of 3.5cm. What is its length?*

	(widths)	(lengths)
(Real court)	7m	24m
(Model)	3.5 cm	?

Children can then either work horizontally to find the scale factor (not a good choice in this case as 7 is not a factor of 24) or work vertically (in this case a better choice as we just need to divide by 2 and change the units)

6. Solve problems involving the relative sizes of two quantities
    - Given the value of one part, find the value of 2, 3, 4 parts and so on
    - Given the value of one part, find the value of the whole
    - Given the value of 2, 3, 4, … parts, find the value of one part
    - Given the value of 2, 3, 4, … parts, find the value of the whole
    - Given the value of the whole, find the value of one part and use this to solve informal sharing problems e.g. *The distance from A to B is four times as far as the distance from B to C. The distance from A to C is 60 cm.*
    - Given the value of the whole, find the value of one part and use this to solve ratio problems *e.g. Jemma and Katie share £60 in the ratio 2:3. Find how much money they receive each.*
    - Solve other problems involving ratio e.g. write the ratio of sides of similar shapes

7. Solve complex problems involving all of the above

Probing Questions

Show me...	Convince me...	What's the same? What's different?	Always, sometimes, never
.... the operations you would carry out to solve this problem "Printing charges for a book are 3p per page and 75p for the cover. I paid £4.35 to get this book printed. How many pages are there in the book?" ... how you would solve this problem: Milly is saving £2.75 a week to buy a pair of jeans. The jeans cost £37. For how many weeks does she need to save? ... how you would solve this problem: In Sports 4 U, there are 18 larges boxes each containing 136 footballs. How many footballs are there altogether? ... how you could find 1/8 of this distance ... how you would represent this problem. Jenny has some books. Miles has 3 times as many as Jenny. Altogether there are 48 books. How many does Miles have?	... that enlarging the sides of a rectangle by a scale factor of 2 does not result in the area increasing by a scale factor of 2. ... that two shapes are similar ...that you get the same answer using a grid method as you do using a column method for multiplication ... what 0.6 would mean on a calculator display if the units were pounds, metres, hours, cars	Proportion, multiplying and adding Ratio, fractions and decimals	... 3-D shapes are similar ... scaling problems can be solved using division and not multiplication ... if one object is 3 times taller than another then the object is 6m taller.

Further Extension

1.

Make up a word puzzle that you could solve with this diagram:

Make up a word puzzle that you could solve with this diagram:

Rich and Sophisticated Tasks

Solve problems involving addition, subtraction, multiplication and division

NRICH: **Always, Sometimes or Never? Number** *** P**

Solve problems involving the relative sizes of two quantities where missing values can be found by using integer multiplication and division facts

NRICH: **Orange Drink** **** P**
NRICH: **Pumpkin Pie Problem** **** P**
NRICH: **Jumping** *** P**
NRICH: **Rectangle Tangle** *** P**
NRICH: **Fraction Fascination** ***** P**

2.
Mum is 28 years older than Anthony. Mum is 4 years younger than Dad. The total age of the three of them is 84 years.

What is Mum's age?

3.
I share equally a length of ribbon between 8 people, and each person gets 0·25m of ribbon.

Can you work out how long the original piece of ribbon was?

4.
Harry and Jim share some marbles in the ratio 3:5.

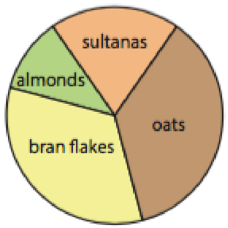
Jim gets 24 more marbles than Harry does.

How many marbles did they share in total between them?

5.
The pie chart shows the ingredients needed to make a breakfast cereal.

120 grams of almonds are used.

Estimate the quantity of each of the other ingredients.



Misconceptions	Teacher Guidance and Notes
Children frequently struggle to decide on the correct operations (and order of these) to solve a worded problem. They may therefore benefit from a visual representation of the problem first. Children often search for additive relationships between quantities rather than multiplicative. For example, they may see a recipe using 300g of flour being scaled up to 600g of flour and think that all the ingredients should be 300g more (rather than doubled).	• This unit covers the final elements of calculation in Stage 6, notably focusing on scaling, ratio and proportion and dealing with these ideas within a range of contexts. • Children have not yet encountered ratio and so this should be introduced carefully. At this stage, the focus is mostly on the informal proportional reasoning associated with scaling up and down as well as sharing in unequal amounts, although there is some direct reference to ratio notation in the KS2 tests so this should be included.

When finding fractions of amounts children may incorrectly divide by the numerator and multiply by the denominator. This is usually a sign that they are following an algorithm rather than thinking through the concept. Sometimes children forget that similar shapes have been enlarged in all dimensions/directions and so forget to apply the scale factor to every length. When solving problems of unequal sharing, some children do not divide by the correct number of parts; they struggle to decide what this is without a diagram.	• Clearly the proportion work and scaling will relate to earlier work on fractions and so the bar model should be revisited here to make these links clear.

Key Assessment Checklist

1. I can solve problems involving all four operations in measures contexts, including those with decimals.
2. I can find a fraction of an amount by relating it to division and multiplication
3. I can find one quantity as a fraction of another quantity, including where the result is an improper fraction
4. I can use a given scale factor to solve a simple scaling problem, knowing when to multiply and when to divide.
5. I can find a scale factor in a scaling problem.
6. I can read and use the vocabulary of ratio and proportion to represent a scaling problem correctly.
7. I can represent a problem involving unequal sharing or grouping and solve it.