Moor Nook CP School

Year 4

Medium Term Plans

Updated March 2022

Overview of Year

Autumn Term	Number and Algebra				Geometry and Measures	
	1. Investigating Number Systems	2. Pattern Sniffing	3. Solving Calculation Problems	4. Generalising Arithmetic	5. Exploring Shape	6. Reasoning with Measures

Spring Term	Number and Algebra			Statistics
	7. Discovering Equivalence	8. Reasoning with Fractions	9. Solving Number Problems	10. Investigating Statistics

Summer Term	Geometry	Number and Algebra		Geometry and Measures	
	11. Visualising Shape	12. Exploring Change	13. Proportional Reasoning	14. Describing Position	15. Measuring and Estimating

Year 4 Overview:		
Unit	**Learning Hours**	**Summary of Key Content**
1. Investigating Number Systems	11	Read Roman numerals to 100, recognise place vale up to 4 digits; 4NPV-2 (Standard & Non-Standard Partitioning); identify, represent and estimate numbers using different representations; round to the nearest 10, 100 and 1000; order and compare numbers beyond 1000 Not in the AET Curriculum: 4NPV-1 Scaling – 100 times the size Not in the AET Curriculum: 4NPV-3 (Locating on a number line) Not in the AET Curriculum: 4NPV-4 (Equal parts on a number line)
2. Pattern Sniffing	10	Count in multiples of 6, 7, 9, 25 and 1000; find 1000 more or less than a given number; recall multiplication tables up to 12x12 4NF-1; use factor pairs and commutativity in mental calculations. (Please refer to Moor Nook's Mental & Written Calculations Policies)
3. Solving Calculation Problems	8	Add and subtract up to 4d using formal methods where appropriate; use inverse operations to check a calculation; solve addition and subtraction 2-step problems (Please refer to Moor Nook's Mental & Written Calculations Policies)
4. Generalising Arithmetic	8	Multiply 2dx1d or 3dx1d using a formal written layout; multiply and divide mentally using place value, known facts etc to help 4NF-2/ 4NF-3; use inverse operations to check a calculation (Please refer to Moor Nook's Mental & Written Calculations Policies)
5. Exploring Shape	8	Identify lines of symmetry 4G-3; identify acute and obtuse angles; compare and order angles up to 180 degrees; compare and classify geometric shapes, including triangles 4G-2 (Equilateral triangles) and quadrilaterals.
6. Reasoning with Measures	8	Estimate, calculate and compare money in £ and p; Perimeter of rectilinear shapes 4G-2; area of rectilinear shapes by counting
7. Discovering Equivalence	10	Not in the AET Curriculum: 4F-1 (Locate mixed numbers on a number line) Recognise and show equivalent families of fractions;count in tenths; recognise tenths from dividing an object into 10 equal pieces and dividing a number by 10; Not in the AET Curriculum: 4F-2 (Convert mixed numbers to improper fractions and vice versa) recognise and write decimal equivalents of any number of tenths or hundredths; recognise and write decimal equivalents to ½, ¼, ¾; order and compare decimals to 2dp; round decimals to the nearest integer.
8. Reasoning with Fractions	8	Add and subtract fractions with same denominator; 4F-3 solve problems involving fractions to calculate quantities, including non-unit fractions
9. Solving Number Problems	12	Divide a (1 or 2d) number by 10 and 100; recap mental multiplication skills;4MD-1 recap formal multiplication; solve problems involving multiplying and adding4MD-2, using the distributive law4MD-3. Solve measures problems. (Please refer to Moor Nook's Mental & Written Calculations Policies)
10. Investigating Statistics	8	Interpret and present data appropriately including bar charts and time graphs Solve problems from bar charts, pictograms, tables etc
11. Visualising Shape	4	Complete a simple symmetric figure

12. Exploring Change	7	Read, write and convert time between 12 and 24 hour clocks (analogue and digital) Solve problems converting between units of time
13. Proportional Reasoning	4 - 8	Revision (and extension if appropriate) of multiplication and division concepts from Units 5 and 9 (Please refer to Moor Nook's Mental & Written Calculations Policies)
14. Describing Position	5	Describe positions on grid in first quadrant as coordinates; describe movements between positions as translations using up/down and left/right; plot specified points and complete to make a polygon 4G-1
15. Measuring and Estimating	6	Convert between different units of measure

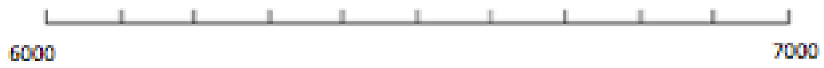
Year 4	Unit 1: Investigating Number Systems	
11 learning hours	This unit introduces the number systems and structures that we use at different levels of the curriculum. At KS1 children are working on the place value system of base 10 with the introduction of Roman Numerals as an example of an alternative system in KS2. Negative numbers and non-integers also come in at this stage and progress into KS3. At KS3 and KS4 we start to look at other ways of representing numbers, including standard form, inequality notation and so on.	
Prior Learning	**Core Learning**	**Learning Leads to....**
➢ tell the time ... using Roman Numerals from I to XII	➢ **read Roman numerals to 100 (I to C) and know that over time, the numeral system changed to include the concept of zero and place value**	➢ read Roman numerals to 1000 (M) and recognise years written in Roman numerals
➢ read and write numbers up to 1000 in numerals and in words ➢ recognise the place value of each digit in a three-digit number (hundreds, tens, ones)	➢ **recognise the place value of each digit in a four-digit number (thousands, hundreds, tens, and ones)**	➢ read, write, order and compare numbers to at least 1 000 000 and determine the value of each digit
➢ identify, represent and estimate numbers using different representations	➢ **identify, represent and estimate numbers using different representations**	
	➢ **solve number and practical problems that involve all of the above and with increasingly large positive numbers**	
➢ compare and order numbers up to 1000	➢ **order and compare numbers beyond 1000**	
	➢ **round any number to the nearest 10, 100 or 1000**	➢ round any number up to 1 000 000 to the nearest 10, 100, 1000, 10 000 and 100 000

Exemplification

1. a) Write these Arabic numerals as Roman numerals: i) 38 ii) 61
   b) Write these Roman numerals as Arabic numerals: i) XXIV ii) LV

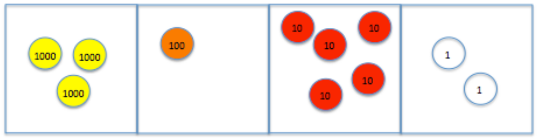
2 a) Write this number using numerals: Eleven thousand, six hundred and seventy
   b) Write this number in words: 3047
   c) What is the value of the digit 7 in the number i) 4875 ii) 7510

3. a) Represent 6130 on this number line



   b) Represent 1275 using base 10

   c) State which number is shown here:



4. Here are some number cards

7	2	4	3	6

a) Choose four cards to make the smallest possible 4 digit number
b) Choose four cards to make the largest possible 4 digit number

5. Write these numbers in order from smallest to largest:
   3057 5370 7350 3750 1375 7530 3705

6. Round 3518 to a) the nearest 10 b) the nearest 100 c) the nearest 1000

Vocabulary

Roman Numerals
numeral
1 I
2 II
3 III
4 IV
5 V
6 VI
7 VII
8 VIII
9 IX
10 X
50 L
100 C
500 D
1000 M
place value
thousand
ten thousand
represent
estimate
round
integer
compare
order

Representation

Roman Numerals (simple)

- Exploring where Roman Numerals are used in real life, for example in dates and times (clocks).
  Predict the meaning of different symbols based on existing knowledge about the clock, years etc. e.g. XII corresponds to 12 so what do you think the X and I could represent? How does IV correspond to 4 when V is 5?

Fluency

1. Convert between simple Roman Numerals and Arabic Numerals
   - convert roman numerals up to 12 to Arabic numerals and vice versa
   - convert single roman numerals up to 100 to Arabic numerals e.g. X and vice versa
   - convert roman numerals of two or more repeated

Probing Questions

Convince me that M > L

Always, Sometimes, Never?
You cannot show decimals using Roman Numerals

• Use (and make) equivalence cards for roman numeral symbols and either Arabic numerals or word versions or visual representations	symbols or stacked symbols to Arabic numerals and vice versa e.g. XXX or LXXV or LIII	
Roman Numerals (more complex) • Using pebbles (or counters) and drawing basic lines on a piece of A4 paper, ask children to work in pairs and challenge each other to show different numbers up to 100 or C on their Roman Calculi. Write each number as a standard numeral and then as a Roman numeral. • Explore this online activity (interactive) to discover the rules of Roman numerals	2. Convert between any Roman Numerals and Arabic Numerals up to 100 ○ convert roman numerals involving the use of an 'IV' to Arabic numerals and vice versa e.g. LIV ○ convert roman numerals involving the use of an 'IX' to Arabic numerals and vice versa e.g. LXIX ○ convert any roman numeral up to 100 to an Arabic numeral or vice versa e.g. XCVII	Convince me that 69 in Roman Numerals is LXVI Convince me which is the correct representation of 99 in roman numerals. IC IX IX XCIX LXXXXIX What's the same and what's different? VI, XVI, LVI, CVI
Representations of numbers • Represent 3 digit numbers (loose and in column format) using: ○ To-scale inseparable hundreds, tens and ones e.g. Base 10, Dienes blocks ○ Not-to-scale hundreds, tens and ones e.g. place value counters, money (£1, 10p and 1p coins), unmarked coloured counters ○ Overlapping place value cards ○ Numerals • Represent 4 digit numbers (loose and in column format) using: ○ To-scale inseparable thousands, hundreds, tens and	3. Convert a given representation to a number (verbal or numerals) ○ recap three digit numbers e.g. 453 ○ recap three digit numbers that are multiples of 10 or 100 e.g. 840/700 ○ recap three digit numbers that incorporate zeroes e.g. 402 or 780 ○ four digit numbers e.g. 4567 ○ four digit numbers that are multiples of 100 or 1000 e.g. 5600 or 6000 ○ four digit numbers that incorporate zeroes e.g. 4003 or 3607 ○ five digit numbers	Show me a number with a 3 in the hundreds column Convince me that there are exactly ten numbers between 2000 and 3000 with a tens digit of 4 and a ones digit of 9

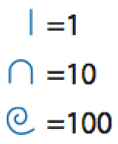
ones e.g. Base 10, Dienes blocks ○ Not-to-scale thousands, hundreds, tens and ones e.g. place value counters, money (£10 notes, £1, 10p and 1p coins), unmarked coloured counters ○ Overlapping place value cards ○ Numerals • Develop sense of size of numbers up to 10 000 using paper strips and paperclips to position e.g. strip represents 0-10 000, where is 2534? What if the strip now represents 0-5000?	4. Convert a given number to a stated concrete or visual representation ○ recap three digit numbers ○ four digit numbers e.g. 4567 ○ four digit numbers that are multiples of 100 or 1000 e.g. 5600 or 6000 ○ four digit numbers that incorporate zeroes e.g. 4003 or 3607 ○ five digit numbers	Show me how we can represent the number 3156 using - place value counters - a number line (marked or blank) Show me where 2351 would be on this scale (blank strip) from - 0-10000 - 2000 - 3000 - 2300 - 2500 - 2350 - 2360 Always, Sometimes, Never? If you take 4 digits, there are 24 different 4-digit numbers that you can create from them (development - 4 different digits or no such restriction)
Partitioning • Using base 10 or equivalent apparatus to split numbers into different combinations and read them aloud. For example 4267 could be partitioned as 4 thousands, 1 hundred, 16 tens and 7 ones.	5. Partition a number into thousands, hundreds, tens and ones and state the value of a given digit within a number ○ Recap three digit numbers ○ Four digit numbers ○ Reverse problem to find number from place value information ○ Partition in a non-standard way (i.e. not just Th, H, T, U) – find two or more ways of partitioning a number	What's the same and what's different? 1, 10, 100, 1000, 10000 True or False? There is one set of base 10 equipment to represent each number Convince me that forty-two hundred and thirty-fourteen is worth the same as 4244
Words and Numerals • Use (and make) word/numeral number cards to help convert between numerals and words	6. Convert a number written in words to numerals and vice versa ○ Recap three digits e.g. four hundred and thirty-seven ○ Recap three digits multiples of 100 e.g. four hundred ○ Recap three digits multiples of 10 e.g. four hundred and thirty ○ Recap three digits with no tens e.g. four hundred and seven	Show me the number two thousand and thirty-four in symbols Show me the number 6903 in words

	○ Four digits multiples of 1000 ○ Four digits, all non-zero ○ Four digits containing zeroes e.g. multiples of 100, 10, numbers with no 10s etc. ○ Five digits	
	7. Recognise matching numerals, words and representations ○ Matching pairs ○ Matching three or more items ○ Matching representations without the numerals present	Always, Sometimes, Never? Numbers that contain a digit of 9 will be greater than those that do not True or False? The representations of 4007 and 4070 are almost the same.
Comparing Numbers • Use apparatus and then visuals and then number cards (abstract) to explore which number is greater when comparing • Use number cards to explore making different four (or five) digit numbers and finding the smallest/largest	8. Compare two numbers to say which is greater, using > or < to notate ○ Recap: two two-digit numbers ○ One three-digit number, one two-digit number ○ Two three digit numbers (unrelated) ○ Two three-digit numbers (similar digits) ○ Mixture of representations/words/numerals	Convince me ... that 4671 < 4716 ... that 6180 > 6159 Show me a number that would make this statement true 8134 > ......
Ordering • Using a washing line to act as a number line and marking key numbers on before positioning a selection of numbers correctly • Suggesting numbers that could lie in between	9. Order numbers from smallest to largest ○ Order three numbers: ▪ Recap: (one and) two-digit numbers only ▪ Three-digit numbers (unrelated) ▪ Three-digit numbers (similar digits) ○ Order four or more numbers (as above) ○ Find a number that lies between two given numbers (2 digits, then 3 digits)	What's the same and what's different? 4562, 2654, 6452, 5246, 6254, 2456 Always, Sometimes, Never? There are 9 integers ? for which 3567 < ? < 3576

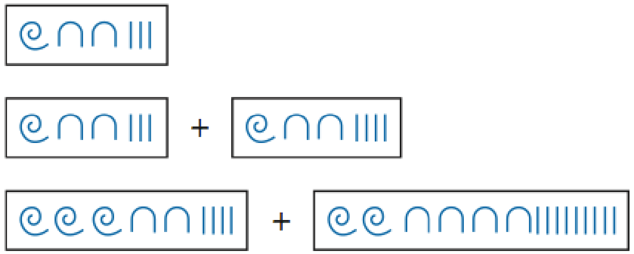
Rounding • Positioning number on marked (and then unmarked) number line to identify neighbouring rounding options • Using number line to investigate when a number is closer to the lower end than the upper end • If finding the lower option is challenging, then represent a number using partitioned equipment e.g. place value counters or place value cards. Then partition the number and keep the pieces required for rounding to generate the lower rounding option. For example, to round 3467 to the nearest 100 make as 3000 + 400 + 60 + 7 and reject the 60 and the 7 to leave 3000 + 400 = 3400. This is the lower option. Then make the higher option by adding one more 100 i.e. 3500.	10. Round a whole number to the nearest 10 ○ round a two digit number to the nearest 10 ○ round a three digit number with tens digit 1-8 to the nearest 10 e.g. 384 ○ round a three digit number with tens digit 0 or 9 to the nearest 10 e.g. 396 or 506 (i.e. where the answer could be a multiple of 100) ○ round a four digit number to the nearest 10 (answer not a multiple of 100 or 1000) ○ round a four digit number to the nearest 10 (answer a multiple of 100 or 1000)	What's the same and what's different? 327, 334, 325, 339
	11. Round a whole number to the nearest 100 or 1000 ○ 3 digits, nearest 100 ○ 4 digits, nearest 100 (answer not a multiple of 1000) ○ 4 digits, nearest 100 (answer a multiple of 1000) ○ 5 digits, nearest 100 ○ 4 digits, nearest 1000 ○ 5 digits, nearest 1000	Convince me that 253 and 329 round to the same number to the nearest 100

Further Extension	**Rich and Sophisticated Tasks**

1. 5000 years ago Egyptians carved number symbols on their tombs:



What is the value of these Egyptian numbers?



2. Match 4600 to numbers with the same value

4600

460 tens
460 hundreds
46 hundreds
4600 ones
46 tens

How many ways can you find to make 5060?

3.
Kiz has these numbers:

1330 1303 1033 1003 1030

He writes them in order from smallest to largest.
What is the fourth number he writes?

4. Find all the different numbers you can make from these digit cards: 1, 3, 4, 5

Recognise the place value of each digit in a four-digit number (thousands, hundreds, tens and ones)

NRICH: Some Games That May Be Nice or Nasty * G
NRICH: Dicey Operations * G
NRICH: The Deca Tree * P
NRICH: Four-digit Targets * P

Round any number to the nearest 10, 100 or 1000

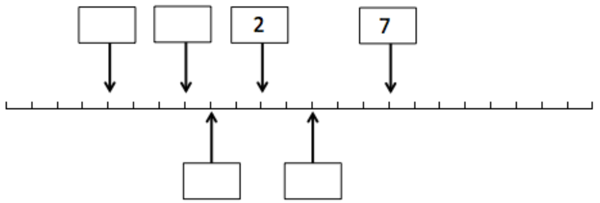
NRICH: Reasoned Rounding * G
NRICH: Round the Four Dice

and 7	
Misconceptions	**Teacher Guidance and Notes**
Children find it had to adapt to the code of roman numerals and they try to translate place value concepts directly. Children think that 49 is IL - breaking the 'adjacent symbol rule' Children think that 40 is XXXX - breaking the '3 max' rule On clocks, sometimes 4 is written as IIII rather iV for aesthetic reasons - this can be confusing as it breaks the rules! Children sometimes write eight thousand as 8 1000 Children struggle if either the hundreds or tens or the units are 'missing' e.g. seven thousand, six hundred and four can be miswritten as 764 or 7640 Children confuse the meaning of < and >, finding it hard to tell which is which.	• When introducing Roman Numerals it can be beneficial to ensure a whole school approach is adopted, ie on displays around clock faces. The history will need to be explored to unpick 'the rules'. Note that these are just conventions rather than things that are innate about maths so make this clear to children. • Children need to understand that we are not calculating with Roman Numerals but making connections to real life and how they are represented today. This is just one alternative number system but there are a multitude of others. Good SMSC opportunity. • When teaching place value use practical resources to expand on different base representations to emphasise the unitised structure of number ie 231 = 2 hundred squares, 3 ten rods and 1 unit/ ones in Base 10. • It is important that children develop their number sense here- they should be able to place numbers on a blank number line including where the scale changes. Try taking a blank paper strip as a scale from 0-1000 and asking children to place 200 on it. Then change the scale to 1-500 and ask them to do the same thing - they should be developing the ability to change the placement based on the scale.

Key Assessment Checklist

1. I can recognise Roman Numerals, identify contexts in which they are used and read/write the numbers 1-10 in Roman Numerals.
2. I can read and write Roman Numerals to 100.
3. I can understand place value of each digit in a 4 digit number as well as partition 4-digit numbers into thousands, hundreds, tens and ones and then in different ways
4. I can read and write numbers in words and numerals
5. I can round any number to the nearest 10, 100 or 1000
6. I can round decimals with one decimal place to the nearest whole number
7. I can solve number/practical problems with numbers up to 10000.
8. I can order and compare numbers beyond 1000, using the signs <, > (and =) to show this comparison.

Year 4	Unit 2: Pattern Sniffing	
10 learning hours	This unit explores pattern from the early stages of counting and then counting in 2s, 5s, and 10s up to the more formal study of sequences. This sequence work progresses through linear sequences up to quadratic, other polynomial and geometric for the most able older students. For children in KS1, this unit is heavily linked to the following one in terms of relating counting to reading and writing numbers. Also in this unit children and students begin to study the properties of numbers and to hone their conjecture and justification skills as they explore odd/even numbers, factors, multiples and primes before moving onto indices and their laws.	
Prior Learning	**Core Learning**	**Extension Learning**
➢ count from 0 in multiples of 4, 8, 50 and 100;	➢ **count in multiples of 6, 7, 9, 25 and 1000**	➢ count forwards or backwards in steps of powers of 10 for any given number up to 1 000 000
	➢ **count backwards through 0 to include negative numbers**	➢ read, write and interpret negative numbers in context
➢ find 10 or 100 more or less than a given number	➢ **find 1000 more or less than a given number**	
➢ recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables	➢ **recall multiplication and division facts for multiplication tables up to 12 × 12**	➢ multiply and divide numbers mentally drawing upon known facts
	➢ **recognise and use factor pairs and commutativity in mental calculations**	➢ identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers ➢ know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers ➢ establish whether a number up to 100 is prime and recall prime numbers up to 19 ➢ recognise and use square numbers and cube numbers, and the notation for squared (^{2}) and cubed (^{3})

Exemplification	Vocabulary
1. Find the next two numbers in each pattern: a) 42, 49, 56, ... b) 16 000, 17 000, 18 000, ... 2. Fill in the missing numbers in the boxes on the number line  3. Find a) 1000 less than 17 465 b) 1000 more than 19 601 4. Complete one missing numbers in each number sentence: a) 9 x 8 = ... b) 6 x .... = 48 c) 36 ÷ .... = 4 d) ...... ÷ 8 = 12 5. a) Find three pairs of factors of 36. b) Calculate 2 x 8 x 5 mentally	negative hundredth fact family factor factor pair commutative multiple

Representation	Fluency	Probing Questions
Counting in 6s, 7s and 9s: • Represent counting in 6s, 7s and 9s using repeated addition with: ○ Numicon ○ Counters on a blank track ○ Counters in groups of 6, 7, 9 ○ Bead strings ○ Placing counter on/Colouring in 100-square • Use a counting stick to represent the first ten multiples of 6, 7 and 9 – explore which values can be found by doubling, tripling etc.	1. Count from in steps of 6, 7 and 9 ○ work out the steps using repeated addition ○ work out some steps using doubling skills ○ count from 0 up to 10th multiple of 6, 7 and 9 with concrete/visual aid ○ count from 0 up to 10th multiple of 6, 7 and 9 without concrete/visual aid ○ count from 0 beyond 10th multiple of 6, 7 and 9	What's the same and what's different? Counting in 6s and Counting in 9s
Counting in 25s and 1000s: • Represent counting in 25s using money (e.g. US dollars – quarters) • Represent counting in 1000s using base 10/Dienes blocks or place value counters or capacities (e.g. litre bottles) or weights	2. Count in steps of 25 and 1000 ○ work out the steps using repeated addition ○ work out some steps using doubling skills ○ count from 0 up to 10th multiple of 25 and 1000	What's the same and what's different? Counting in 25s, Counting in 100s and Counting in 1000s

• Use a counting stick to represent the first ten multiples of 25 and 1000 – explore which values can be found by doubling, tripling etc.	with concrete/visual aid ○ count from 0 up to 10th multiple of 25 /1000 without concrete/visual aid ○ count from 0 beyond 10th multiple of 25 (and 1000)	
Counting back/Negatives • Explore a thermometer and temperatures to investigate counting backwards through zero. • Use a counting stick to represent counting around zero – mark the end number first and count backwards. You can also give each child a number line or 'counting stick' to use kinaesthetically alongside the teacher model. • Use the Tug of War nrich game to explore the ideas of negatives	3. Count backwards through 0 to negative numbers ○ read and write a negative number ○ count in sequence from 0 through the negative numbers ○ label negative numbers on a number line where zero is shown or known ○ count back a specified amount from a given number to arrive at a negative answer	Convince me that if I start on 5 and count back 8 places I will end up at -3 What's the same and what's different? 3, 2, 1, 0, -1, -2, -3
1000 more and less • Use place value apparatus (e.g. base 10, Dienes, place value counters) to represent a number and add/subtract a 1000 piece to/from it • Use a number line to represent 1000 more and 1000 less (can partition the 1000 to break at the next multiple of 1000)	4. Find 1000 more than a number ○ Using base 10 or place value counters ○ Mentally, by increasing the numbers of 1000s by one ○ Examples beyond 1000 ○ Bridging over multiples of 10000 e.g. 1000 more than 9845	Show me 1000 more than 4567 Show me 1000 more than 12045 What's the same and what's different? 167, 1167, 2167, 3167 Always, Sometimes or Never True? When I find 1000 more than a number, only one digit will change
	5. Find 1000 less than a number ○ 4 digits, with apparatus/visual aids (e.g. base 10 or place value counters) ○ 4 digits, mentally (by decreasing the numbers of 100s by one) ○ examples beyond 10000	Show me 1000 less than 4567 Show me 1000 less than 12045 Always, Sometimes or Never True? When I find 1000 less than a number, only one digit will change

	○ bridging over multiples of 10000 e.g. 1000 less than 10876	
Times tables • Represent a times table multiplication calculation in multiple ways: e.g. 6 x 9 as: ○ 9 groups of 6 objects ○ repeated addition using 9 numicon 6s ○ 9 lots of 6-rods (Cuisenaire) ○ **as an array** made of 9 rows each of 6 counters/dots • Represent a times table division calculation with unknown answer: e.g. 24÷ 6 = ... as • 24 objects grouped in 6s • 24 objects grouped into an array (columns of 6) • Represent a times table division with unknown divisor in multiple ways e.g. 54 ÷ .... = 9 as ○ 54 objects shared into 9 piles ○ 54 objects grouped into an array (with rows of 9) • Represent a times table division with unknown dividend using an array e.g. .... ÷ 7 = 5 as ○ groups of 7 counters in columns until there are 5 columns altogether (i.e. 5 counters per row)	6. Find times table multiplication facts (up to 12s) ○ By representing the calculation concretely to deduce the answer ○ By representing the calculation visually to deduce the answer ○ By relating the calculation to another known calculation and counting on/back or doubling etc ○ By beginning to recall key facts	Always, Sometimes or Never True? Multiples of 6 are also multiples of 2 and of 3 Always, Sometimes or Never True? Numbers in the nine times table have digits that add up to 9
	7. Find and begin to recall times table division facts (up to 12s) ○ Unknown answer ○ Unknown divisor ○ Unknown dividend ○ Beginning to recall key facts	Convince me that there are 9 possible pairs of numbers a and b where a x b = 36
	8. Find the other related facts when given one times table multiplication or division fact: ○ Given a multiplication fact, state the equivalent multiplication fact and two related division facts ○ Given a division fact, state the equivalent division fact and two related multiplication facts	Show me the fact family for 7 x 8 Convince me that a fact family will always have four facts
Factors • Dividing a number by 2, 3, 4, etc using hoops and counters (or other objects). Recording the successful numbers and	9. Find factors and pairs of factors of a number ○ by attempting to divide by 1, 2, 3, 4, ... practically or abstractly and recording the successful	Show me a factor pair that makes 18 Show me two factor pairs that make 20 Show me a number with an odd number of

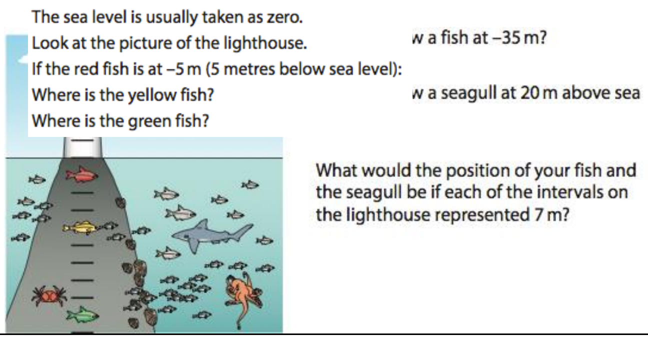
counting the number in each hoop to find the paired number. Recording this as a multiplication statement. • Build arrays to represent factors. For example, build an array with 24 counters and identify factors as possible numbers of rows or columns that will make 24. Find as many different arrays for 24 as you can. Can you make an array with 24 counters that has 5 rows? What does this mean about 5 and its relationship to 24?	numbers ○ in pairs by matching the divisor with the resulting quotient ○ by finding the factor paired with a given factor	factors What's the same and what's different? 1, 2, 3, 4 What's the same and what's different? 3, 6, 12, 18
Commutativity • Build an array to represent, for example, 6 x 7 and then build one for 7 x 6. Compare these arrays to show why multiplication is commutative. • Exploring whether the same works for division using arrays	10. Use factor pairs and commutativity in mental calculations ○ to find the product of three numbers (using pairs of factors and commutativity) e.g. 8 x 2 x 9 is the same calculation as 2 x 72 (because 8 x 9 is a known times table and you can reorder due to commutativity) ○ when multiplying multiples of 10 e.g. 30 x 9 using 3 x 9.	Convince me that a x b gives me the same answer as b x a Convince me that 7 x 2 x 6 is the same calculation as 2 x 42

Further Extension

1. Gemma is counting in 25s from 0. Which of these numbers will she say?

990 550 125 755 150

2.



Rich and Sophisticated Tasks

Count backwards through zero
NRICH: Swimming Pool

Recall multiplication and division facts for multiplication tables up to 12x12
NRICH: Multiplication Square Jigsaw * G P
NRICH: Shape Times Shape * P
NRICH: Table Patterns Go Wild! ** I
NRICH: Let's Divide Up! * P
NRICH: Carrying Cards * P
NRICH: Light the Lights Again * G P
NRICH: Multiples Grid * I
NRICH: Zios and Zepts * P
NRICH: Times Tables Shifts * G P

3. What is the relationship between these calculations?

   6 × 4 x 7; 4 x 6 x 7; 8 x 3 x 7; 7 x 12 x 2

4.

Multiply a number by itself and then make one factor one more and the other one less. What happens to the product?

E.g.

4 × 4 = 16	6 × 6 = 36
5 × 3 = 15	7 × 5 = 35

What do you notice? Will this always happen?

Misconceptions

Pupils forget to include 0 when counting – they may also struggle to understand its role as neither a positive nor a negative number.

When counting in multiples, many children believe that you stop after the 10^{th} or 12^{th} multiple (due to times table practice) – they do not see that multiples are infinite.

When dealing with negatives, children position them incorrectly in the same order as positive numbers. They believe the negative number line looks like

Pupils struggle to find 1000 more when bridging a 10,000 - e.g. 1000 more than 9647

Pupils forget about 1 and the number itself being factors. Pupils may try to use a non-integer as a factor e.g. 2.5

Pupils know that multiplication is commutative but they struggle to use it in questions by spotting pairs of numbers in a multiplication string that could be easily combined.

Teacher Guidance and Notes

- This stage requires children to master all times tables up to 12x12. In reality, this will require more time than solely this unit but the unit provides the opportunity to explore the concepts behind the times tables and to begin the process of memorisation.
- Use a counting stick to help children learn their times tables (multiplication and division facts) as well as to start to see how they relate to each other.
- Note that the number zero is neither positive nor negative
- The expectations of this stage are that children count from 0 in multiples of 6, 7, 9, 25 and 1000 – however, it worth exploring and practising counting in these multiples from other starting numbers also to develop fluency.
- In Stage 4 it is not expected that children can find all the factors of a given number – however, this is worth encouraging where possible as it provides nice reasoning and challenge tasks to try to find all the pairs.
- Ensure 10/100 more and less are secure before approaching 1000 more or less.
- Use the fact family concept to get children to find the associated facts e.g. for 8x4=32 you would also write 4x8=32, 32÷4=8, 32÷8=4.

Key Assessment Checklist
1. I can count in steps of 6, 7, 9 from 0
2. I can count in steps of 25 and 100 from 0; I can explain how the pattern of 25s and 1000s are related to 100s
3. I can find 1000 more or 1000 less than a given number
4. I can count backwards from a positive number using negative numbers after 0.
5. I can count forwards and backwards in hundredths, saying the whole number for every ten tenths
6. I can give the fact family for any multiplication up to 12x12 (or associated division); I can use these families to solve problems
7. I can find factor pairs of a number using times table facts
8. I can complete mental calculations using factor pairs to help me

Year 4	Unit 3: Solving Calculation Problems
8 learning hours	This unit explores the concepts of addition and subtraction at KS1 building to wider arithmetic skills including multiplication at late-KS2. It is strongly recommended that teachers plan this unit for KS1/KS2 with direct reference to the calculation policy! At KS3 students are developing calculation into its more general sense to explore order of operations, exact calculation with surds and standard form (which have been introduced in Inv Number Systems briefly) as well developing their skills in generalising calculation to algebraic formulae. They need to substitute into these formulae and calculate in the correct order to master this strand. The formulae referenced are examples of the types of formula they will need to use, but the conceptual understanding for these formulae will be taught elsewhere in the curriculum.

Prior Learning	Core Learning	Learning Leads to....
• add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction • estimate the answer to a calculation and use inverse operations to check answers	• **add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate** • **solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why** • **estimate and use inverse operations to check answers to a calculation**	• add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction) • use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy

Exemplification

1. Calculate
a) 6456 + 2187 b) 7264 – 3509

2. There are 3470 young football players in the local area. 1682 of them are boys. How many of them are girls?
a) Amy draws a diagram to help answer the problem. Which is the correct diagram?

3470	
Girls	1682

Girls	
1682	3470

1682	
3470	Girls

b) Solve the problem

3. Lianne estimates the answer to 3682 – 1215 as 2300.
Do you agree with Lianne? Explain your answer

Vocabulary

add	how many more?
and	take (away)
more	leave
make	how many left?
sum	less
total	fewer
altogether	difference between
score	equals
double	is the same as
one more	minus
two (ten) more	number sentence
plus	order
equals	calculate
hundred	column subtraction
ten	estimate
one	inverse
exchange	operation
column digit	check
columnar	
column addition	

<table>
<tr><th>Representation</th><th>Fluency</th><th>Probing Questions</th></tr>
<tr><td>Mental Addition
Representing addition as counting or jumping on (augmentation) using a number line (jumping in 1000s, 100s, 10s and 1s)</td><td>1. Add a four-digit number and ones/tens/hundreds mentally (up to 10 000)
four-digit number + 1000
four-digit number + multiple of 1000
four-digit number + one-digit number (not crossing a ten)
four-digit number + one-digit number (not crossing a ten)
four-digit number + 10
four-digit number + multiple of 10 (not crossing a hundred)
four-digit number + multiple of 10 (crossing a hundred)
four-digit number + 100
four-digit number + multiple of 100 (not crossing a thousand)
four-digit number + multiple of 100 (crossing a thousand)</td><td>Show me two numbers with a sum of 4215

Convince me that if I add a multiple of 1000 to this number, the hundred, tens and ones digits will stay the same.

Always, Sometimes, Never?
Adding 5 to a number that ends in 6 will result in a number that ends in 1.

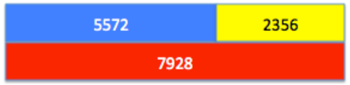
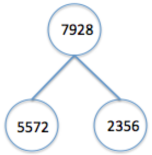
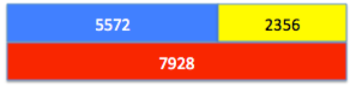
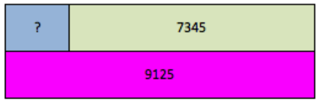
Always, Sometimes, Never?
The sum of three odd numbers is even.</td></tr>
<tr><td rowspan="2">Written Addition
Representing numbers using place value counters or equivalent [thousands, hundreds, tens and ones] then combining and finding the total value (aggregation) (exchanging ten 1s for one 10 or ten 10s for one 100 or ten 100s for one 1000 as required when bridging)
(see calculation policy for more details)</td><td>2. Add a four-digit number and a three-digit number
No exchange required e.g. 2452 + 537
Exchange required from ones to tens e.g. 2452 + 539
Exchange required from tens to hundreds e.g. 2452 + 587
Exchange required from hundreds to thousands e.g. 2452 + 715
Multiple exchange e.g, 2452 + 789</td><td>Show me a 4-digit number and a 3-digit number with a sum of 2170.
What about a 4-digit number less than 2000?

Always, Sometimes, Never?
Adding 8 to a number that ends in 2 will result in a multiple of 10.</td></tr>
<tr><td>3. Add a four-digit number and a four-digit number
No exchange required e.g. 2452 + 5237
Exchange required from ones to tens e.g. 452 + 239
Exchange required from tens to hundreds e.g. 452 + 287
Exchange required from hundreds to thousands e.g. 2452 + 3717
Multiple exchanges required from both ones to tens and from tens to hundred e.g, 2452 + 5769</td><td>Show me two numbers with a sum of 5000

Show me
... two numbers that are easy to add
... two numbers that are hard to add

Always, Sometimes, Never?
A four digit number add a four digit number gives an eight digit number</td></tr>
<tr><td>Mental Subtraction
Representing subtraction as counting or</td><td>4. Subtract ones/tens/hundreds/thousands from a four-digit number mentally</td><td>Show me two numbers with a difference of 2000</td></tr>
</table>

jumping back (reduction) using a number line (jumping in 1000s, 100s, 10s and 1s) • Representing subtraction as a comparative difference between two sets of objects using number lines with both numbers marked and difference found	• four-digit number - 1000 • four-digit number - multiple of 1000 • four-digit number - one-digit number (not crossing a ten) • four-digit number - one-digit number (not crossing a ten) • four-digit number - 10 • four-digit number - multiple of 10 (not crossing a hundred) • four-digit number - multiple of 10 (crossing a hundred) • four-digit number – 100 • four-digit number – multiple of 100 (not crossing a thousand) • four-digit number – multiple of 100 (crossing a thousand)	Always, Sometimes, Never? Subtraction makes a number smaller
Written Subtraction • Representing first number using place value counters [thousands, hundreds, tens and ones] then removing or taking away the second number and finding the resulting value (partitioning) (exchanging one 10 for ten 1s or one 100 for ten 10s or one 1000 for ten 100s as required when bridging) (see calculation policy for more details)	5. Subtract a three-digit number from a four-digit number • No exchange required e.g. 5675 - 454 • Exchange required from tens to ones e.g. 5675 - 259 • Exchange required from hundreds to tens e.g. 5675 – 582 • Exchange required from thousands to hundreds e.g. 5675 - 713 • Multiple exchange required e.g. 5675 - 489	Show me a 3-digit number that can be subtracted from 3412 to give: - an answer greater than 3000 - an answer less than 3000 - an answer of 2768 Always, Sometimes, Never? The difference of two odd numbers is even
	6. Subtract a four-digit number from a four-digit number • No exchange required e.g. 5675 - 3254 • Exchange required from tens to ones e.g. 5675 - 2359 • Exchange required from hundreds to tens e.g. 5675 – 3281 • Exchange required from thousands to hundreds e.g. 5675 - 3812 • Multiple exchanges required e.g. 5675 - 2886	Show me ... two numbers that are easy to subtract ... two numbers that are hard to subtract Always, Sometimes, Never? A four digit number subtract a four digit number gives a three digit number What's the same and what's different? 2285 + 3126; 3126 + 2285; 5411 – 2285; 5411 – 3126 3126 – 2285 2285 + 5411 3126 + 5411

Problems as Additions and/or Subtractions • Representing addition problems using: ○ the bar model  ○ a part-part-whole model 	7. Interpret a word problem correctly as an addition or subtraction calculation and solve • represent and solve an addition word problem using a bar model • represent and solve a subtraction word problem using a bar model • represent and solve an addition/subtraction word problem using a part-part-whole model • represent and solve an addition/subtraction word problem using a number line • represent and solve a two-step addition and/or subtraction word problem	Show me the four number facts that this bar model shows  Show me the other calculations that you know the answer to if I tell you that 2348 + 5417 = 7765
Missing Number Problems • Using a bar model or part-part-whole model to represent the calculation to decide whether to add or subtract e.g. ? + 7345 = 9125 	8. Solve missing number problems involving addition or subtraction • a+ b = ? • a + ? = b • ? + a = b • a – b = ? • ? – a = b • a - ? = b	What's the same and what's different? • ? – a = b • a - ? = b What's the same and what's different? addition; subtraction
Estimation • Use place value counters or other place value equipment to represent a number and then round it to the nearest 1000, 100 (or even 10) to allow easy mental addition or subtraction.	9. Estimate the answer to an addition or subtraction calculation • addition - numbers close to multiples of 1000 e.g. 5962 + 2135 • subtraction - numbers close to multiples of 1000 e.g. 5962 - 2135 • addition - numbers close to multiples of 100 e.g. 2596 + 4213 • subtraction - numbers close to multiples of 100 e.g. 6596 - 4213 • addition – by rounding to nearest 10 e.g. 5449 + 3219 • subtraction by rounding to nearest 10 e.g 5671 - 3358	Convince me that 5962 + 2135 has an answer of approximately 8000.
Checking • Use the bar model to represent a problem to explore inverse calculations	10. Find the inverse calculation to an addition or subtraction and use it to check an answer • give fact family for any given addition or subtraction calculation • find inverse (addition) - state checking calculation, estimate, calculate exactly • find inverse (subtraction)- state checking calculation, estimate, calculate exactly	Show me how you could check whether 6281 + 2376 = 8657 using another calculation Convince me that addition and subtraction are opposites

Further Extension

1.

Fill in the missing digits.

1 □ 3 + 6 □ = 200

1 □ 5 □ + 300 = 1557

5 □ 28 – 44 □ = 4788

□ □ □ 0 – 2468 = 5092

2.

Identify the missing numbers in these bar models. They are not drawn to scale.

1000		
	353	354

2000		
493		754

Select your own numbers to make this bar model correct.

5000		

3.

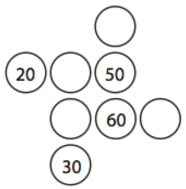
Fill in the empty boxes to make the equations correct.

7 □ 1 + □ 3 □ = 999

7 □ 1 + □ 3 □ = 1000

4.

Complete this diagram so that the three numbers in each row and column add up to 140.



Now create your own diagram with a total of 250.

Rich and Sophisticated Tasks

Solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why

NRICH: [Money Bags] ** P
NRICH: [Amy's Dominoes] ** P
NRICH: [Fifteen Cards] * P I
NRICH: [Sealed Solution] ** P
NRICH: [Roll These Dice] ** I

5. (This is a reasoning rather than calculation task)

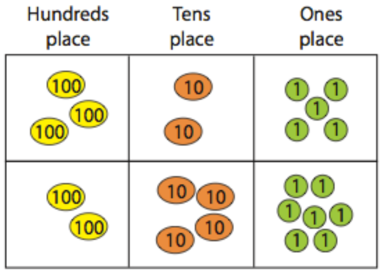
Write >, = or < in each of the circles to make the number sentence correct.

1023 + 24 + 24 ○ 1023 + 48

1232 – 232 ○ 1355 – 252

1237 – 68 + 32 ○ 1242 – 69 + 31

6.



$$\begin{array}{r} 325 \\ +\,247 \\ \hline \end{array}$$

Sam has completed these calculations, but he is incorrect.
Explain the errors he has made.

$$\begin{array}{r} 325 \\ +\,247 \\ \hline 581 \\ \hline \end{array} \qquad \begin{array}{r} 355 \\ -\,247 \\ \hline 112 \\ \hline \end{array}$$

Misconceptions	Teacher Guidance and Notes
Children struggle to interpret whether to add or subtract from the language used. Children can find 'How many more/less?' particularly troublesome as it relates to ordinal values of numbers and relationships. Children struggle to add numbers when their place value understanding is weak. If they do not read a number like '4352' as 4 thousands, 3 hundreds, 5 tens and 2 ones then they struggle to combine the ones, tens, hundreds and thousands from two numbers appropriately. When performing columnar addition, children may forget to include the hundreds, tens or hundreds they have generated from earlier exchanges. They may also fail to exchange them at all and thus end with a two-digit numbers in the 1s column etc. When subtracting, children will sometimes subtract the larger number from the smaller initially. When performing columnar subtraction, children may exchange from the wrong column or fail to exchange altogether (instead just finding the difference between the digits in the column, even where the second one is greater than the first).	• The aim of this unit for these children is to develop security in the formal processes of addition and subtraction and with this more fluid use broader problems and contexts. Simultaneously they should be developing efficiency of mental methods when appropriate. Therefore, encourage children to look at the numbers in a calculation before commencing to decide if they can do it in their head, with jottings or whether they need to use a written method. • At this level you should aim to use place value counters with children as a representation (or money) but if you need to, go back to objects where the value of the numbers is more obvious e.g. dienes or numicon. • Ensure children are going through the full exchange process when adding or subtracting i.e. picking up 10 one counters and swapping them for a ten counter or vice versa. They should then 'regroup' and ensure that the tens and ones are in the right columns to be combined. • To begin to embed the written routines of the calculation policy, it is advised that children work in pairs with one child manipulating the equipment and saying what they are doing aloud while the other child records the calculation using the column method so that they learn that

Children may also fail to correctly record the exchange and thus not reduce the tens, for example, by one so that the answer is 10 too high.

Children find calculations where multiple exchanges must be made particularly hard e.g. 4678 + 3945 because the notation becomes unwieldy. Similarly subtractions such as 2304 - 1789 cause issues because of the need to carry out a chain reaction of exchange. In these instances you may need to resort to equipment, even where the child does not need it for 'standard' calculations.

Children often do not see difference as a representation of subtraction because take away is emphasised so much. They need to see subtraction represented in this way also to challenge this.

The equals sign is not always correctly interpreted as 'has the same value as' by children, who may see it as 'the answer is'.

Some children may use the incorrect operation when checking and fail to realise that they need to use the inverse - this is more pronounced when subtracting.

When completing missing number problems and using representations of a problem, children sometimes incorrectly arrange a number sentence e.g. if they are told that a + b = c they incorrectly say that a – b = c etc

the column method is just a written representation of the practical process (rather than a 'different' method) - see the videos at the NCETM for examples of this. https://www.ncetm.org.uk/resources/40532
To help with setting out calculations in columns use large squared paper or laminated grids and mini-WB pens.

- The pitch of this unit is numbers up to 10 000, but of course these ideas an be extended beyond 10 000 for those children who are confident working with in this area.
- At this Stage, it is important to introduce a wide range of problems, contexts and situations involving addition and subtraction. The representations of the bar model are particularly crucial and the properties of inverses as applied to solve missing number problems should be directly addressed.
- Try to model the wide range of language used to signify addition and subtraction – see vocabulary list above. The children ultimately need to be able to recognise that a problem is an addition problem from the language (and same for subtraction).
- Use 'sum' only to mean an addition calculation – use the word 'calculations' to mean mixed operation computations
- Challenge issues with the use of the = sign by looking at examples where the question is on the right e.g. ? = 2514 + 7288 as well as balance problems in Further Extension e.g. 6143 + 2614 = ? + 3271
- Language is critical in this learning process - make sure you use and insist on the correct terminology for place value e.g. 4123+3456 would involve twenty add fifty, not two add five. Also insist on children describing their steps orally e.g. I need to add seven ones and 5 ones which makes twelve ones. So I will exchange 10 of these ones for a ten and regroup (put the ten in the right column).

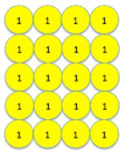
Key Assessment Checklist

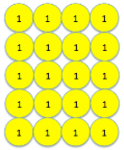
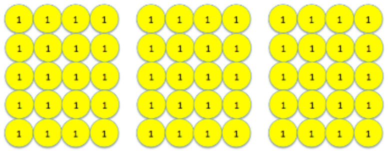
1. **I can add two numbers up to four digits using a columnar method**
2. **I can subtract two numbers up to four digits using a columnar method**
3. I can estimate the answer to addition and subtraction calculations involving four digits
4. I can use the inverse operation to check answers to addition and subtraction calculations
5. I can solve 2 step addition and subtraction problems choosing the correct operation and using the most appropriate methods

Year 4	Unit 4 : Generalising Arithmetic
8 learning hours	This unit is focused on developing fluency in the manipulation of number. At primary level this is focused on arithmetic itself and the methods for four operations particularly; however, this is naturally generalised to thinking about rules of arithmetic more widely at secondary level i.e. algebra. These aspects have been paired together intentionally to help teachers describe algebra as simply a generalisation of number. It is expected that teachers will go back to arithmetic to help students see where the 'rules' of algebra come from. Note that the greyed out content is covered previously and hence is not required content here unless of concern.

Prior Learning	Core Learning	Learning Leads to...
➢ write and calculate mathematical statements for multiplication (and division) using the multiplication tables that they know, including for 2-digit numbers times 1-digit numbers, using mental and progressing to formal methods ➢ solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects	➢ **use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers** ➢ **multiply two-digit and three-digit numbers by a one-digit number using formal written layout**	➢ multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers ➢ multiply and divide whole numbers and those involving decimals by 10, 100 and 1000

Exemplification	Vocabulary	
1. Calculate a) 200×6 b) $420 \div 6$ c) $6 \times 3 \times 5$ d) $12 \div 1$ e) 4×0 2. Calculate a) 42×7 b) 576×4	multiply times product lots of groups of array by	product grid partitioning compact column divide quotient

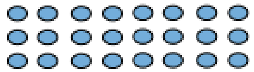
Representation	Fluency	Probing Questions
Place Value and Multiplication • Building an array using place value counters to represent, for example, 4 x 5  Then replacing the 1s counters with 10s to explore 40 x 5	1. Use place value to find a related multiplication fact mentally • multiple of 10 x single digit e.g. 30×6 • multiple of 100 x single digit e.g. 300×6 • multiple of 1000 x single digit e.g. 3000×6 • multiple of 10 x multiple of 10 e.g. 30×60	What's the same and what's different? $4 \times 3; 40 \times 3; 400 \times 3;$ $4 \times 30; 4 \times 300$

 and so on with 100s, 1000s etc.		
Place Value and Multiplication • Building an array using place value counters to represent, for example, $20 \div 5$  Then replacing the 1s counters with 10s to explore $200 \div 5$ 	2. Use place value to find a related division fact mentally • multiple of 10 ÷ single digit e.g. $180 \div 6$ • multiple of 100 ÷ single digit e.g. $1800 \div 6$ • multiple of 1000 ÷ single digit e.g. $18000 \div 6$ • ext: multiple of 10 ÷ multiple of 10 e.g. $180 \div 60$ • ext: multiply of 100 ÷ multiple of 10 e.g. $1800 \div 60$	
Multiplying 3 numbers • Representing the first multiplication as an array and then using repeats of this array to show the second multiplication e.g. 4 x 5 x 3 could look like  • Rearranging these images to help develop the sense of commutativity of multiplication to show that this is the same as 12 x 5 or as 4 x 15 and so on	3. Multiply three numbers together mentally • three single digits • two-digit x 1-digit x 1-digit • two-digit x multiple of 10 x 1-digit • examples including multiplying by 1 • examples including multiplying by 0	Show me three numbers with a product of 72 What's the same and what's different? $7 \times 6;\ 7 \times 2 \times 3;\ 8 \times 7;$ $2 \times 4 \times 7;\ 2 \times 2 \times 2 \times 7$ Always, Sometimes, Never? A number multiplied by 0 gives an answer of 0 Always, Sometimes, Never? A number divided by 0 gives an answer of 0 Always, Sometimes, Never? A number multiplied by 1 gives an answer of 1

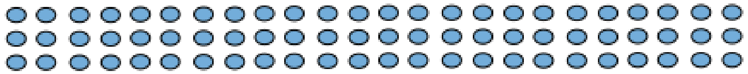
<table>
<tr><td></td><td></td><td>Always, Sometimes, Never?
A number divided by 1 gives an answer of itself</td></tr>
<tr><td>

Multiplying 2-digits by 1-digit – Informal Methods

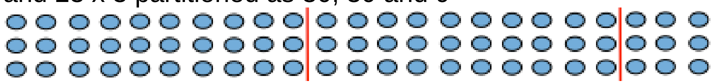
- Building arrays to represent a calculation as repeated addition, for example 8 x 3



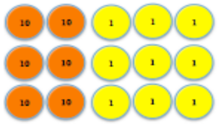
- Exploring larger arrays and how these can be partitioned into easier-to-manage pieces (i.e. multiples of 10). For example, here is 23 x 3



and 23 x 3 partitioned as 30, 30 and 9



- Building arrays using place value counters



- Generalising the array using a grid (area) representation

	20	3
3	60	9

60 + 9 = 69

</td><td>4. Multiply a 2-digit number by a single digit using an informal method</td><td>Convince me that 17 x 3 is the same as the sum of 10x3 and 7x3

Convince me that 14 x 6 will give a different answer to 16 x 4

What's the same and what's different?
45x5, 25x9, 15x15, 10x20</td></tr>
<tr><td>

Formal Methods

- Linking the grid representation to expanded formal method and then to compact method

</td><td>

5. Multiply a 2-digit number by a single digit using a formal method
   - no exchange e.g. 32×3
   - exchange from 1s to 10s e.g. 26×3
   - exchange from 10s to 100s e.g. 41×7
   - exchange in both columns e.g. 54×6

</td><td>Show me a two digit number and one digit number you can multiply to give an 8 in the ones column

What's the same and what's different?
45x9, 25x4, 15x7, 10x8

Always, Sometimes, Never?</td></tr>
</table>

$\begin{array}{rrl} 2 & 3 & \\ & 3 & \times \\ \hline & 9 & \\ 6 & 0 & + \\ \hline 6 & 9 & \end{array}$ $\quad$ $\begin{array}{rrl} 2 & 3 & \\ & 3 & \times \\ \hline 6 & 9 & \end{array}$		A two digit number multiplied by a one number gives an answer that is a two digit number.
3-digit x 1-digit - Informal Methods • Building arrays using place value counters e.g. 234 x 4 • Generalising the array using a grid (area) representation	6. Multiply a 3-digit number by a single digit using an informal method	What's the same and what's different? 243 x 7 and 247 x 3 What's the same and what's different? 297 x 3 and 300 x 3 - 3 x 3
3-digit x 1-digit - Formal Methods • Linking the grid representation to expanded formal method and then to compact method	7. Multiply a 3-digit number by a single digit using a formal method • no exchange e.g. 132×3 • exchange from 1s to 10s e.g. 231×3 • exchange from 10s to 100s e.g. 271×3 • exchange from 100s to 1000s e.g. 812×4 • multiple exchanges e.g. 562×7	Show me a three digit number and one digit number you can multiply to give a 5 in the ones column Always, Sometimes, Never? A three digit number multiplied by a one number gives an answer that is a three digit number
Recognising Multiplication Problems • Using a bar model to represent a problem as repeated addition or scaling	8. Recognise and solve a simple multiplication word problem • example of groups e.g. Melanie has 6 bags that each contain 37 sweets. How many sweets does she have altogether? • example of scaling e.g. A tree is 58cm tall. It grows to three times this height over the next year. How tall is it now?	Convince me that if I know that 468 / 4 is 117, then I can check I am right by calculating 4 x 117

Further Extension	Rich and Sophisticated Tasks
1. True or false? $7 \times 6 = 7 \times 3 \times 2$ $7 \times 6 = 7 \times 3 + 3$ Explain your reasoning. Can you write the number 30 as the product of 3 numbers? Can you do it in different ways?	Use the numbers 0-9 once each to complete these calculations correctly $\begin{array}{rrrl} & 4 & 2 & \\ & & \square & \times \\ \hline 1 & 2 & 6 & \end{array}$ $\quad$ $\begin{array}{rrrl} & \square & 3 & \\ & & 5 & \times \\ \hline 2 & 1 & \square & \end{array}$ $\quad$ $\begin{array}{rrrl} & 6 & \square & \\ & & 4 & \times \\ \hline 2 & 6 & 8 & \end{array}$ $\quad$ $\begin{array}{rrrrl} & 3 & 5 & \square & \\ & & & 7 & \times \\ \hline \square & 5 & 1 & 3 & \end{array}$ $\quad$ $\begin{array}{rrrrl} & 2 & 8 & 1 & \\ & & & \square & \times \\ \hline \square & 6 & \square & 6 & \end{array}$ [NRICH: What's in the box?]

2.

Place one of these symbols in the circle to make the number sentence correct:
>, < or =.

Explain your reasoning.

8×50	◯	50×8
8×50	◯	80×5
300×3	◯	5×200

3.

Multiply a number by itself and then make one factor one more and the other one less. What happens to the product?

E.g.

$4 \times 4 = 16$ $6 \times 6 = 36$

$5 \times 3 = 15$ $7 \times 5 = 35$

What do you notice? Will this always happen?

Misconceptions	Teacher Guidance and Notes
Children sometimes struggle to partition correctly when dividing up an array or using the grid method. Weak times tables can lead to errors in larger calculations e.g. 40 x 7 is dependent on the knowledge of 4 x 7 When using the formal written method, children sometimes struggle to deal with situations where they need to exchange ones for a ten etc. and may forget to 'add in' any of these extra tens, hundreds etc in the next column Finding related facts to those already containing 0s can cause errors e.g. 200 x 5 can be incorrectly stated as 100 Children make errors when multiplying (or dividing) by 1 (and 0).	• This unit is focused on the skill of multiplication (although there is minor reference to using the division elements of times tables and related facts) • The aim is to develop skills in formal multiplication by a single digit, but there is a significant amount of conceptual development to do first • Children have met multiplication in Year 2/3 in a more informal way but this is the first time they have progressed to formal methods with exchanging etc. However, they should be familiar with arrays and partitioning. • For further guidance, see the calculation policy as well as the NCETM videos for exemplification! https://www.ncetm.org.uk/resources/40532 • When teaching multiplication is important that children understand the two different representations i.e. 'lots of'/'groups of' and 'scaling'. We often pay more attention to the former and hence problems involving the latter are not always even recognised as multiplication. Therefore include word problems linked to scaling as well as simple those representing 'groups of' to ensure children recognise these as multiplications. There is

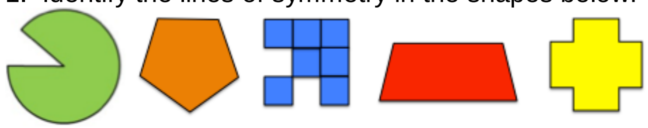
	more focus on the problem solving elements with multiplication and division later in the year. • It is advised that you use a consistent meaning for a multiplication expression i.e. a x b means a multiplied by b and is represented by a objects (in a horizontal line) replicated in b rows. Thus the array for axb will be different for the array bxa (although they will contain the same number of dots). • It is critical that children can multiply single digits together (i.e. they know their times tables!) so this could be an additional focus in this unit (alongside regular practice)

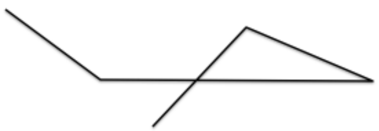
Key Assessment Checklist

1. I can calculate related multiplication facts using place value
2. I can calculate related division facts using place value
3. I can multiply numbers mentally
4. I can multiply a two-digit number by a one-digit number informally (using practical equipment or a representation to help me).
5. I can multiply a two digit number by a one digit number using a formal written method
6. I can multiply a three digit number by a one digit number using a formal written method

Year 4	Unit 5: Exploring Shape
8 learning hours	In this unit children and students explore the properties of shapes, both 2D and 3D. At KS1 this is focused on common shape names and basic features of vertices, sides etc. but this then develops to classifying quadrilaterals and triangles in KS2. Alongside this focus children begin to explore angle and turn in KS2 and develop this to more formal angle rules through Stages 5, 6, 7, 8. Older students begin to explore the field of trigonometry, encountering first Pythagoras' Theorem, then RA-triangle trig before finally looking a the sine rule and cosine rule.

Prior Learning	Core Learning	Learning Leads to...
➢ recognise angles as a property of shape or a description of a turn ➢ identify right angles, recognise that two right angles make a half-turn, three make three quarters of a turn and four a complete turn; identify whether angles are greater than or less than a right angle ➢ identify horizontal and vertical lines and pairs of perpendicular and parallel lines ➢ Draw 2D shapes and make 3D shapes; recognise 3D shapes in various orientations and describe them.	➢ **identify lines of symmetry in 2-D shapes presented in different orientations** ➢ **identify acute and obtuse angles and compare and order angles up to two right angles by size** ➢ **compare and classify geometric shapes, including quadrilaterals and triangles, based on their properties and sizes**	➢ know angles are measured in degrees: estimate and compare acute, obtuse and reflex angles ➢ use the properties of rectangles to deduce related facts and find missing lengths and angles ➢ distinguish between regular and irregular polygons based on reasoning about equal sides and angles.

Exemplification	Vocabulary	
1. Identify the lines of symmetry in the shapes below:  2. a) Look at the diagram	line of symmetry symmetrical vertical horizontal mirror line mirror image angle right angle	right-angled triangle scalene triangle quadrilateral rectangle square, trapezium kite



(i) label an acute angle A (ii) label an obtuse angle B

b) Put these angles in order of size, from smallest to largest

3. Explain the difference between
a) an equilateral triangle and an isosceles triangle
b) a quadrilateral and a pentagon
c) a square and a rhombus

acute
obtuse
greater than
less than
compare
order
shape
2D
3D
side
vertex/vertices
property/ies
triangle
equilateral
isosceles
scalene
right-angled
quadrilateral
square
rectangle
rhombus
parallelogram

irregular
oblong
pentagon
hexagon
octagon
decagon
polygon
circle
parallel
perpendicular
equal
diagonal
properties
Carroll diagram
Venn diagram
criterion, criteria
sort, classify

Representation	Fluency	Probing Questions
Symmetry • Exploring symmetry in designs and other objects. For example, look at the symmetry of different flags • Folding paper shapes to identify (and test) possible lines of symmetry • Using tracing paper to identify (and test) possible lines of symmetry on images that cannot be folded • Symmetry ITP programme	1. Identify lines of symmetry in 2D shapes in any orientation • square • equilateral triangle • isosceles triangle • rectangle • kite • delta/arrowhead • rhombus • parallelogram • regular pentagon • other regular polygons • other isosceles shapes e.g. isosceles trapezium or isosceles pentagon • compound shapes • shapes made from arrangements of many squares e.g. heptominoes	Show me a shape with exactly two line of symmetry ... 1 line .... no lines Show me the lines of symmetry of this shape Show me a shape with the same number of lines of symmetry as this shape

Angles • Making right angles using paper strips and paper fasteners (or geostrips) and then making the angle smaller or bigger • Overlaying geostrips oe to a drawn angle and then moving to another angle to make a comparison • Making different angles on geoboards with elastic bands and then ordering them • Sorting angles into acute, right and obtuse categories	2. Categorise angles as acute, right or obtuse • identify right angles from a set of angles • identify right angles in shapes and diagrams • identify angles less than a right angle (i.e. acute) from a set of angles • identify angles less than a right angle (i.e. acute) in shapes and diagrams • identify angles more than a right angle (i.e. obtuse) from a set of angles • identify angles more than a right angle (i.e. obtuse) in shapes and diagrams • sort sets of angles into categories of acute, right and obtuse • identify all the acute, right and obtuse angles in a shape or diagram	Show me an obtuse angle, a right angle, an acute angle Convince me that all acute angles are smaller than all obtuse angles What's the same and what's different? acute angle, right angle, obtuse angle
	3. Compare and order angles up to 2 right angles • compare an angle with a right angle and say which is greater • compare one acute and one obtuse angle and say which is greater • compare two acute angles and say which is greater • compare two obtuse angles and say which is greater • order three angles from least to greatest • order four or more angles from least to greatest	Show me an angle that is less than this one/greater than this one Always, Sometimes, Never? Two acute angles together make an obtuse angle.
	4. Notate on and read diagrams correctly • equal lengths using a mark across the side • (second pairs of equal lengths) • parallel lines using arrows • (second pairs of parallel lines) • equal angles marked • (second pairs of equal angles) • perpendicular lines marked with a right angle • lines of symmetry shown using a dashed line	Show me a shape with three sides of equal length. And another…. What's the same and what's different? Parallel line markings and equal length markings

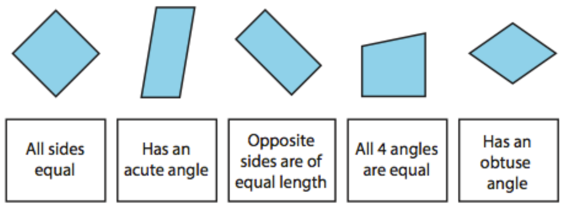
Properties of Triangles
• Making as many different triangles as possible on a geoboard (the quadrilaterals etc)
• Visualising shapes as described and then sketching. For example: *Imagine a*
	5. Compare and classify triangles
	• describe and compare side, angle and symmetry properties
	  ○ equilateral
	  ○ isosceles
	  ○ scalene
	  ○ right-angled
		Show me a triangle that is equilateral
		
		Convince me that an isosceles triangle has one line of symmetry
		
		Always, Sometimes, Never?
		Right-angled triangles are scalene

large, white equilateral triangle on the table in front of you. Take a smaller, red equilateral triangle and push it into the top corner of the white triangle. Now take a second red equilateral triangle and push it into one of the other corners of the white triangle. Without saying anything, quickly draw the white shape that is left uncovered.	• given definition or properties, identify the triangle • provide a definition given the name of the triangle	triangles.

Properties of Quadrilaterals
• Making as many different quadrilaterals as possible on a geoboard
• Playing picture battleships with shape cut outs in pairs. One child makes a shape with their shape cut outs. They then describe it to their partner, who has to build it from their matching shape cut outs.
• Creating a quadrilateral family tree
	6. Compare and classify quadrilaterals
	• describe and compare side, angle and symmetry properties
	  ○ square
	  ○ rectangle
	  ○ parallelogram
	  ○ rhombus
	  ○ kite
	  ○ trapezium
	• given definition or properties, identify the quadrilateral
	• provide a definition given the name of the quadrilateral
		Show me the quadrilateral family tree
		
		Convince me that a square is a rectangle
		
		Convince me that a shape with four right angles exactly is a rectangle
		
		What's the same and what's different? square, rectangle, oblong
		
		What's the same and what's different? parallelogram, rectangle, rhombus
		
		What's the same and what's different? rhombus and a square
		
		Always, Sometimes, Never?
		Squares are parallelograms

Properties of 2D shapes
• Making shapes using string and pegs outside (or people!) and investigating their symmetry
• Sorting shapes into hoops using a given criterion e.g. has at least one right angle
• Using hoops to create Venn diagrams for sorting shapes
• Using the Polygon ITP to explore shapes with ICT
• Use geogebra to construct shapes with given properties.
	7. Compare and classify other 2D shapes
	• describe and compare side, angle and symmetry properties
	  ○ regular pentagon, hexagon, octagon, decagon
	  ○ isosceles pentagons etc.
	  ○ circle
	  ○ semicircle
	  ○ other shapes of interest!
	• given definition or properties, identify the shape
	• provide a definition given the name of the shape
		Show me a shape that is a polygon.
		Show me a shape that is not a polygon.
		
		Convince me that a circle is not a polygon
		
		What's the same and what's different? trapezium, rectangle, circle
		
		Always, Sometimes, Never?
		Pentagons have 5 lines of symmetry

• Playing 'Guess My Shape' using 20 questions format (can be done nicely using a shape fan so that all children put forward a guess after each new fact is revealed) • Playing 'I like' with properties of shapes. Each child draws or chooses a shape. The teacher then picks out shapes that (s)he 'likes' and children must guess the property that (s)he is looking for. [can be narrowed down to just quadrilaterals if desired]		
Properties of 3D Shapes • Sorting shapes into hoops using a given criterion e.g. has at least one right angle • Using hoops to create Venn diagrams for sorting shapes	8. Compare and classify 3D shapes • describe and compare faces, vertices and edges (and properties of these) ○ cube ○ cuboid ○ prisms ○ pyramids ○ cone ○ cylinder ○ sphere ○ hemisphere ○ other shapes of interest • given definition or properties, identify the shape • provide a definition given the name of the shape	Always, Sometimes, Never? Cuboids are prisms What's the same and what's different? Pyramid and prism What's the same and what's different? Surface and face Show me a shape with 5 faces

Further Extension	Rich and Sophisticated Tasks
1. Below are five quadrilaterals: a rectangle, a rhombus, a square, a parallelogram and an unnamed quadrilateral. Write the names of each of the quadrilaterals. Draw lines from each shape to match the properties described in the boxes below.  	**Identify lines of symmetry in 2-D shapes presented in different orientations** **NRICH: Let Us Reflect * P** **NRICH: Stringy Quads ** P** **NRICH: Counters in the Middle * G P** **Compare and classify geometric shapes, including quadrilaterals and triangles, based on their properties and sizes** **NRICH: Nine-pin Triangles *** I** **NRICH: Cut it Out *** P** **NRICH: Sorting Logic Blocks * G** **NRICH: What Shape? * G P** **NRICH: Shapes on the Playground ** P**

2.

Captain Conjecture says that a quadrilateral can sometimes only have three right angles.

Do you agree?

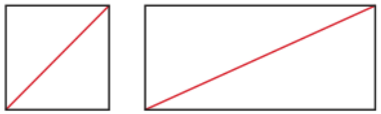
Explain your reasoning.

3.

Tom says, 'In each of these shapes the red line is a line of symmetry.'

Do you agree?

Explain your reasoning.



4. Use 5 squares to build your own pentominoes. How many can you find? Mark on their lines of symmetry and classify them using this!

NUFFIELD AMP: Symmetry

BOWLAND assessments: Three of a Kind

Misconceptions

Children confuse the process of finding lines of symmetry with that of halving and quartering a shape.
They may also be drawn particularly towards vertical and horizontal lines of symmetry, sometimes missing those at an angle.
Similarly, some children 'see' diagonal lines of symmetry that are not there in reality because their eyes cannot process whether there is a 'match' with both sides of the picture at this angle.

When describing angles, some children interchange acute and obtuse - they also don't realise that right angles are part of this progression i.e. your angle is either acute OR right OR obtuse if it is less than a half turn.

When analysing the properties of shapes, children are not always precise with their mathematical language. They may say ‘a rectangle has four parallel sides’ when in fact they mean that it ‘has two pairs of parallel sides’.

Teacher Guidance and Notes

- In Stage 4 children should be extending their repertoire of shapes as well as their ability to use the properties of the shapes to explain their classifications and to derive facts.
- The expectation here is that children can identify lines of symmetry in any orientation and not just vertical or horizontal.
- It is easier to both see and to test out symmetry for the human brain if the mirror line is vertical (because of our eye formation) so encourage children to turn the paper or object so they can see it in this way to make their judgments.
- The pitch for the angle work in this stage is simply classification, comparison to right angles and ordering. Degrees are not introduced until Stage 5 and so should be avoided here.
- Encourage children to see shapes as families rather than as individuals e.g. there is a polygon family, within which there is a triangle family, a quadrilateral family, a pentagon family and so on. These mini-families

Children do not always realise that shapes can belong to more than one classification e.g. a rectangle is a parallelogram, a quadrilateral and a polygon A rhombus provides a particular challenge as some children assume it is a square in a different orientation. Often due to overexposure, many children assume that all polygons are regular and find it very hard to visualise irregular pentagons, hexagons etc. Children find 3D shapes hard to visualise and hence to count the faces, edges and vertices accurately.	break down themselves e.g. quadrilaterals may be parallelograms (and then either rhombuses or rectangles or neither) and so on. • Focus on mathematical language so that children are using words like sides, vertices, symmetry, parallel, perpendicular, right angles and so on • Make links to word origins, root words and prefixes (quad = four, tri = three). The names of shapes sometimes relate to the side properties and sometimes ot the angle e.g. rectangle (right-angled) or hexagon (six-sided) or isosceles [equal legs] • Use lots of sorting activities practically initially to help develop the 'testing against a criterion' behaviour and the thoroughness needed • Bring in representations of sorting later e.g. caroll diagrams and venn diagrams.

Key Assessment Checklist

1. I can identify lines of symmetry in 2D shapes
2. I can identify and describe all possible lines of symmetry in a 2D shape (horizontal, vertical, diagonal)
3. I can identify and distinguish between acute, right and obtuse angles
4. I can order and compare angles up to two right angles
5. I can recognise and describe the properties of 'famous' quadrilaterals
6. I can recognise and describe the properties of 'famous' triangles
7. I can say what is the same and different about 2D shapes, sorting them by their properties.
8. I can sort 2D and 3D shapes by their properties using Venn diagrams.

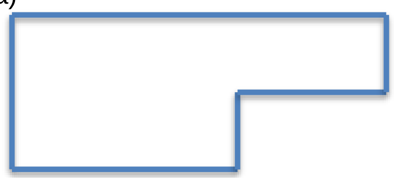
Year 4	Unit 6 : Reasoning with Measures
8 learning hours	This unit focuses on mensuration and particularly the concepts of perimeter, area and volume. Primary children are also working on money concepts at this stage, while older secondary students develop mensuration into volume and surface area of challenging shapes, applying Pythagoras' Theorem and trigonometry also in combination with these problems. Note the focus on reasoning within this unit: it is common for children to complete routine problems involving mensuration but this unit is about the developing a secure conceptual understanding of these ideas that they can apply to a wide range of problems and contexts. The opportunity to use and build on earlier number work is built into this unit and it is expected that children apply their arithmetic skills, for example, in these problems.

Prior Learning	Core Learning	Learning Leads to...
➢ measure the perimeter of simple 2-D shapes	➢ **measure and calculate the perimeter of a rectilinear figure (including squares) in centimetres and metres** ➢ **find the area of rectilinear shapes by counting squares**	➢ measure and calculate the perimeter of composite rectilinear shapes in centimetres and metres ➢ calculate and compare the area of rectangles (including squares), and including using standard units, square centimetres (cm^2) and square metres(m^2) and estimate the area of irregular shapes
➢ add and subtract amounts of money to give change, using both £ and p in practical contexts	➢ **estimate, compare and calculate different measures, including money in pounds and pence**	

Exemplification

1. Find the perimeter of:

a)

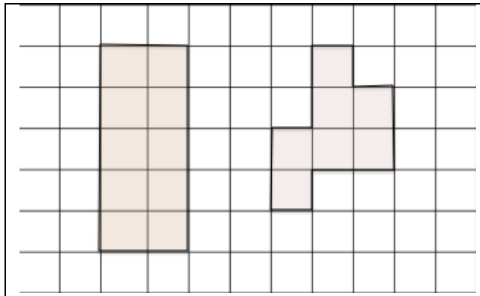


b) this rectangle (not to scale)

2. Find the area of these shapes:

Vocabulary

measure length width height calculate perimeter distance edge metre, m; centimetre, cm; (millimetre, mm) accurate approximate area	money pounds; pence; £; p price change total '2' of per better value

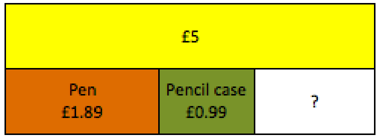
 3. a) Mo buys 2 birthday cards for 85p each and box of chocolates for £2.29. He pays using a £5 note. How much change will Mo receive? b) Emily needs to buy 2kg of pasta. She can buy a 2kg bag for £2.49 or she can buy several 500g bags for 59p each. What should Emily do? Explain your answer.	squares estimate	

Representation	Fluency	Probing Questions
Perimeter • Measuring the side lengths of a shape using rulers, tape measures, trundle wheels, metre rules etc. • Drawing over each square edge of a shape shown on a squared grid to count the number of square edges to find the perimeter. (Could do this using an acetate overlay) • Walking round the outside of a shape and chanting the lengths aloud before summing them to find the perimeter.	1. Find the perimeter of a shape by measuring • rectangle/square, lengths whole number of centimetres • rectangle/square, lengths whole number of metres • triangle, lengths whole number of centimetres/metres • other polygon, lengths whole number of centimetres/metres • rectangle/square, lengths whole number of millimetres	What's the same and what's different? Measure; Estimate; Calculate
	2. Find the perimeter of a shape by calculating • rectangle, shown on squared grid • square, shown on squared grid • rectangle, shown to scale (not on grid) • square, shown to scale (not on grid) • rectangle, not shown to scale, length and width given • square, not shown to scale, length given • rectangle, length and width described in words • square, length described in words	Show me two different rectangles with a perimeter of 18cm. Convince me that if you know the side lengths of a rectangles, you can work out its perimeter quickly without measuring. Convince me that you can find the width of a rectangle if you know its length and its perimeter

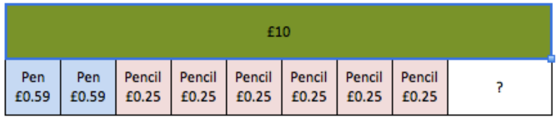
	3. Find a shape with a specified perimeter • any shape (polygon) with total side lengths as specified • square, whole number length • rectangle, one side given, whole number lengths • rectangle, no sides given, whole number lengths	Show me a shape with a perimeter of 16cm. Always, Sometimes, Never? The perimeter of a square is 4 times its length Always, Sometimes, Never? The area of a shape is an even number. Always, Sometimes, Never? The perimeter of a shape is the same as its area. Always, Sometimes, Never? Larger packs are better value than smaller packs
Area • Using acetate overlays showing square centimetres to count the squares inside a given shape. • Dividing a shape into square centimetres by drawing the on to help find its area. • Arranging a given number of square centimetres into a shape to find a shape with a given area. Do this with an array to form a rectangle.	4. Find the area of a rectilinear shape by counting squares • rectangle, shown on squared grid • square, shown on squared grid • composite rectilinear shape, on squared grid • rectangle, shown to scale (not on grid but can overlay grid) • square, shown to scale (not on grid but can overlay grid) • composite rectilinear shape, shown to scale (not on grid but can overlay grid) • rectangle, not shown to scale, length and width given – to be drawn on a grid • square, not shown to scale, length given – to be drawn on a grid • rectangle, length and width described in words - to be drawn on a grid • square, length described in words - to be drawn on a grid	Show me a shape with an area of 12 square cm Show me all the shapes you can find with an area of $5cm^2$ Convince me that the area of a 4cm by 5cm rectangle is $20cm^2$, regardless of how you count What's the same and what's different? A 6cm by 6cm square and a 7cm by 5cm rectangle. What's the same and what's different? Perimeter and Area Always, Sometimes, Never? A square has less area than a rectangle.

Money

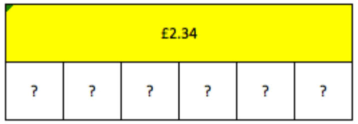
- Representing money problems using the bar model
  E.g.



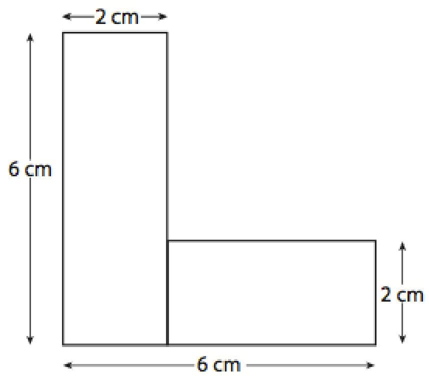
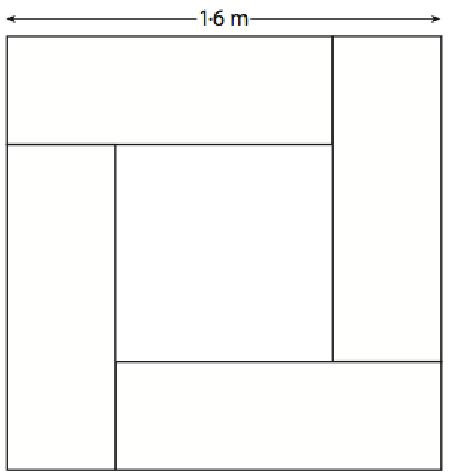
- Multiplication examples e.g. a pen costs 59p and a pencil costs 25p.
  How much change do you get from £10 if you buy 2 pens and 6 pencils?



- Division examples e.g. 6 pens cost £2.34. How much is each one?



5. Solve addition and subtraction money problems • find total price (all pence) • find total price (£ and pence) • find change given total price and money paid • find difference between two amounts • find change given individual item costs and money paid • find difference between two prices • say whether someone has sufficient money to purchase items listed	What's the same and what's different? Total; Sum; Cost; Change; Difference; Altogether; More Than
6. Solve money problems involving multiplication • find total cost involving multiple items of same price • find total cost involving multiple items of same price more than once • find change given cost of individual items, number of each item and money paid • find best value between smaller item and larger item by multiplying price of smaller item a sufficient number of times	Show me how you would calculate the total cost of three pens that cost £1.29 each
7. Solve money problems involving division • find cost per item given total and number of items • share a bill between people • find cost of a specific item given total cost, items purchased, costs of the others and number of each purchased • find cost per item given change provided, amount paid and number of items • find best value between smaller item and larger item by dividing price of larger item by an appropriate number	Show me how you can share £7.80 between three people evenly Show me the cost per book if 6 identical books cost £14.34 What's the same and what's different? Change from £5 buying 3 pens costing 89p each; Price per magazine if four magazines cost £9.32

Further Extension	Rich and Sophisticated Tasks
1. The shape below is made from two rectangles. Identify the perimeter of each of the two rectangles. How many 1 cm squares would fit into the smaller rectangle? How many more squares fit into the larger rectangle?  2. The rectangular tiles here are three times as long as they are wide. What is the perimeter of the centre square?  	Find area of rectilinear shapes by counting squares **NRICH: Torn Shapes * P I** **NRICH: Twice as Big? * P** Estimate, compare and calculate different measures, including money in pounds and pence **NRICH: Discuss and Choose * P**

3.
Sophie would like to build a rectangular patio in her garden. She wants the area of her patio to be $24m^2$.

What to do:

- Think about the possible sizes that Sophie's patio could be. Write these down.
- Draw some designs using these sizes.
- Draw these to a scale of 1cm = 1m.
- Use another piece of paper if you need more room.
- Measure accurately using your ruler. Label the measurements
- Once you have drawn your rectangles, check to make sure the areas are correct.
- Work out the perimeters of each shape

4.
Sam has been given a large area of land. He would like to build a stable for his horse on part of it. He wants it to be rectangular with a perimeter of 50m.

What to do:

- On paper work out some of the possible areas for Sam's stable. Write them down.
- On a piece of squared paper, sketch some designs using these sizes.
- Use the scale of 1cm = 1m. Remember to label them.
- Once you have drawn your rectangles, check to make sure the perimeters are correct.
- Work out the areas of each shape in the most efficient way you can.

5.
Which would you rather have, 3 × 50p coins or 7 × 20p coins?

Explain your reasoning.

Misconceptions	Teacher Guidance and Notes
When finding perimeters by counting squares, children often lose count and cannot remembered where they started from. Frequently, children count the squares around the edge of a shape, rather than the lengths and this causes them to miss out one length at each corner (because they see it as just one square, although it is in fact occupying two edges). When finding the perimeter of a rectangle, children may forget to include the length and width twice each, instead adding only the numbers provided on the diagram. When measuring, some children have difficulty in measuring objects longer than the ruler. They may also make an error by misaligning the end of the ruler to the end of the line, instead of aligning 0 to the end of the line. For many children, there is a lack of concept of area as number of unit squares needed to fill a space. They do not recognise that we choose the size of the squares to measure in, but that, when we have chosen, we need to stick to squares of that size and state this in our answer. This weak conceptual understanding can cause confusion between area and perimeter. When calculating costs, children may confuse when to add and subtract. If purchasing repeated items, children may also fail to realise that they can use multiplication to help them. Children who struggle to represent a problem visually are the most likely to make these errors of interchanging operations. Weak arithmetic including poor recall of times tables may mask issues around understanding of money. Specifically, some children may lack the sharing and grouping concepts of division and so struggle to solve money problems involving these skills.	• This unit has two foci: perimeter/area and money. • Measuring skills are extended here to develop accurate measurements and applied specifically to perimeter. Ensure that children can both measure a perimeter from an accurate drawing and calculate it from a sketch or description. • In Stage 4 we are completing only early work on area and hence considering only rectilinear shapes (i.e. shapes with all right-angles – rectangles and squares and their composites). Note that finding the perimeter of a composite rectilinear shape is an explicit objective of Stage 5, so we really are focused here on developing confidence in perimeters of rectangles and squares only. • Similarly, with area the focus is on the concept of area as the number of squares needed to fill the space. If possible try to consider squares other than solely square centimetres. Once again, we only consider rectilinear shapes, so triangles etc are beyond the specification of the stage. If appropriate, children can begin to explore more efficient ways of counting the squares (ie in rows or columns using counting in ns or multiplication) that will lead them towards the generalisation for the area of a rectangle. • Money work is now linking to multiplication and division and the application of these skills to a wider range of problems. Teachers will need to take into account children's mastery of number work and calculation particularly. It may be necessary to revisit division and/or multiplication with some students, not forgetting that it is important to link division with concrete activities, involving both interpretations: sharing and grouping.

Key Assessment Checklist
1. I can measure perimeters accurately in mm, cm or m and calculate a perimeter from given measurements. 2. I can solve problems involving the perimeter of squares and rectangles using mm, cm and m. 3. I can find the area of a square or rectangle by counting the cm squares it takes to fill the shape, and I can work out the area of a right angled triangle by treating it as half a rectangle 4. I can begin to explain why the area of a rectangle is length x width by referring to counting squares in rows or columns 5. I can solve simple money problems involving addition and subtraction in pounds and pence. 6. I can use multiplication to calculate the cost of buying several of the same thing and combine this with addition and subtraction to get total costs and change 7. I can use division to calculate shares of a bill or how many of the same thing can be bought for an amount 8. I can estimate the cost of several items or the number that can be bought with a given amount by rounding prices to easier amounts

<table>
<tr><th>Year 4</th><th colspan="2">Unit 7 : Discovering Equivalence</th></tr>
<tr><td>10 learning hours</td><td colspan="2">This unit explores the concepts of fractions, decimals and percentages as ways of representing non-whole quantities and proportions.
For the youngest children, the work is focused on fractions and developing security in recognising and naming them.
At KS2 this then builds to looking at families of fractions and decimals and percentages.
At secondary level this is extended to more complex % work and equivalence with recurring decimals and surds.</td></tr>
<tr><th>Prior Learning</th><th>Core Learning</th><th>Learning Leads to...</th></tr>
<tr><td>➢ compare and order unit fractions, and fractions with the same denominators;
➢ recognise and show, using diagrams, equivalent fractions with small denominators</td><td>➢ recognise and show, using diagrams, families of common equivalent fractions

➢ count up and down in tenths; recognise that tenths arise from dividing an object into 10 equal parts and in dividing one-digit numbers or quantities by 10
➢ count up and down in hundredths; recognise that hundredths arise when dividing an object by one hundred and dividing tenths by ten

➢ recognise and write decimal equivalents of any number of tenths or hundredths

➢ recognise and write decimal equivalents to 1/4,1/2, ¾

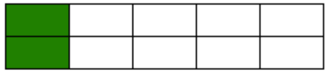
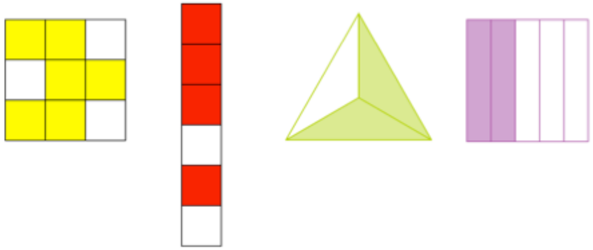
➢ round decimals with one decimal place to the nearest whole number

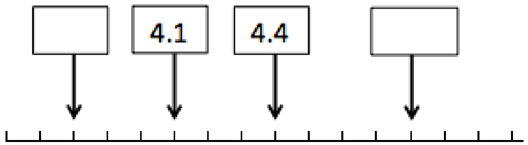
➢ compare numbers with the same number of decimal places up to two decimal places</td><td>➢ identify, name and write equivalent fractions of a given fraction, represented visually, including tenths and hundredths

➢ count up and down in hundredths; recognise that hundredths arise when dividing an object by one hundred and dividing tenths by ten
➢ read and write decimal numbers as fractions [for example, 0.71 = 71/100]

➢ recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents

➢ solve problems which require knowing percentage and decimal equivalents of 1/2, 1/4, 1/5, 2/5, 4/5 and those fractions with a denominator of a multiple of 10 or 25.</td></tr>
</table>

Exemplification	Vocabulary
1. a) What fraction does this diagram represent?  b) What equivalent fraction does this diagram represent?  c) Draw the next diagram to show the next equivalent fraction in the family d) Which of these diagrams is the odd one out? Explain your answer  2. Use the correct sign, > or <, to complete the □ in these number sentences a) 17.6 □ 16.7 b) 4.37 □ 4.73 c) 24.68 □ 24.8 3. a) Write 3 tenths as a decimal b) Write $\frac{47}{100}$ as a decimal b) 0.08 is the same as ...... tenths and .....hundredths 4. a) Write ¾ as a decimal b) 0.5 is equivalent to the fraction ...... 5. Look at this number line which is marked in tenths. Complete the missing numbers by counting up and down in tenths.	fraction numerator denominator part whole per family pattern equivalent decimal tenth hundredth place value decimal place decimal point round greater than less than



6. Round to the nearest whole number:
a) 13.8 b) 214.5 c) 1.4

<table>
<tr><th>Representation</th><th>Fluency</th><th>Probing Questions</th></tr>
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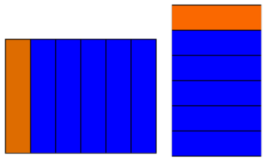
Fractions

- Folding (and colouring) paper circles to represent a unit (and then non-unit) fraction to compare two or more fractions (and hence order them).
- Similarly using these or pre-made versions (e.g. magnetic pieces) to discover equivalent fractions
- Folding (and colouring) paper strips to represent a unit (and then non-unit) fractions to compare two or more fractions (and hence order them)
- Representing fractions using the bar model (vertically and horizontally) e.g. 1/6



- Producing own fraction wall or fraction-fan to help identify equivalent fractions

- Positioning fractions on a number line (washing line) including beyond 1. Linking the number line to the bar model from 0-1 etc.
- Folding paper strips vertically (rectangles) to represent a fraction and then folding horizontally to discover

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1. Recognise and represent fractions
   - recognise and name a unit fraction from a representation
   - recognise and name a proper fraction from a representation
   - recognise and name an improper fraction from a representation of greater than one whole
   - produce a diagram to represent a unit fraction
   - produce a diagram to represent a proper fraction
   - produce a diagram to represent an improper fraction
   - say whether or not a given fraction has been correctly chosen to represent a diagram

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Show me where 1/10 sits on the number line

Show me what comes next 7/10, 8/10, 9/10, ...

Show me how you can show 3/10 of this shape? of this number? on the number line? as a decimal?

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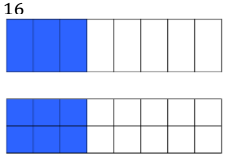
2. Produce equivalent fractions
   - find equivalent fractions to a unit fraction by splitting up a diagram into more parts e.g. $\frac{3}{8}$ and $\frac{6}{16}$



   - find equivalent fractions to a proper fraction by splitting up a diagram into more parts
   - produce a sequence of equivalent fraction diagrams for a unit fraction
   - produce a sequence of equivalent fraction diagrams for a proper fraction

</td>
<td>

Show me an equivalent fraction to 2/10

Show me what comes next in this pattern: 3/10, 6/20, 9/30, 12/40, ...

Always, Sometimes, Never? Equivalent fractions form a number pattern when you write them in a list

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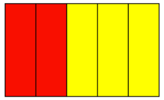
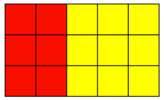
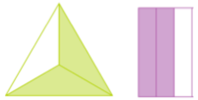
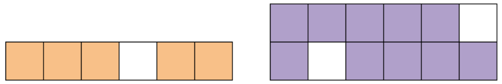
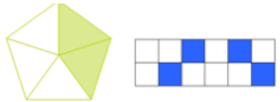
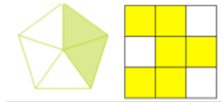
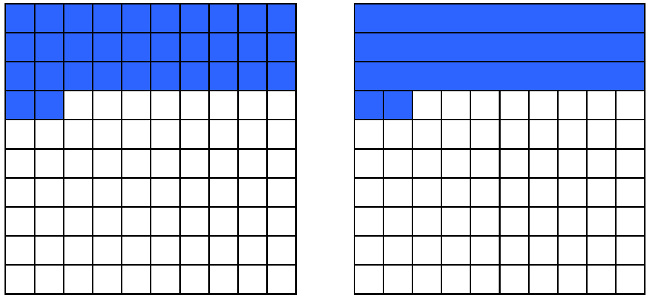
</table>

equivalent fractions and the proportional link between numerators and denominators For example, for $\frac{2}{5}$ is equivalent to $\frac{6}{15}$   • Splitting the same shaded diagram up in multiple ways to show that the overall fraction shaded does not change • NRICH: Matching Fractions (Pelmanism) http://nrich.maths.org/8283/note • Fractions ITP (Nat Strat) http://www.taw.org.uk/lic/itp/fractions.html • Fraction manipulatives - exploring equivalence http://donnayoung.org/math/fraction.htm • Fraction models and support questions - http://www.annery-kiln.eu/gaps-misconceptions/all-images.html	3. Recognise equivalent fractions • identify equivalent fractions from diagrams with the same number of parts in the whole  • identify equivalent fractions from diagrams when parts need to be combined but the structure is the same  • identify equivalent fractions from diagrams when parts need to be combined and the structure is different  • say whether two fractions shown in diagrams are equivalent or not • complete a diagram to make two fractions equivalent	Convince me that $\frac{8}{12}$ is equivalent to $\frac{2}{3}$ Convince me that the fractions shaded are not equivalent  Convince me that 20/100 is equivalent to two tenths (in more than one way!) What's the same and what's different? 
Decimals • Counting or chanting to represent the tenths supported by both a number line as well as a proportion (e.g. circles cut into ten equal pieces). • Using a counting stick to consider what comes next when counting up or down in tenths (or any fraction e.g sevenths) • Building numbers from place value counters • Using overlapping partitioning cards to construct and deconstruct numbers • Comparing two numbers by constructing, partitioning and analysing place by place. • Representing decimals using tenth strips and hundredth squares to show why, for example, 32 hundredths is the same as 3 tenths and 2 hundredths. See electronic resource using this representation	4. Represent and read decimals up to 1 decimal place • read a decimal <0 with 1 decimal place as a number of tenths • interpret a diagram showing tenths as a decimal or fraction • write a decimal<0 with one decimal place as a fraction with denominator 10 • write a fraction with denominator 10 as a decimal • know that 3 tenths, for example, comes from splitting 3 into ten equal parts • count up in tenths from any number of tenth, reading each multiple of ten tenths as a whole number e.g. *twenty-eight tenths, twenty-nine tenths, thirty tenths or three* • count down in tenths from any number of tenths, reading each multiple of ten tenths as a whole	Show me a decimal that would make this statement true 5.6 < ......

 • Exploring the position of numbers on a scale (e.g. on geogebra) • Develop sense of small (decimal) numbers using paper strips and paperclips to position e.g. strip represents 0-1, where is 0.3? 0.03? 0.13? 0.33? What if strip is now 0-0.1?	number e.g. *twenty-two tenths, twenty-one tenths, twenty tenths or two* • read and write decimals >1 with one decimal place as an improper fraction	
	5. Represent and read decimals up to 2 decimal places • read a decimal <0 with 2 decimal places as a number of hundredths • interpret a diagram showing hundredths as a decimal or fraction • write a decimal <0 with 2 decimal places as a fraction with denominator 100 • write a fraction with denominator 100 as a decimal with 2 decimal places • know that 10 hundredths are equivalent to 1 tenth	Always, Sometimes, Never? When you write a fraction with a denominator of 100 as a decimal, the decimal will have two decimal places Always, Sometimes, Never? If you have two decimals, the longer decimal will be worth more than the shorter decimal
	6. Recall and use equivalences between fractions and decimals • know and show that 0.5 is equivalent to ½ • know and show that 0.25 is equivalent to ¼ • know and show that 0.75 is equivalent to ¾ • know that 0.1 is equivalent to 1/10 • solve simple problems involving these equivalences	Convince me that 1/4 = 0.25 What's the same and what's different? 7/10, 0.7, 70/100, 14/20 What's the same and what's different? 1/4, 1/2, 0.5, 0.25, 3/4, 2/4, 0.75 What's the same and what's different? tenth, 1/10, 0.1, ÷ 10, 10/100
	7. Compare and order decimals • decimals <0, 1 decimal place • decimals >0, 1 decimal place • decimals < 0, 2 decimal places • decimals < 0, 2 decimal places • decimals < 0, 1 or 2 decimal places (mixed) • decimals >0, 1 or 2 decimal places (mixed)	Convince me that 0.8 > 0.59

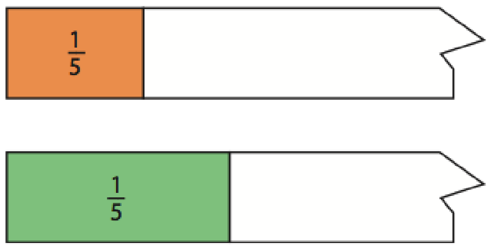
Rounding • Positioning number on marked (and then unmarked) number line to identify neighbouring rounding options • Using number line to investigate when a number is closer to the lower end than the upper end	8. Round a decimal with 1 decimal place to the nearest whole number • decimals <0 to either 0 or 1 • know that 0.5 rounds up to 1 • decimals >0	What's the same and what's different? 2.7, 3.4, 2.5, 3.9

Further Extension

1.
How many ways can you express $\frac{2}{8}$ as a fraction?

2.
Two paper strips are ripped. Identify which original paper strip is longer.

Explain your answer.

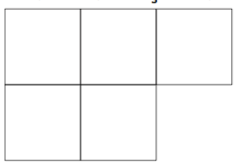


3.
8 girls share 6 bars of chocolate equally.
12 boys share 9 bars of chocolate equally.

Clare says each girl got more to eat as there were fewer of them.
Rob says each boy got more to eat as they had more chocolate to share.

Explain why Clare and Rob are both wrong.

4.
If the picture represents $\frac{1}{3}$ of a shape, draw the whole shape.



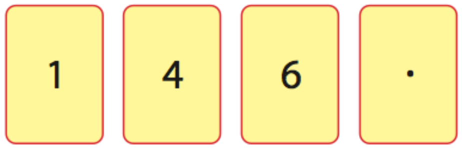
Rich and Sophisticated Tasks

Recognise and show, using diagrams, families of common equivalent fractions
NRICH: Fractional Triangles * P
NRICH: Bryony's Triangle * P
NRICH: Fair Feast * P

Round decimals with one decimal place to the nearest whole number
NRICH: Round the Dice Decimals 1 * P I

5.

Using these cards can you make a number between 4·1 and 4·61?



What is the smallest number you can make using all four cards?
What is the largest number you can make using all four cards?

Misconceptions

Some children struggle to read a fraction from a diagram. This is usually because they do not understand how the whole and the shaded parts relate to each other. Some children do not fully recognise that the parts of the whole must be of equal size. They also do not see the denominator as an indicator of the number of parts in the whole and use it directly to order fractions, believing that fractions with a larger denominator are bigger.

Additionally, some children do not realise that for fractions to be equivalent the proportions of shaded parts must be the same.

At this stage, many children find it hard to recognise equivalent fractions by the numbers themselves (although some will see a pattern) and so need a visual representation to spot equivalent fractions.

Pupils may confuse 'tens' and 'tenths' and similarly 'hundreds' and 'hundredths'. Occasionally children may expect the first place value column after the decimal point to be called the ‘unitths’ or the ‘oneths’ rather than the tenths.

When counting in tenths, some children may find it hard to use the whole numbers when a multiple of ten is reached because they do not recognise that 10 tenths makes a whole.

Children read decimals incorrectly saying 'three point forty-two' instead of 'three point four two'.

When ordering children think that 'longer' decimals are larger e.g. they presume that 3.14 > 3.4

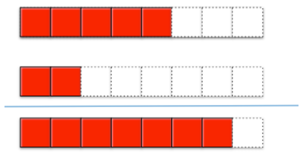
Teacher Guidance and Notes

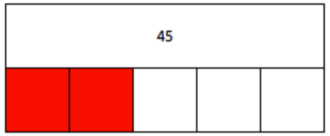
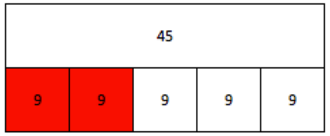
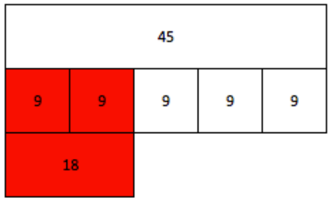
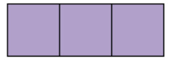
- As with all fraction units in all stages, it is essential that children understand the role played by the numerator and denominator in a fraction. Specifically, that the denominator tells us the number of parts in the whole and the numerator tells us the number of parts that we are working with. Strongly model the language of part and whole throughout in order to embed these concepts.
- The expectation in Stage 4 is that children are still using diagrams to represent and work with fractions. They are not expected to work with equivalent fractions in numeric form only.
- The fourth small step requires children to just 'know' the equivalences for common fractions - focus on speed recall here. In Stage 5 and 6 children explore the process of division within a fraction to arrive at the decimal equivalents of common fractions.
- There is a clear link to money with decimals with two decimal places and this can be exploited to help children grasp the concepts of ordering and rounding.
- Try to use the language of place value with decimals as well as with integers e.g. 4 tenths and 2 hundredths OR 42 hundredths and to use the usual apparatus to represent these numbers in different ways e.g. place value counters

Key Assessment Checklist
1. I can recognise representations of equivalent fractions
2. I can draw diagrams to show equivalent fractions
3. I can count up and down in tenths
4. I can group sets of objects into tenths by splitting them into 10 equal groups
5. I can write tenths as decimals by using their place value headings; I can write hundredths as decimals by using their place value headings
6. I can write the decimal equivalents of 1/4, 1/2 and ¾
7. I can round decimals with one decimal place to the nearest whole number
8. I can order and compare numbers with up to two decimal places, using the signs <, > (and =) to show this comparison.

Year 4	Unit 8 : Reasoning with Fractions	
8 learning hours	This unit progresses from the development of the understanding of non-whole items at the lowest end to flexibility and fluency with calculations involving fractions for older primary students. This knowledge is then applied within the secondary curriculum to the topic of probability, thus providing a clear context in which the skills of adding and multiplying fractions particularly are needed. It is critical that pupils develop confidence and security in understanding and manipulating fractions as well as flexibility in representing a number as a fraction or as a decimal, percentage, diagram etc. Note that once fraction calculations are mastered here, they should be used in following units as examples just as other numbers are in order to keep the skills fresh.	
Prior Learning	**Core Learning**	**Learning Leads to...**
➢ add and subtract fractions with the same denominator within one whole [for example, 5/7 + 1/7 = 6/7]	➢ **add and subtract fractions with the same denominator** ➢ **solve problems involving increasingly harder fractions to calculate quantities, and fractions to divide quantities, including non-unit fractions where the answer is a whole number** ➢ **solve simple measure and money problems involving fractions and decimals to two decimal places**	➢ add and subtract fractions with the same denominator and denominators that are multiples of the same number ➢ multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams

Exemplification	Vocabulary	
1. Calculate a) $\frac{3}{8}+\frac{2}{8}$ b) $\frac{7}{10}-\frac{3}{10}$ c) $\frac{5}{6}+\frac{5}{6}$ d) $\frac{11}{4}-\frac{5}{4}$ 2. a) Calculate $\frac{1}{5}$ of 35 b) Calculate $\frac{2}{3}$ of 33 3. 1 metre of fabric costs £2.40. AJ needs one piece of fabric of 0.35 m and one piece of fabric of 0.4m How much will AJ's fabric cost?	fraction numerator denominator part whole sum difference	equivalent fractions fraction of .... divide share unit fraction proportion tenth hundredth

Representation	Fluency	Probing Questions
Adding and Subtracting Fractions • Using the bar model to add and subtract fractions with the same denominator $\frac{5}{8}+\frac{2}{8}$  or $\frac{7}{8}$ in total	1. Add proper fractions with the same denominator • add two unit fractions with the same denominator e.g. $\frac{1}{3}+\frac{1}{3}$ • add two proper fractions with the same denominator e.g. $\frac{2}{9}+\frac{3}{9}$ • add two proper fractions with the same denominator, simplifying the answer e.g. $\frac{5}{8}+\frac{1}{8}$ • add two proper fractions with the same denominator, crossing the next whole to give an improper fraction $\frac{5}{8}+\frac{7}{8}$	Show me how you can use a bar model to add 3/8 to 7/8 Show me two fractions with a sum of 5/7 Convince me that 1/7 + 5/7 = 6/7
	2. Subtract proper fractions with the same denominator • subtract a unit fraction from a proper fraction with the same denominator e.g. $\frac{6}{7}-\frac{1}{7}$ • subtract two proper fractions with the same denominator e.g. $\frac{6}{7}-\frac{4}{7}$ • subtract two proper fractions with the same denominator, simplifying the answer e.g. $\frac{5}{6}-\frac{1}{6}$ • subtract a proper fraction from an improper fraction e.g. $\frac{13}{8}-\frac{3}{8}$ • subtract a proper fraction from an improper fraction, crossing the next whole e.g. $\frac{7}{5}-\frac{3}{5}$	Show me how you could use a bar model to subtract 3/8 from 7/8 Show me two fractions with a difference of 5/6
Word Problems • Use the bar model to represent the word problem visually	3. Solve problems involving adding and subtracting fractions • word problems - addition • word problems – subtraction • word problems – combinations • missing number problems (using inverse operations)	Always, Sometimes, Never? When adding or subtracting fractions you need to add both the denominator and the numerator Always, Sometimes, Never? When adding or subtracting fractions the denominators always need to be the same

Finding Fractions of an Amount • Drawing the bar model to represent the problem For example, to find $\frac{2}{5}$ of 45 - Represent $\frac{2}{5}$  - Then show this as equal to 45  - Then share 45 between each of the 5 pieces (i.e. 45 ÷ 5)  - Then find the total value of the shaded section, that is 18. 	4. Calculate a fraction of an amount • unit fraction • non-unit fraction, two-..... ths • non-unit fraction, three+-....ths • improper fraction	Show me how to find 1/6 of £42 Show me what is wrong in this calculation... 2/3 of 36 is 6 because $36 \div 2 = 18$ and $18 \div 3 = 6$
	5. Given the value of a fraction of the amount, calculate the original amount • unit fraction (by multiplying) • non-unit fraction (by dividing and then multiplying) • improper fraction	Show me the whole if this is 1/3 
	6. Solve problems involving calculation of fractions of amounts • represent a problem visually • word problems e.g. numbers of people • word problems involving measurements • word problems involving money • combinations of problems e.g. amount remaining after two fractions removed	Convince me that 2/3 of 24 is 16 Convince me that 1/4 of 30 metres is 7.5m Always, Sometimes, Never? You find a tenth of a number by removing its final zero
Equivalence • Exploring fractions with a denominator of 10/100 to find equivalences as well as thinking of how fractions can be turned into equivalent fractions with a denominator of 10/100	7. Solve problems involving combinations of fractions and decimals to 2dp • solve comparison word problems between decimals and fractions • se equivalences to make calculations easier e.g. 0.25 = ¼	Convince me that finding 1/10 of a quantity is the same as dividing by 10

Further Extension	Rich and Sophisticated Tasks
1. True or false? $\frac{1}{5}+\frac{2}{5}=\frac{3}{5}$ $\frac{1}{5}+\frac{2}{5}=\frac{3}{10}$ $\frac{1}{5}+\frac{2}{5}=\frac{6}{10}$ Explain your reasoning. 2. Peter wrote down two fractions. He subtracted the smaller fraction from the larger and got $\frac{1}{8}$ as the answer. Write down two fractions that Peter could have subtracted. Can you find another pair? 3. Insert the symbol >, < or = to make each statement correct. $\frac{2}{5}$ of 5 ◯ $\frac{1}{4}$ of 4 $\frac{1}{7}$ of 7 ◯ $\frac{2}{7}$ of 14 $\frac{2}{3}$ of 9 ◯ $\frac{1}{3}$ of 18 Make up three similar statements using >, < or =. 4. Captain Conjecture says, 'To find a tenth of a number I divide by 10 and to find a fifth of a number I divide by 5.' Do you agree? Explain your reasoning. 	Solve problems involving increasingly harder fractions to calculate quantities, and fractions to divide quantities, including non-unit fractions where the answer is a whole number **NRICH: Andy's Marbles ** P** **NRICH: Fractions in a Box ** P-** **NRICH: Chocolate ** P I**

Misconceptions	Teacher Guidance and Notes
When adding or subtracting fractions children may add (or subtract) both the numerators and denominators. This is because they do not recognise that the denominator indicates the number of parts of the whole and so treat the fractions as 4 'whole numbers' to be added together. Some children struggle to see the link between proper fractions and the unit fractions of which they are multiples. Therefore, they find it difficult to calculate fractions of amounts such as 2/3 because they do not realise this is simply double 1/3	• This unit applies the work of Unit 7 in representing fractions to the calculation process when adding, subtracting and finding fractions of an amount • Children may still need further development of their skills in representing a fraction in multiple ways so that they can then combine these to calculate. • It is strongly recommended that a school adopt a consistent approach to representing fractions using the (vertical) bar model, which can then be supplemented by additional representations as appropriate. • As previously, ensure you model the use of language such as denominator and numerator and part and whole as much as possible to secure these concepts • Make connections with other areas of maths where fractions are used for example when describing turns, calculating measures for recipes, calculating journey times and fuel consumption, working out results of sales offers with money and comparing prices.

Key Assessment Checklist

1. I can add fractions with the same denominator.
2. I can subtract fractions with the same denominator.
3. I can solve problems involving fractions to calculate quantities where the answer is a whole number
4. I can use my knowledge of fractions to divide quantities to solve problems involving whole numbers
5. I can solve problems involving non-unit fractions to calculate quantities, where the answer is a whole number
6. I can use my knowledge of non unit fractions to divide quantities to solve problems involving whole numbers
7. I can solve measure problems involving fractions and decimals to 2 decimal places
8. I can solve money problems involving fractions and decimals to 2 decimal places

Year 4	Unit 9 : Solving Number Problems	
12 learning hours	This unit continues pupils' earlier study of arithmetic (and algebra for secondary students). At Key Stage 1 children are working on multiplication (and division in Stage 2) as a way to represent repeated addition and scaling (and repeated subtraction – grouping - and sharing) At Key Stage 2 children are developing skills in applying their arithmetic to more complex problems. At secondary level and in Stage 6, students begin to find unknown values by applying inverse operations. Equations of all types including quadratic and simultaneous are covered in later stages.	
Prior Learning	**Core Learning**	**Learning Leads to...**
➢ write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods ➢ solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects	➢ divide a two-digit or three-digit number by one digit number ➢ **find the effect of dividing a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as ones, tenths and hundredths** ➢ **solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects** ➢ **solve simple measure and money problems involving fractions and decimals to two decimal places**	➢ multiply and divide whole numbers and those involving decimals by 10, 100 and 1000 ➢ multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers ➢ divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context ➢ solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes ➢ solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign ➢ solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates ➢ solve problems involving number up to three decimal places

Exemplification

1. Calculate using a written method
a) $98 \div 7$
b) $384 \div 6$
c) $87 \div 5$

2. a) Find the value of Δ in each of these statements and write your answer in the box

	1s		0.1s
$4 \div 10 = \Delta$		.	
$78 \div 10 = \Delta$		.	

b) Find the value of Δ in each of these statements and write your answer in the box

	1s		0.1s	0.01s
$82 \div 10 = \Delta$		.		
$912 \div 100 = \Delta$		.		

3.
a) Three children calculated 8 x 9 in different ways:

Amy $8 \times 9 = 8 \times 8 + \blacksquare = \blacksquare$	Bob $8 \times 9 = 9 \times 9 - \blacksquare = \blacksquare$	Chloe used the commutative law: $8 \times 9 = \blacksquare \times \blacksquare = \blacksquare$

Complete the calculations to fill in the values of the missing numbers marked ■

b) Tom ate 9 grapes at the picnic. Sam ate 3 times as many grapes as Tom.
How many grapes did they eat altogether?

4.
a) An empty box weights 0.5 kg. Ivy puts 10 toy bricks inside it and the box now weighs 2kg. How much does each brick weigh?
b) Which would you rather have? Three 50p coins or seven 20p coins? Explain your answer.

Vocabulary

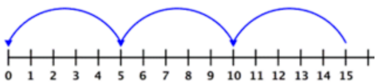
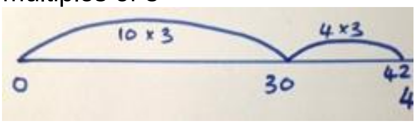
division
divide
divided by
divisor
dividend
quotient
remainder
shared between
grouping and sharing

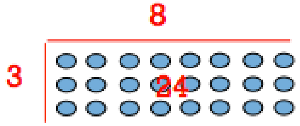
place value: 100s,10s, 1s, 0.1s, 0.01s
hundreds, tens, ones, tenths, hundredths
exchange

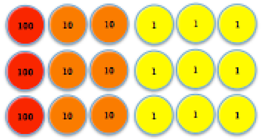
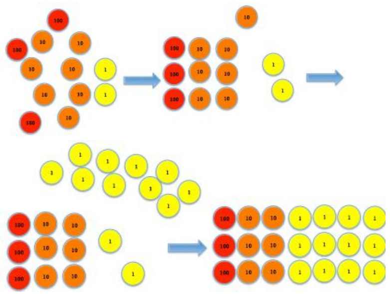
partition
distribute
recombine
digits

commutative (law)
distributive (law)

solve
problem
represent
array
grid
scaling
bar model

Representation	Fluency	Probing Questions
Multiplying • Using arrays, then grid representations as in Unit 5.	1. Recap: multiply a 2-digit or 3-digit number by a single digit • no exchange e.g. 132×3 • exchange from 1s to 10s e.g. 231×3 • exchange from 10s to 100s e.g. 271×3 • exchange from 100s to 1000s e.g. 812×4 • multiple exchanges e.g. 562×7	Show me how you could represent 73 x 6 using an array? a grid method? two calculations? What's the same and what's different? ... grid, array, partitioned calculation, column method, bar model
Dividing (→ Mental Methods) • Using a bead string/Numicon for grouping/repeated subtraction  (24 beads then grouped into 3s produces 8 groups) • Using a number line to show repeated subtraction to see how many groups fit inside e.g. 15 ÷ 5  • Using a number line to show partitioned grouping. For example, $42 \div 3$ can be found by considering known multiples of 3  • Partitioning a larger number to divide each part and then recombine For example: 72 ÷ 3 60 ÷ 3 + 12 ÷ 3 20 + 4	2. Divide a 2-digit or 3-digit number by a 1-digit number mentally (with jottings) • within times table e.g. $72 \div 8$ • beyond times table but each digit a multiple of the divisor e.g. $96 \div 3$ • divide a multiple of 10 by a single digit e.g. $80 \div 4$ or $180 \div 3$ • divide a 2-digit number by 1-digit number using partitioning. e.g. $72 \div 3$ by partitioning 72 into 60 and 12 • divide a 3-digit number by a 1-digit number using simple partitioning e.g. $327 \div 3$ by partitioning 327 into 300 and 27 • divide a 3-digit number by a 1-digit number using repeated partitioning e.g. $357 \div 3$ by partitioning 357 into 300 and 57 and then into 300, 30 and 27 • ext: divide a 3-digit number by a 1-digit number using repeated, more challenging partitioning e.g. $756 \div 6$ by partitioning 756 first into 600 and 156 and then into 600, 120 and 36	What's the same and what's different? divisor; dividend; quotient; remainder
	3. Use the distributive law • to partition a multiplication calculation into two (or more) calculations • to partition a division calculation into two (or more) calculations • to simplify a partitioned multiplication calculation e.g. $23 \times 7 + 17 \times 7$ which can be recombined to give 40×7 • to simplify a partitioned division calculation	What's the same and what's different? ... 40x7 + 2x7, 47x2, 42 x 7 and 40x2+7x2 Show me the single calculation that is equivalent to 20x4 + 5x4

<table>
<tr>
<td rowspan="2">Dividing (→ Written Methods)
• For a calculation p ÷ q, grouping a set of p counters into groups of size q, arranging these groups as an array. For example, for 24 ÷ 3, count out 24 counters and arrange in columns of 3.... then read off the answer of 8 as the number of columns


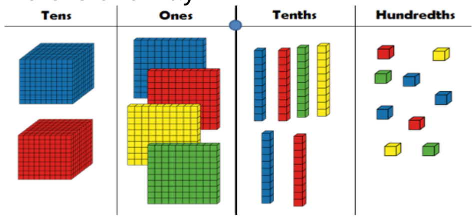
• Building a number using place value counters and grouping them into groups that are the size of the divisor, before arranging these groups as an array to explore the partitioning approach. For example, 369 ÷ 3

• Repeating the above, but exchanging remaining counters for 10 counters of the next size down before continuing to group. For example, 372 ÷ 3

• Recording these methods to arrive at compact division</td>
<td>4. Divide 2-digit number by a 1-digit number using a written method
• no exchange necessary e.g. $84 \div 4$
• one exchange from tens to ones e.g. $72 \div 3$
• example with a remainder e.g. $87 \div 6$</td>
<td>Show me a division with a remainder
Show me a division without a remainder</td>
</tr>
<tr>
<td>5. Divide a 3-digit number by a 1-digit number using a written method
• no exchange necessary e.g. $848 \div 4$
• situation where hundreds digit is less than divisor e.g. $355 \div 5$
• example with a remainder e.g. $756 \div 5$
• one exchange from hundreds to tens e.g. $805 \div 5$
• one exchange from tens to ones e.g. $642 \div 3$
• two exchanges e.g. $714 \div 6$</td>
<td>Show me how you divide 684 ÷ 4 using place value counters? using a written method? using a mental method?

Show me two numbers that are easy/hard to divide

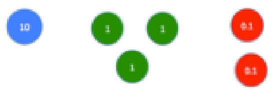
$125 \div 5, 98 \div 4, 145 \div 9, 126 \div 6$</td>
</tr>
</table>

Dividing by 10 and 100

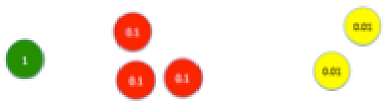
- Using base 10 to represent decimals
  Here is one way



- Using a hundred-grid to show why 2 tenths is the same as 20 hundredths etc
- Using place value counters to represent decimals (you can use unlabelled counters and given children a key)
  For example, here is 13.2



- Exchanging each counter for one that is ten times smaller. For example to calculate 13.2 ÷ 10,
  the 10 becomes a 1
  the 1s become 0.1s
  the 0.1s become 0.01s



  which makes 1.32
- Using a place value grid to help automate this process by imagining the above to arrive at the shortcut of making the 1s into 0.1s and so on

	1s ones		0.1s tenths	0.01s hundredths
43 ÷ 10 = Δ		.		
728 ÷ 100 = Δ		.		

6. Divide a 1-digit or 2-digit number by 10
   - multiple of 10 divided by 10
   - 2-digit number divided by 10
   - 1-digit number divided by 10

Convince me that 65 ÷ 10 = 6.5

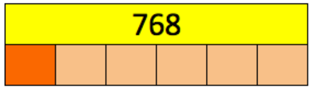
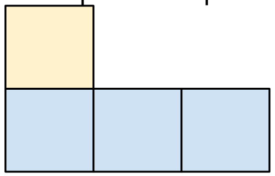
Always, Sometimes, Never?
When you divide a number by 10, you remove one zero from the end

7. Divide a 1-digit or 2-digit number by 100
   - multiple of 100 divided by 100 (even though this is 3-digits)
   - 2-digit number divided by 100 e.g. 87
   - multiple of 10 divided by 100
   - 1-digit number divided by 100

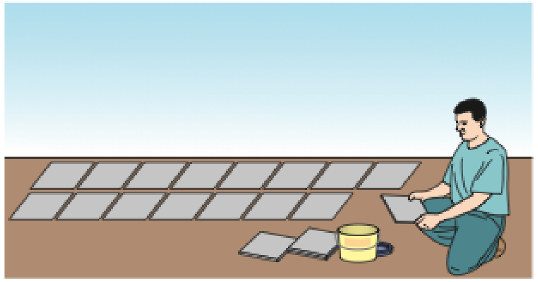
Show me
... 24 ÷ 10
... 24 ÷ 100
... 124 ÷ 10
... 124 ÷ 100
... 240 ÷ 10
... 240 ÷ 100

Convince me that 230 ÷ 100 = 2.3

Always, Sometimes, Never?
When you divide a number by 100, you will end up with a number with hundredths in

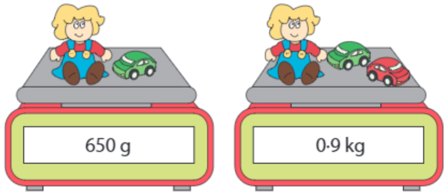
Representing and Solving Problems: • Using the bar model to represent a word problem. For example, 768 shared between 6  • Using the bar model to represent and solve correspondence problems 	8. Recognise and solve a simple division problem • word problem – sharing language e.g. 684g flour to make 6 cupcakes. How much flour is in each cupcake? • word problem – grouping language e.g. 825 people enter a quiz. There are 5 people in each team. How many teams will there be? • remainder problems – an account has £92 in it. If you spend £5 per day, after how many days will the money run out? • finding unit fractions of an amount e.g. find an sixth of 96	Convince me that I will need 8 minibuses to take 136 children on a trip using minibuses that seat 17 children each. Convince me that 7 is a factor of 917
	9. Solve missing number problems involving multiplication and division • find missing answer e.g. $78 \div 3 = \blacksquare$ or $24 \times 6 = \blacksquare$ • know that multiplication and division are inverses of each other • find a missing number from a multiplication by dividing (by a single digit) e.g. $6 \times \blacksquare = 318$ or $\blacksquare \times 7 = 217$ • find a missing dividend by multiplying by the (single-digit) divisor e.g. $\blacksquare \div 4 = 62$	Show me how you can represent this problem: Jodie has 8 crates containing 24 bottles. How many bottles does she have in total? What's the same and what's different? ... the number that is 4 times bigger than 23, 23 x 4, 4 lots of 23, the product of 4 and 23
	10. Solve correspondence and scaling problems • Find an amount ... times as large/long/heavy as a given amount • Find the starting amount given the final amount and the knowledge that it is .... times as large/long/heavy as the original • Given the value of x items, find the value of 1 item and hence the value of y items (by dividing and then multiplying) • Given the value of x items, find the value of a multiple of x items directly by multiplying. e.g. 6 cakes weigh 84g. How much do 30 cakes weigh?	Convince me that if I know the cost of 3 items, I can find the cost of 48 items by doing a multiplication

	11. Solve measure and money problems involving multiplication and division • Given cost of one item, find cost of multiple items • Given length/weight/capacity of one item, find length/weight/capacity of multiple items • Given total cost, find cost of one item • Given total length/weight/capacity, find length/weight/capacity of one item • Work in mixed units on the above	Always, Sometimes, Never? A calculation involving division will have a remainder
	12. Solve measure and money problems involving fractions and decimals • Find a fraction of an amount of money or a length/weight/capacity • Given lengths, weights or capacities with decimals e.g. 2.25m, find sums, difference, products and quotients.	Always, Sometimes, Never? Division is the inverse of multiplication

Further Extension	Rich and Sophisticated Tasks
1.  Roger has 96 patio slabs. Using all of the slabs find three different ways that he can arrange the slabs to form a rectangular patio. 2. Multiply a number by itself and then make one factor one more and the other one less. What happens to the product? E.g. $4 \times 4 = 16$ $6 \times 6 = 36$ $5 \times 3 = 15$ $7 \times 5 = 35$ What do you notice? Will this always happen?	Recall multiplication and division facts for multiplication tables up to 12x12 **NRICH: Multiplication Square Jigsaw * G P** **NRICH: Shape Times Shape * P** **NRICH: Table Patterns Go Wild! ** I** **NRICH: Let Us Divide! * P** **NRICH: Carrying Cards * P** **NRICH: Light the Lights Again * G P** **NRICH: Multiples Grid * I** **NRICH: Zios and Zepts * P** **NRICH: Times Tables Shifts * G P** Solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects **NRICH: A Square of Numbers * G P** **NRICH: What do you Need? * P** **NRICH: This Pied Piper of Hamelin ** P** **NRICH: Follow the Numbers * P I** **NRICH: What's in the Box? * P** **NRICH: How Do You Do It? * P** **NRICH: Ip Dip * I**

3.

How much does the car weigh in grams?
How much does the doll weigh in grams?



4.

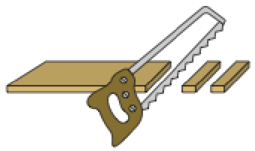
Sid and Sam share some money. Sid gets twice as much as Sam. Tick the coins which Sid might take.

Is there more than one way of sharing the coins?

5.

A 50 cm length of wood is cut into 4 cm pieces.

How many 4 cm pieces are cut and how much wood is left over?



NRICH: Journeys in Numberland * I

Misconceptions

Children are generally confident using the distributive law to partition a multiplication calculation although they may be less experienced with division. However, they often do not realise they can use it to recombine a calculation too.

Exchanging causes an issue for some children when using formal division methods – they may forget to carry over any remainder or forget what the remainder actually is.

Some children struggle when the first digit of the dividend is less than the divisor because they don't see how to exchange it all (or carry the whole thing over to the next column). They may carry the divisor over, rather than the first digit of the dividend.

Teacher Guidance and Notes

- This unit is focused on the remaining elements of four operations not already explored this year. Note that the national curriculum for Year 4 is relatively light in coverage of division and hence it has been included additionally here to ensure there is a smooth bridge between the division of 2-digit numbers by 1-digit numbers in Stage 3 to the division of 4-digit numbers by 1-digit numbers in Stage 5.
- There is also the opportunity to relate multiplication and division to each other and to use them when solving more complex problems including with measures and decimals.
- By this stage children should be working on or confident with ALL times tables - therefore, when solidifying multiplication processes, ensure they encounter numbers from across these times tables. Refer to the calculation policy for more detail on the progression of these concepts.

In division, children get confused when there is a remainder within the calculation and may forget to use it or may put the remainder itself as the answer. Children can sometimes think that dividing by 10 means taking the zero off the end and multiplying by 10 means adding it. They do not always relate multiplying and dividing to the place value and unitisation of a number e.g. 24 x 10 is 2 tens and 4 ones multiplied by 10 which will be 2 hundreds and 4 tens or 240. This can lead to errors where a decimal point is needed and not used or vice versa. Some children still experience confusion over tenths and tens, hundreds and hundredths - they may not correctly label columns as a sign of this. When carrying out more complex multiplications, some children will fail to realise that multiplication is commutative and struggle to use the times tables that they know to tackle a related question. Children may struggle to represent scaling and correspondence problems visually (because they don't conform to the 'lots of' imagery that some children focus on for multiplication) Children find it hard to separate how you can 'make' a number by both ADDING and MULTIPLYING - they may lean towards additive relationships more than multiplicative e.g. they may not have understanding of how 24 can be made of 10 and 14 as well as 20 and 4 (and other examples).	• If children are not yet confident, you may wish to devote some additional time to this aspect within this unit, although it is not directly listed as a fluency step. • It is recommended that you use place value equipment to secure children's understanding of both division by a single digit and by 10 or 100 (resulting in a decimal). • You need to expose the children to a lot of different problem solving using multiplication (and addition and division) in this unit • Try to encourage children to represent the problem first to decide which calculations to do and then to carry these out below. The Bar Model is really useful as a consistent way of representing problems (be they word problems, real life problems or more abstract problems). You may need to model this first to help children see how to use it - see the guidance at the NCETM on the bar model here https://www.ncetm.org.uk/resources/44568

Key Assessment Checklist

1. I can multiply a 2-digit or 3-digit numbers by a single-digit number
2. I can divide a 2-digit (or simple 3-digit) number mentally with jottings
3. I know and can use the distributive law to partition a multiplication or division or recombine one that has been partitioned.
4. I can divide a 2-digit number by a single-digit number using a written method
5. I can divide a 3-digit number by a single-digit number using a written method
6. I can divide a single-digit or two-digit number by 10 or 100 to get a decimal answer
7. I can solve problems involving multiplication and division, including word problems and missing number problems
8. I can solve problems involving measures and money as well as scaling and correspondence by multiplying and dividing

Year 4	Unit 10 : Investigating Statistics	
8 learning hours	In this unit children and students explore the collection, representation, analysis and interpretation of data. It covers a range of calculations of central tendency and spread as well as multiple charts and graphs to represent data. As it is the only unit directly exploring statistics, it is critical that children have time to explore the handling data cycle here and to focus sufficient time on interpreting their results.	
Prior Learning	**Core Learning**	**Learning Leads to...**
➢ interpret and present data using bar charts, pictograms and tables ➢ solve one-step and two-step questions [for example, 'How many more?' and 'How many fewer?'] using information presented in scaled bar charts and pictograms and tables	➢ **interpret and present discrete and continuous data using appropriate graphical methods, including bar charts and time graphs** ➢ **solve comparison, sum and difference problems using information presented in bar charts, pictograms, tables and other graphs**	➢ solve comparison, sum and difference problems using information presented in a line graph ➢ complete, read and interpret information in tables, including timetables

Exemplification

1. Here is a table of the average temperature for each month of last year:

Month	1	2	3	4	5	6	7	8	9	10	11	12
Average Temp (°C)	6	7	10	12	16	18	21	22	18	14	10	7

Answer the questions below and explain your reasoning:

- On average what was the hottest month of the year?
- In which months was the average temperature below 10°C?
- In which months would you choose to go outside without your coat on?

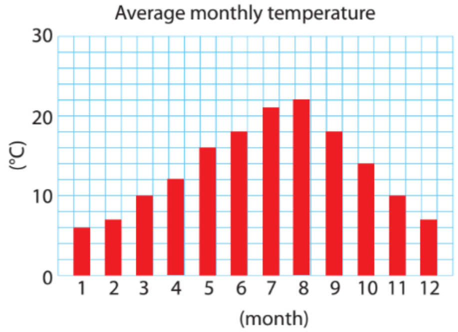
Choose another way to represent the data.

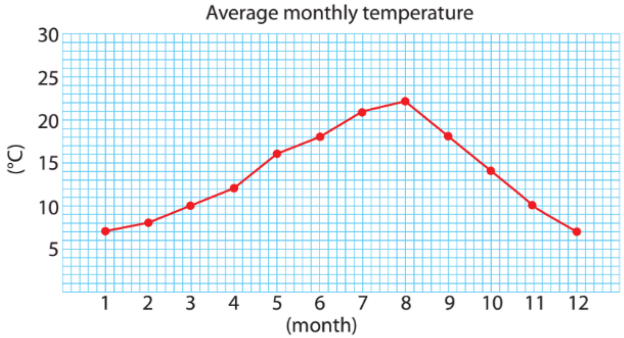
Vocabulary

data	most
discrete	least
continuous	sum
bar chart	difference
class intervals	compare
frequency	construct
diagram	interpret
line graph	
time graphs	
trend	

2. These two graphs represent the same data.

What's the same? What's different?





Which graph is better?

Explain your reasoning.

3. Use the ages below to complete the grouped frequency table.

12, 18, 25, 17, 22, 30, 28, 14, 14, 35, 17, 21

Age	Frequency
11 to 20	
21 to 30	
31 to 40	

Why is it useful to sort the categories into groups?

Representation	Fluency	Probing Questions
Discrete and continuous data • Be able to explain in their own words that discrete is data that can't be broken down into smaller measurements (e.g. favourite food, shoe size) • Be able to explain in their own words that continuous data can be broken down or measured in smaller increments (e.g. age, height, time)	1. Discrete vs. continuous • identify discrete data • identify continuous data • explain why a recording might be continuous • identify things that we record as discrete but are actually continuous (e.g. age)	What's the same and what's different? discrete and continuous data

• Represent the benefit of grouping continuous data into measurable increments. To do this maybe get students to collect age of each other in days (http://jalu.ch/coding/days/en or http://www.howlonghaveibeenalivefor.com/) and then also in years (like normal). Then get them to draw a bar chart to see which is better for analysing.		
Bar Charts etc. • Making a human frequency diagram by using each child to represent one piece of data and grouping them together.	2. Construct a bar chart or frequency diagram • Leave gaps between bars for discrete data • Grouped data frequency diagrams should be touching	Show me ... a sketch of a bar chart ... a sketch of a frequency diagram True or False? When drawing a bar chart you want to make the step size as small as possible
	3. Interpret a bar chart or frequency diagram • Interpret bar charts to find totals in multiple categories	What's the same and what's different? bar chart and frequency diagram True or False? Bar charts have bars that do not touch

Time Graphs vs. Bar Charts
• Students need to see that time graphs are great for seeing trends over time
• A good way is to give them some time based data (e.g. ice cream sales over a year) and get them to plot a bar chart and a time graph and discuss which shows a trend better
• If you have access to a measuring cylinder/beaker and a jug of water - it's a great visual representation:
  ○ Start a timer on the board and fill up the cylinder for 10 seconds slowly at a steady pour
  ○ Note the height/volume and wait for 20 seconds then slowly pour out about half for another 10 seconds
  ○ Record these values in a table with 10 second
	4. Construct a time graph
	• Identify if data is suitable for a time graph
	• Plot points correctly in a time graph
	• Join up points with a ruler
		Convince me that a line graph is the best to use for this data (temperature each month)

	5. Interpret a time graph • Use time graphs to answer question such as 'how much did the value rise between month 2 and month 3?'	What's the same and what's different? bar chart and line graph

<table>
<tr>
<td>increments so you have something like this:

Time	Volume
0s	0ml
10s	200ml
20s	200ml
30s	200ml
40s	100ml
50s	100ml

Plot this as a bar chart and ask some questions such as how much water was in the jug after 5 seconds?
Get students to identify that a time graph is better for continuous data such as this</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Solving Problems
Exploring lots of real life charts to gain experience at reading them and finding information</td>
<td>6. Solve problems by reading relevant information from a graph</td>
<td>Which chart would be best to display:
- A person's height from age 0 to age 20.
- A person's pulse rate during the data.
- A class' favourite colour.
- The pupils' favourite music from a year group at school.
- The sales of ice creams at a shop over a month in July.
- Votes for all the celebrities in a tv talent contest for one show.
- Votes for one celebrity in a tv talent contest for a series of shows.</td>
</tr>
</table>

Further Extension

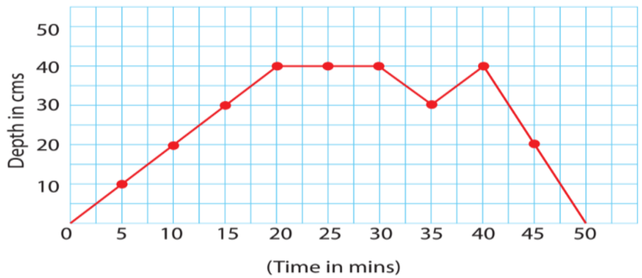
1. Here is a table of the average temperature for each month of last year:

Month	1	2	3	4	5	6	7	8	9	10	11	12
Average Temp (°C)	6	7	10	12	16	18	21	22	18	14	10	7

Write the word 'true', 'false' or 'unknown' next to each statement, giving an explanation for each response.

- I would need to wear my coat outside in January.
- The hottest day of the year was in August.
- A temperature of –2 was recorded in January.
- Choose two other ways to represent the data.

2. Make up a story that fits the graph.



Rich and Sophisticated Tasks

Solve comparison, sum and difference problems using information presented in bar charts, pictograms, tables and other graphs

NRICH: **Venn Diagrams** *** P**
NRICH: **More Carroll Diagrams** *** P**
NRICH: **Plants** **** I**

Misconceptions

Children find it hard to see the difference between discrete and continuous data - often because the way we measure and record continuous data makes it sort of discrete when we write it down e.g. heights are continuous because they can take any value but if we are measuring the nearest cm then they can't take ANY value in our study and so they are to some extent now discrete!

Children forget that bar charts should have gaps between them (as the data is discrete) and frequency diagrams have bars that touch as the data is continuous.

Children use bars for line graphs and vice versa

Teacher Guidance and Notes

- Be aware that a bar chart represents discrete data and as such the bars should not touch each other. When the data is continuous, a frequency diagram (later histogram) should used and the bars will touch because the categories connect. At this level, there will mostly be bar charts as any continuous data takes the form of a time series that can be plotted using a line graph instead.
- Make sure children have a chance to explore what type of graph would be appropriate for a specific set of data and question. They need to understand what a line graph gives you that a bar chart/frequency diagram doesn't and know the sort of situations where you would naturally use one.
- It can be good to get children to come up with their own axes and scales as a challenge - and even to compare two data set using bar charts.

Key Assessment Checklist
1. I can recognise discrete and continuous data and come up with appropriate categories for it
2. I can record discrete or continuous data in a frequency table
3. I can construct a bar chart or frequency diagram to represent discrete or continuous data correctly.
4. I can interpret a bar chart, including reading several different values to answer a more complex question in the context of the original problem.
5. I can explain and understand the limitations where data is grouped.
6. I can construct line graphs and time graphs correctly.
7. I can interpret a line graph and a time graph
8. I can select appropriate charts and read a range of charts to solve comparison problems.

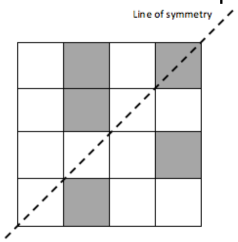
Year 4	Unit 11 : Visualising Shape
4 learning hours	In this unit children focus on exploring shapes practically and visually. There is an emphasis on sketching, constructing and modelling to gain a deeper understanding of the properties of shapes. It is therefore necessary to secure the practical skills at the same time as using them to explore the shapes in questions. At secondary level students are developing their skills in construction and the language/notation of shape up to the understanding, use and proof of circle theorems.

Prior Learning	Core Learning	Learning Leads to...
➢ draw 2-D shapes and make 3-D shapes using modelling materials; recognise 3-D shapes in different orientations and describe them	➢ **complete a simple symmetric figure with respect to a specific line of symmetry**	➢ draw given angles, and measure them in degrees (o) ➢ identify 3-D shapes, including cubes and other cuboids, from 2-D representations

Exemplification

1.
a) Complete this image to produce a symmetrical shape

b) Shade two more squares so that the dashed line becomes a line of symmetry



Vocabulary

reflect	circle
symmetry	square
symmetrical	triangle
vertical	rectangle
horizontal	rectangular
line of symmetry	pentagon
mirror line	hexagon
lines	octagon
draw	corners
straight	sides
shapes	
dimensional	

Representation

Symmetry

- Folding shapes to find lines of symmetry or to test whether given lines are lines of symmetry
- Using a mirror along a mirror line to produce the other half of a symmetrical image visually

Fluency

1. Recap: Identify a line of symmetry in a shape or design (horizontal, vertical or diagonal at 45º)
   - standard shapes: square, equilateral triangle, isosceles triangle, rectangle, kite, delta/arrowhead, rhombus, (parallelogram), regular pentagon, other regular polygons, other isosceles shapes e.g. isosceles trapezium or isosceles pentagon
   - compound shapes

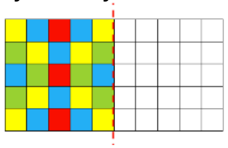
Probing Questions

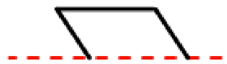
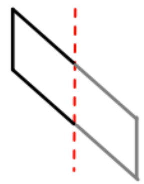
Show me a shape that is symmetrical

Show me a shape that has 2 lines of symmetry

Show me a line of symmetry on a triangle

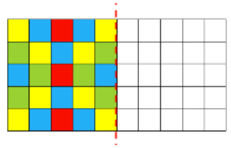
Convince me that a square has more

• Folding shapes broken into squares along a line of symmetry and colouring squares in to produce a symmetrical image • Folding an image along a mirror line and printing over to produce the mirror image to complete a symmetrical diagram (can also be done with paint e.g. butterflies) • Using tracing paper to identify (and test) possible lines of symmetry on images that cannot be folded • Using coloured tiles to form a mosaic pattern with a given line (or lines) of symmetry  • Symmetry ITP programme	• patterns/designs • shapes made from arrangements of many squares e.g. heptominoes • designs on squared grid with some shapes	than one line of symmetry Convince me that a rectangle doesn't have more than 2 lines of symmetry Convince me that an equilateral triangle has more than one line of symmetry Always, Sometimes, Never? The number of lines of symmetry is the same as the number of sides on the shape
	2. Use a line of symmetry to produce a symmetrical pattern (own design) • vertical mirror line • horizontal mirror line	Show me a pattern that is symmetrical Can you show me one with two lines of symmetry? Show me a shape that has 1 line of symmetry Show me a picture that has some symmetry in it

	3. Use a line of symmetry to complete a symmetrical image (outline on one side of the line given)
	• image on one side of mirror line, not touching
	  ○ vertical mirror line
	  ○ horizontal mirror line e.g.
	• image on one side of mirror line, touching line
	  ○ vertical mirror line
	  ○ horizontal mirror line e.g.
	
	• image on both sides of mirror line
	  ○ vertical mirror line
	  ○ horizontal mirror line
		Convince me that this image has not been completed correctly to produce a shape with a line of symmetry as shown
		
		Always, Sometimes, Never?
		Quadrilaterals have four lines of symmetry

4. Use a line of symmetry to complete a symmetrical pattern (some squares shaded in grid
    - shading on one side of the line
        - vertical mirror line
        - horizontal mirror line
        - diagonal mirror line
    - shading on both sides of line
        - vertical mirror line
        - horizontal mirror line
        - diagonal mirror line
    - specific number of squares to be shaded

Convince me that there is only one way to complete the symmetrical pattern with the mirror line shown

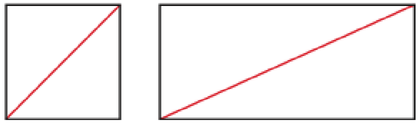


Further Extension

1.
Draw some 2-D shapes that have:
- no lines of symmetry
- 1 line of symmetry
- 2 lines of symmetry.

2.
Tom says, 'In each of these shapes the red line is a line of symmetry.'
Do you agree?

Explain your reasoning.



Rich and Sophisticated Tasks

Complete a simple symmetric figure with respect to a specific line of symmetry
NRICH: Symmetry Challenge * I**
NRICH: School Fair Necklaces ** I

(Identify lines of symmetry in 2-D shapes presented in different orientations)
NRICH: Let Us Reflect * P
NRICH: Stringy Quads ** P
NRICH: Counters in the Middle * G P

Misconceptions

Symmetry:
Children cannot always see the lines of symmetry and should be encouraged to rotate the shapes/images to help them to spot them (as this is easier when the lines are vertical). Similarly they may need to fold 2D shapes to test out their theories.
Conversely, children often believe there is symmetry where there is in fact not

Teacher Guidance and Notes

- Children have looked at symmetry earlier this year during the unit on Exploring Shape. This work considered identifying lines of symmetry of a shape and using this to describe the properties of the shape.
- This unit is now focused on actively constructing the rest of an image to produce a symmetrical shape/design
- Note that at this stage children are expected to be able to identify a

e.g. down the diagonal of a rectangle. Reflection: Children may not realise or comprehend the nature of inversion that a reflection creates - therefore they think that the shape/figure is to be repeated the other side of the mirror line instead of reflected. There will similarly be some confusion when doing the opposite (e.g. moving left of the mirror to reflect a line that moves to the right). Children struggle when the mirror line is not vertical and so may find it hard to predict where the shape will go using the line. A line that touches the shape is harder to work with than an example with a gap between the shape and the mirror line.	diagonal mirror line but <u>not</u> to use one to produce an image of their own. They may be asked to complete a shaded pattern with a diagonal mirror line as a most challenging task. • There are a number of links to other curriculum areas including art, religious studies, design technology and many more that can be exploited in this unit. • Children working at greater depth could begin to think about the symmetry within 3D shapes to deepen their understanding further. What does symmetry mean for a 3D shape? Is it a mirror line or is it a plane?

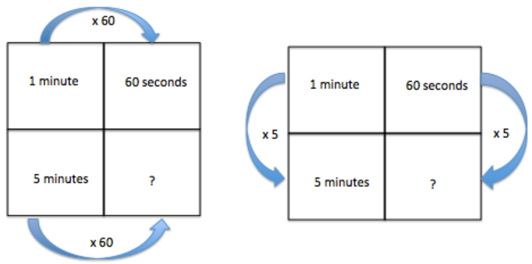
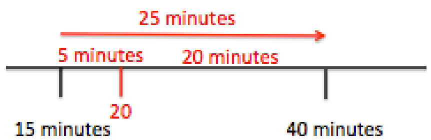
Key Assessment Checklist

1. I can recognise symmetry in patterns, identify lines of symmetry in 2D shapes and use this to help describe 2D shapes
2. I can reflect a simple shape or pattern across a vertical or horizontal mirror line
3. I can reflect a simple shape or pattern across a diagonal mirror line
4. I can complete a symmetrical picture (1 line of symmetry)

Year 4	Unit 12: Exploring Change	
7 learning hours	For primary pupils this unit focuses on the measures elements of time and co-ordinates. There is a progression from sequencing and ordering through telling the time formally to solving problems involving time. The co-ordinate work flows in the secondary students' learning focused on the relationships between co-ordinates. Key objectives include the use of y=mx+c for straight lines, the use of functions and the graphing of more complex functions.	
Prior Learning	**Core Learning**	**Learning Leads to...**
➢ tell and write the time from an analogue clock, including using Roman numerals from I to XII, and 12-hour and 24-hour clocks ➢ know the number of seconds in a minute and the number of days in each month, year and leap year ➢ compare durations of events [for example to calculate the time taken by particular events or tasks] ➢ estimate and read time with increasing accuracy to the nearest minute; record and compare time in terms of seconds, minutes and hours; use vocabulary such as o'clock, a.m./p.m., morning, afternoon, noon and midnight	➢ **read, write and convert time between analogue and digital 12- and 24-hour clocks** ➢ **solve problems involving converting from hours to minutes; minutes to seconds; years to months; weeks to days**	➢ solve problems involving converting between units of time

Exemplification	Vocabulary
1. a) Write these times in 24-hour format (i) Quarter past five in the morning (ii) 10:25 pm b) Write these times in 12-hour format (i) Ten to six in the morning (iii) 19:30 2. a) A soldier starts an exercise at 18:00 on Friday 4th June. The exercise finishes at 14.40 on Sunday 6th June. Calculate the duration of the exercise in days, hours and minutes. b) A human pregnancy usually lasts for 40 weeks. How many days is this?	24 hour clock 12 hour clock analogue digital am/pm convert duration difference second, minute, hour day, week, month, year, leap year

Representation	Fluency	Probing Questions
Time • Comparing different clocks, both analogue and digital and both 12-hour and 24-hour. • Exploring the 24-hour analogue clock at the Greenwich Observatory  • Labelling a clock with key words, roman numerals, multiples of 5, fractions and 24-hour hours. Then counting round the clock and moving the hands to match either in 12-hour format e.g. 1:00, 1:05, 1:10, 1:15, ... or in analogue format e.g. one o'clock, five past one, ten past one, quarter past one, twenty past one, twenty-five past one, ...    It can be nice to do this using different overalys for the outer labels so you can shift from Roman numerals, to 24-hour clock, to past/to descriptors, to multiples of 5 and so on. • Making clocks using paper plates, card sticks and split pins for hands • Using manipulative clocks to show and read times (preferably mini-clocks for each child and a larger one for the teacher) • Making human clocks using arms	1. Recap: read and show times in 12-hour format • read the time from a clock face and record in 12-hour • draw hands on a clock face to show a 12-hour time	Show me another way of writing 12 o'clock ... and another ... and another
	2. Read and write times in 24-hour format • understand structure of 24 hour • equate hours in 12-hour format with 24-hour format e.g. 2:00pm and 14:00 • read the time from a clock face and record in 24-hour • draw hands on a clock face to show a 24-hour time	Convince me that 20:40 is the same as 8:40pm
	3. Convert between time formats • 24-hour to 12-hour • 12-hour to 24-hour (am) • 12-hour to 24-hour (pm) • clock face to 12-hour or 24-hour • 12-hour or 24-hour to clock face	What's the same and what's different? 12-hour watch; 24-hour watch; analogue watch What's the same and what's different? 18:20; 6:20 am; 8:20 pm

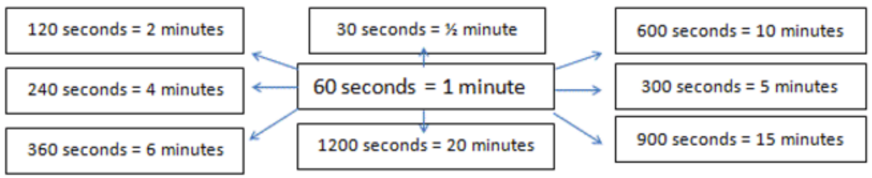
Converting Times • Using 2 x 2 proportion grids to scale up and convert For example, to find the number of seconds in 5 minutes, either vertically or horizontally:  	4. Convert between seconds and minutes as well as minutes and hours • know there are 60 minutes in an hour • calculate the numbers of minutes in a given number of hours • convert hours to minutes where number of hours is not whole e.g. 2 ½ hours • know there are 60 seconds in a minute • convert whole minutes to seconds	Show me a time that is equivalent to 300 minutes Convince me that there are 300 seconds in 5 minutes.
	5. Convert between hours and days • know there are 24 hours in a day • convert whole days to hours • convert non-whole days to hours	What's the same and what's different? 2 days, 20 hours, 48 hours
	6. Convert between days and weeks and months and years • know there are 7 days in a week • convert whole weeks to days • know there are 52 weeks in a year • know how many days are in each month • know there are 12 months in a year • convert whole years to months • know there are 365 (or 366) days in a (leap) year • ext: convert years to days	Always, Sometimes, Never? Four weeks is longer than a month What's the same and what's different? 30 days, a month, June, July (more than one answer/justify) Convince me that there are 48 months in 4 years
Durations • Using a number line to find time intervals and durations  • Exploring bus or train timetables to identify durations of journeys • Looking at TV guides to calculate durations	7. Find the difference between times given in a range of (different) units • times in same units (e.g. minutes, days, months) • times in mixed units (e.g. 3 minutes and 80 seconds) • times and dates (e.g. 10:30 on 7th May and 21:00 on 12th May)	Convince me that half an hour isn't the same as 50 minutes Convince me that there are 3 days and 2 hours between 14:30 on 3rd February and 4:30 pm on 6th February

Further Extension

1. Brent and Chris were gardening. They started at 13:25. Brent finished at 15:55. Chris carried on for another hour and ten minutes. For how long was Chris gardening?

2. Adnan spent 1 hour 55 minutes at the gym. She left at 16:30. When did she get there?

3. Produce an equivalence diagram for conversions.
For example:



Rich and Sophisticated Tasks

Read, write and convert time between analogue and digital 12- and 24-hour clocks
NRICH: Wonky Watches ** P
NRICH: Watch the Clock *** P

Misconceptions

Some children may have insecure knowledge of reading the time and number, particularly counting in 5s. Similarly, there may be a misconception of working in base 10 with time that leads to issues around the use of 60 minutes in an hour, for instance. Thus they may believe that there are, for example, 100 seconds in a minute, 100 minutes in an hour and so on.

There may be confusion of am and pm, especially with noon, which should be shown as 12pm, and midnight, which should be shown as 12am. Similarly, the use of am for early morning may be an issue - some children believe that am is when it is light and pm is when it is dark.

The 24-hour clock can be problematic also. Some children find it hard to convert times because they add 10 instead of twelve e.g. they think 1pm is the same as 10 hours + 1 hour so will be 11:00 rather than 12 hours + 1 hour or 13:00.

Additionally, children may forget the 4th digit in 24-hour format writing, for example, 2:15 instead of 02:15.

When starting to work out time periods, children may revert back to addition as if they were working in base 10.

Leap years can cause some confusion, particularly with the rationale.

Teacher Guidance and Notes

- Children encountered 12-hour and 24-hour clock formats in Stage 3 and hence the emphasis here is on rapid conversion and usage.
- Additionally, this unit focuses on making conversions between units of time by finding, for example, the number of seconds in 5 minutes and calculating more complex time durations involving mixed units or dates and times.
- As in earlier stages, integrate work on time into daily routines and activities to ensure confidence is developed. For example, introduce a mental time question each morning; ensure it is worded in various ways eg, If I left at 3.15pm and the journey took 35 minutes how long would it take? The starting time was 3.15pm and the finish time was 3.50pm, how liong did it take me? I walked for 35minutes and arrived at 3.50pm, what time did I depart?
- Converting between the different time intervals requires re-emphasis of the number of minutes in an hour etc. Starters relating to 60 and 12 can help to improve speed of calculations with this non-base 10 setting.

Key Assessment Checklist

1. I can confidently read and read times using analogue and digital time, including 24hr clock.
2. I can convert between 12-hour and 24-hour clock rapidly.
3. I can solve problems that need me to convert hours to minutes.
4. I can solve problems that need me to convert minutes to seconds.
5. I can solve problems that need me to convert years to months.
6. I can solve problems that need me to convert weeks into days.

Year 4	Unit 13: Proportional Reasoning
4 - 8 learning hours	In this unit pupils explore proportional relationships, from the operations of multiplication and division on to the concepts of ratio, similarity, direct and inverse proportion. For primary pupils in Stages 1-3, this is focused on developing skills of division. Stages 4 and 5 revisit the whole of calculation to broaden to all four operations in a range of contexts and combination problems; the emphasis here is really on representing and then solving a problem using their calculation skills, not just calculating alone. In Stage 6 the real underpinning concepts of proportion and ratio develop. Secondary pupils begin to formalise their thinking about proportion by finding and applying scale factors, dividing quantities in a given ratio and fully investigating quantities in direct or inverse proportion, including graphically.

Prior Learning	Core Learning	Learning Leads to...
➢ recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables ➢ write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods ➢ solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects	➢ **recall multiplication and division facts for multiplication tables up to 12 × 12** ➢ **use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers** ➢ **multiply two-digit and three-digit numbers by a one-digit number using formal written layout** ➢ divide a two-digit or three-digit number by one digit number ➢ **find the effect of dividing a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as ones, tenths and hundredths** ➢ **solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects** ➢ **solve simple measure and money problems involving fractions and decimals to two decimal places**	➢ multiply and divide numbers mentally drawing upon known facts ➢ multiply and divide whole numbers and those involving decimals by 10, 100 and 1000 ➢ multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers ➢ divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context ➢ solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes ➢ solve problems involving addition, subtraction, multiplication and division

		and a combination of these, including understanding the meaning of the equals sign ➢ solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates of change.

Exemplification	Vocabulary
1. Complete one missing numbers in each number sentence: a) 9 x 8 = ... b) 6 x .... = 48 c) 36 ÷ .... = 4 d) ...... ÷ 8 = 12 2. Calculate a) 200×6 b) $420 \div 6$ c) $6 \times 3 \times 5$ d) $12 \div 1$ e) 4×0 3. Calculate a) 42×7 b) 576×4 4. Calculate using a written method a) $98 \div 7$ b) $384 \div 6$ c) $87 \div 5$ 5. a) Find the value of Δ in each of these statements and write your answer in the box	addition multiplication division divide place value digits partition 1s/ones tenths hundredths distributive law solve problem represent array grid scaling bar model factor, quotient inverse brackets long multiplication compact (short) multiplication

	1s		0.1s
4 ÷ 10 = Δ		.	
78 ÷ 10 = Δ		.	

b) Find the value of Δ in each of these statements and write your answer in the box

	1s		0.1s	0.01s
82 ÷ 10 = Δ		.		
912 ÷ 100 = Δ		.		

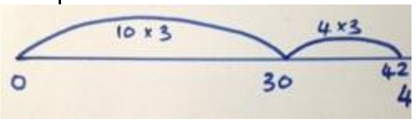
6. a) Three children calculated 8 x 9 in different ways:

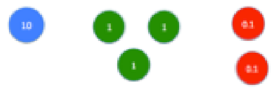
Amy	Bob	Chloe used the commutative law:
$8 \times 9 = 8 \times 8 + \blacksquare = \blacksquare$	$8 \times 9 = 9 \times 9 - \blacksquare = \blacksquare$	$8 \times 9 = \blacksquare \times \blacksquare = \blacksquare$

Complete the calculations to fill in the values of the missing numbers marked ■

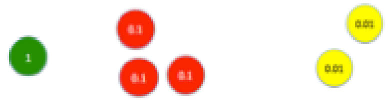
b) Tom ate 9 grapes at the picnic. Sam ate 3 times as many grapes as Tom.
How many grapes did they eat altogether?

7. a) An empty box weights 0.5 kg. Ivy puts 10 toy bricks inside it and the box now weighs 2kg. How much does each brick weigh?
b) Which would you rather have? Three 50p coins or seven 20p coins? Explain your answer.

Representation	Fluency (See Units 5 and 9 for further details)	Probing Questions
Multiplying/Dividing (→ Mental Methods) • Using a number line to show partitioned grouping. For example, $42 \div 3$ can be found by considering known multiples of 3  • Partitioning a larger number to divide each part and then recombine For example: 72 ÷ 3 60 ÷ 3 + 12 ÷ 3 20 + 4	1. Instantly recall and use multiplication and division facts for the multiplication tables up to 12x12 2. Calculate simple mental multiplications and divisions of 2-digit numbers by single digits	Convince me that if I know the 6 times table, I can find the numbers in the 3 times table Convince me that dividing by 24 is the same as dividing by 12 and 2 Show me the single calculation that is equivalent to 20x4 + 5x4 Convince me that 15 x 9 = 135 Convince me that 14 x 6 will give a different answer to 16 x 4
Multiplying by 0 and 1/Dividing by 1 • Building arrays to represent a number multiplied by 1 and then divided by 1 to see why the original number is the answer • Recognising that there is no array for a multiplication of 0 hence the answer is 0	3. Mentally calculate the result of multiplication by 0 or 1, division by 1 as well as the product of three numbers	Show me that any non zero number x 0 = 0 Show me that any non zero number + 0 does not = 0
Dividing by 10 and 100 • Using place value counters to represent decimals (you can use unlabelled counters and given children a key) For example, here is 13.2	4. Divide a 1-digit or 2-digit number by 10 or 100	Always, Sometimes, Never? When you divide a number by 10, you remove one zero from the end Always, Sometimes, Never?

<table>
<tr>
<td>


Exchanging each counter for one that is ten times smaller
For example to calculate 13.2 ÷ 10,
the 10 becomes a 1
the 1s become 0.1s
the 0.1s become 0.01s

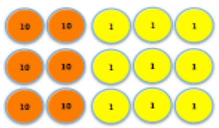


which makes 1.32

Using a place value grid to help automate this process by imagining the above to arrive at the shortcut of making the 1s into 0.1s and so on

</td>
<td></td>
<td>When you divide a number by 100, you will end up with a number with hundredths in</td>
</tr>
<tr>
<td>
Multiplication

Building arrays using place value counters



Generalising the array using a grid (area) representation

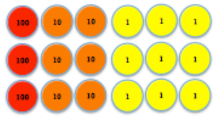
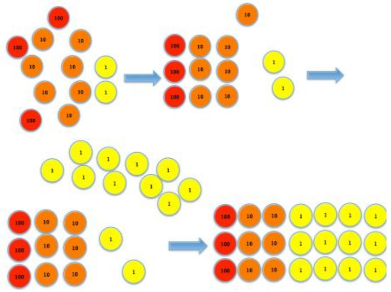
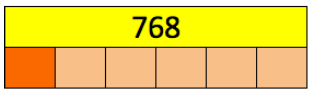
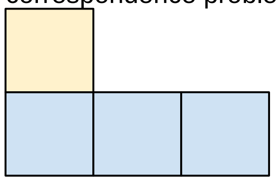
<table>
<tr><td></td><td>20</td><td>3</td></tr>
<tr><td>3</td><td>60</td><td>9</td></tr>
</table>

60 + 9 = 69

Linking the grid representation to expanded formal method and then to compact method

$$\begin{array}{rr} 2 & 3 \\ & 3 \;\times \\ \hline & 9 \\ 6 & 0 \;+ \\ \hline 6 & 9 \end{array} \qquad \begin{array}{rr} 2 & 3 \\ & 3 \;\times \\ \hline 6 & 9 \end{array}$$
</td>
<td>5. Multiply a 2-digit or 3-digit number by a single digit using a formal method</td>
<td>Show me how you can calculate 46 x 7 using
- a grid method
- partitioning
- a column method
- using near multiples

Convince me that 17 x 6 = 102</td>
</tr>
</table>

Division • Building a number using place value counters and grouping them into groups that are the size of the divisor, before arranging these groups as an array to explore the partitioning approach. For example, 369 ÷ 3  • Repeating the above, but exchanging remaining counters for 10 counters of the next size down before continuing to group. For example, 372 ÷ 3  • Recording these methods to arrive at compact division	6. Divide a 2-digit or 3-digit number by a single digit using a written method	What's the same and what's different? 18 ÷ 2, 18 ÷ 3, 18 ÷ 4, 18 ÷ 5 Always, Sometimes, Never? I can test if a division is correct by multiplying my answer by the number I was dividing by Show me a three digit number that is divisible by 3
Representing and Solving Problems: • Using the bar model to represent a word problem. For example, 768 shared between 6  • Using the bar model to represent and solve correspondence problems 	7. Recognise and solve single operation problems (including correspondence and scaling problems)	Show me how you can represent this problem: Jodie has 8 crates containing 24 bottles. How many bottles does she have in total? Convince me that that if I know that 468 / 4 is 117, then I can check I am right by calculating 4 x 117
	8. Recognise and solve multi-step problems	Show me a solution to: a number ÷ 6 = another number x 5 Show me a three digit number x 3 = a number with a 6 in the units column

Further Extension

1.
True or false?

$7 \times 6 = 7 \times 3 \times 2$

$7 \times 6 = 7 \times 3 + 3$

Explain your reasoning.

Can you write the number 30 as the product of 3 numbers?

Can you do it in different ways?

2.
Place one of these symbols in the circle to make the number sentence correct:
>, < or =.

Explain your reasoning.

8×50	◯	50×8
8×50	◯	80×5
300×3	◯	5×200

3.
Sally has 9 times as many football cards as Sam. Together they have 150 cards.
How many more cards does Sally have than Sam?

Rich and Sophisticated Tasks

Recall multiplication and division facts for multiplication tables up to 12x12

NRICH: Multiplication Square Jigsaw * G P
NRICH: Shape Times Shape * P
NRICH: Table Patterns Go Wild! ** I
NRICH: Let Us Divide! * P
NRICH: Carrying Cards * P
NRICH: Light the Lights Again * G P
NRICH: Multiples Grid * I
NRICH: Zios and Zepts * P
NRICH: Times Tables Shifts * G P

Solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects

NRICH: A Square of Numbers * G P
NRICH: What do you Need? * P
NRICH: This Pied Piper of Hamelin ** P
NRICH: Follow the Numbers * P I
NRICH: What's in the Box? * P
NRICH: How Do You Do It? * P
NRICH: Ip Dip * I
NRICH: Journeys in Numberland * I

Misconceptions

Children sometimes make errors when multiplying by 1 or 0, saying that 3 x 0 = 3, for example. They find it hard to imagine zero lots of 4 or 3 multiplied by 1, for example. When multiplying 3 digits together children can forget to use the product of the first calculation for the 2nd part..

Children may think that dividing by 10 means taking the zero off the end and multiplying by 10 means adding it. They do not relate multiplying and dividing to the place value and unitisation of a number e.g. 24 x 10 is 2 tens and 4 ones multiplied by 10 which will be 2 hundreds and 4 tens or 240. This can lead to errors where a decimal point is needed and not used or vice versa.

Teacher Guidance and Notes

- This unit provides an opportunity to revisit and strengthen earlier work on calculations, particularly for multiplication and division.
- However, if these skills are already strong, there is no need to go through the concepts from first principles in full – rather, there can be a greater emphasis on working with solving increasingly complex problems.
- Greater guidance on these objectives is provided in Units 5 and 9 if required.
- Note that division is not explicitly referenced in the Year 4

When carrying out more complex multiplications, some children will fail to realise that multiplication is commutative and struggle to use the times tables that they know to tackle a related question. Children may struggle to represent scaling and correspondence problems visually (because they don't conform to the 'lots of' repeated addition imagery that some children focus on for multiplication) Children find it hard to separate how you can 'make' a number by both adding and multiplying - they may lean towards additive relationships more than multiplicative e.g. they may not have understanding of how 24 can be made of 10 and 14 as well as 6 and 4 (and other examples). When looking at a scaling problem, children may automatically view it as an additive relationship. E.g. this tree is 4m tall, this one is 12m tall so they see that as 8m taller and not 3 times as tall.	national curriculum but is included here in order to continue to consolidate and extend children's skills so they are Stage 5-ready by the end of the year.

Key Assessment Checklist

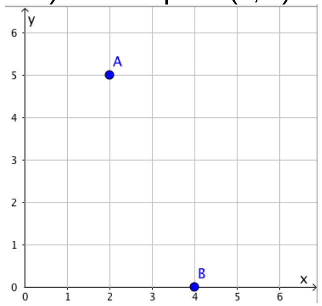
1. I can instantly recall and use multiplication and division facts for the multiplication tables up to 12x12
2. I can calculate simple mental multiplications and divisions of 2-digit numbers by single digits
3. I can mentally calculate the result of multiplication by 0 or 1, division by 1 as well as the product of three numbers
4. I can divide a 1-digit or 2-digit number by 10 or 100
5. I can multiply a 2-digit or 3-digit number by a single digit using a formal method
6. I can divide a 2-digit or 3-digit number by a single digit using a written method
7. I can recognise and solve single operation problems (including correspondence and scaling problems)
8. I can recognise and solve multi-step problems

Year 4	Unit 14: Describing Position
5 learning hours	In this unit pupils explore how we can communicate position and movement mathematically. They look at transformations from simple turns to reflection/rotation/enlargement/translations up to similar shapes generated by enlargements, co-ordinate systems and ultimately vectors

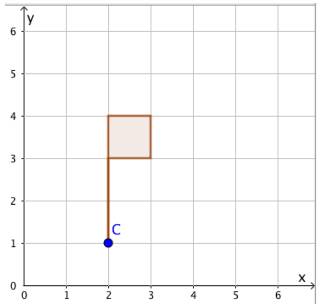
Prior Learning	Core Learning	Learning Leads to...
➢ use mathematical vocabulary to describe position, direction and movement, including movement in a straight line and distinguishing between rotation as a turn and in terms of right angles for quarter, half and three-quarter turns (clockwise and anti-clockwise)	➢ **describe positions on a 2-D grid as coordinates in the first quadrant** ➢ **describe movements between positions as translations of a given unit to the left/right and up/down** ➢ **plot specified points and draw sides to complete a given polygon**	➢ identify, describe and represent the position of a shape following a reflection or translation, using the appropriate language, and know that the shape has not changed

Exemplification

1. a) Write down the coordinates of points A and B
   b) Plot the point (5, 3)



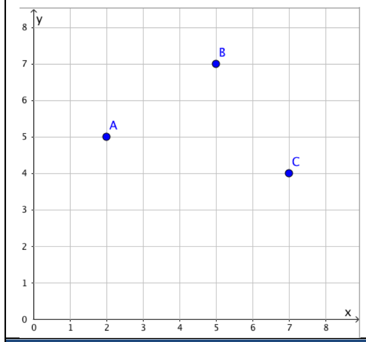
2. The flag shape shown here is translated 4 squares to the right and 2 squares up.
What are the new coordinates of point C?



Vocabulary

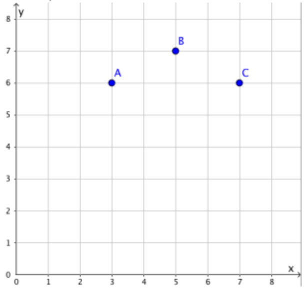
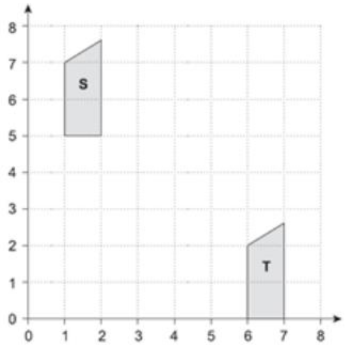
plot
coordinate
point
(x, y)
axes
x-axis
y-axis
origin
first quadrant
horizontal
vertical
2-D
translate
translation
across
left/right
up/down
polygon
vertex/vertices
regular (and irregular)

equilateral triangle
isosceles triangle
right-angled triangle
square
rectangle

3. A, B and C are three vertices of a square. What are the coordinates of the fourth vertex?  	parallelogram rhombus kite pentagon hexagon octagon

Representation	Fluency	Probing Questions
Coordinates and Translation • <u>People Points:</u> ○ Using large axes on the playground or hall floor, with each child representing/standing on a point. Starting by carrying out the journey from (0,0) moving horizontally and then vertically. ○ Joining points (children!) using string or washing line to help form polygons. The fourth child can then move to where they think the last vertex is and the class can view the shape formed to check it is correct. If necessary, the child can adjust their position. ○ When translating, children forming the shape can move en masse across and up/down to see where the vertices end up. • <u>Dynamic Geometry Software</u> ○ Using dynamic geometry software like geogebra to plot points and join them to form a polygon. Children can	1. Use coordinates to describe positions and to plot points in the first quadrant • plot a point (not on axes) e.g. (4, 3) • describe a point (not on axes) e.g. (2, 5) • plot a point on axis e.g. (2, 0) or (0, 5) • describe a point on axis e.g. (0, 3) or (1, 0) • know that (0,0) is also called the origin	Convince me that that the coordinates in the first quadrant will always be positive What's the same and what's different? (3, 4); (3, 2); (3, 6); (1, 3); (3, 0); (3, 3); (5, 3) Always, Sometimes, Never? Coordinates on the axes contain a 0
	2. Use coordinates to plot a set of points to form a polygon • plot a set of given points and connect to form a polygon (and name the shape) • plot own points to form a given shape e.g. rectangle or pentagon or triangle and describe the coordinates of the vertices • given all vertices except one, plot these vertices and plot/deduce coordinates of final vertex (one possibility only) • say whether someone's choice of coordinates for the last vertex of a polygon is right or wrong and explain why • plot a set of given points, joining them in order, to produce a pattern or design or picture	Show me the coordinates of 4 points that will form a square Show me four co-ordinates that form a parallelogram

then move the last vertex around to see what effect that has on the shape. o Predicting where a point will be by drawing on the interactive whiteboard and then revealing using Geogebra to check the answer. o Dragging the shape to complete a translation to see what the new coordinates of the vertices are.	3. Carry out translations as movements up/down and left/right • translate a shape a given number of squares right or left and redraw it • give coordinates of the new vertices • translate a shape a given number of squares up or down and redraw it • give coordinates of the new vertices • translate a shape both horizontally and vertically • give coordinates of the new vertices	Convince me that a translated shape cannot be a reflection of the original object Convince me that if you translate a shape 3 squares to the right then all the coordinates increase by 3 in the x coordinate. What's the same and what's different? translation across 2; translation up 2; translation down 2; translation right 2; translation left 2
	4. Describe movements between positions as translations of a number of squares up/down and left/right • describe the translation following a horizontal movement only (using left or right) • describe the translation following a vertical movement only (using up or down) • describe the translation following a horizontal and vertical movement	Always, Sometimes, Never? A translated shape will be the same size as the original Always, Sometimes, Never? A translation moves shapes further away from the origin
	5. Solve problems involving coordinates • continue patters on coordinate grids, predicting next sets of coordinates • find missing coordinates • find final vertices of polygons where there is more than one possible answer	What's the same and what's different? (6, 2); (4, 2); (5, 3); (5, 0) Always, Sometimes, Never? Coordinates on a vertical line have the same y-coordinate

Further Extension	Rich and Sophisticated Tasks
1. A, B and C are three vertices of a kite.  What could the coordinates of the fourth vertex be? Can you find another possible solution? How many solutions are there? 2. Describe the translation to get from shape S to shape T.  What is the translation to get from shape T to shape S? What do you notice about your two answers?	Describe positions on a 2-D grid as coordinates in the first quadrant **NRICH:** **Coordinate Challenge** *** P** **NRICH:** **Eight Hidden Squares** **** P** Plot specified points and draw sides to complete a given polygon. **NRICH:** **A Cartesian Puzzle** *** P**

Misconceptions	Teacher Guidance and Notes
There is a tendency to reverse co-ordinates both when plotting and reading - sometimes this is because children cannot correctly identify the x-axis and the y-axis, sometimes it is due to incorrectly remembering a rule to go across first and then up. Children may not realise the importance of equal divisions between points on the axes (especially between 0 and 1) – this will be clear if they have to draw their own axes. Be aware of issues around co-ordinates on the axes themselves - children may find the 0 off-putting so make a point of addressing this. Children find the word translation confusing (mostly due to its linguistic meaning dominating in their minds). They may confuse this word with the word 'transformation'. Some children will distort a shape when translating it, not realising that the size and proportions of it should be preserved. Children often measure the distance between the two end points of shape, rather than looking at the movement between corresponding vertices on the original and new shape. Some children may confuse left and right.	• This unit represents children's first encounter with coordinates. • At this stage they need only work in the first quadrant with positive numbers. • It is valuable to give pupils the opportunity to draw their own axes as well as providing pre-drawn axes as, whilst time-consuming, this activity may reveal issues around understanding of scale etc. • Pupils need to be aware that the horizontal is the x-axis and the vertical is the y-axis. Be very way of using ideas such as "along the corridor and up the stairs." as these can be misremembered (after all, there may be no reason not to go up the stairs and then along the corridor!). Try to focus on the reason for this i.e. we do the x-axis first before the y-axis in alphabetical order. Once this is clear a shortcut can be established but try not to start with the shortcut. • It is also worth observing whether pupils translate every point and then connect them or whether they translate one and then use the congruence of the images (even if they don't say it like this!) to predict the remaining points. This shows an implicit understanding of the preservation of length under translation. • There are lots of good games to explore co-ordinates in detail e.g. battleships

Key Assessment Checklist

1. I can identify a point in the first quadrant using coordinates; I can plot a point when given its coordinates in the first quadrant
2. I can form a polygon using coordinates, including finding the coordinates of the last vertex.
3. I can solve problems using coordinates axes in the first quadrant
4. I can carry out a translation as a combination of a horizontal and vertical shift
5. I can describe translation as a combination of a horizontal and vertical shift