Moor Nook CP School

Year 2

Medium Term Plans

Updated March 2022

Overview of Year

Autumn Term	**Number and Algebra**				**Geometry and Measures**	
	1. Investigating Number Systems	2. Pattern Sniffing	3. Solving Calculation Problems	4. Generalising Arithmetic	5. Exploring Shape	6. Reasoning with Measures

Spring Term	**Number and Algebra**		**Statistics**
	7. Discovering Equivalence 8. Reasoning with Fractions	9. Solving Number Problems	10. Investigating Statistics

Summer Term	**Geometry**	**Number and Algebra**		**Geometry and Measures**	
	11. Visualising Shape	12. Exploring Change	13. Proportional Reasoning	14. Describing Position	15. Measuring and Estimating

Year 2 Overview:		
Unit	**Approx Learning Hours**	**Summary of Key Content**
1. Investigating Number Systems	8	Reading and writing numbers in words and numerals to 100 Recognise place value; compare and order numbers 2NPV-1 including standard and non-standard partitioning Not within the AET curriculum: Locating 2 digit numbers on a number line 2NPV-1
2. Pattern Sniffing	8	Counting in 2s, 3s and 5s; recall and use times table facts for these Arrange patterns; recognise odd and even numbers
3. Solving Calculation Problems	11	Recall addition and subtraction facts to 20 2NF-1 (within 10); derive them to 100 Add and subtract numbers using objects and pictures up to 2d+2d 2AS-1 (Bridging through 10) Show commutativity of addition (and non-commutativity of subtraction) (Please refer to Moor Nook's Mental & Written Calculation Policies)
4. Generalising Arithmetic	10	Solve addition and subtraction problems with 2-digit numbers 2AS-2/ 2AS-3/ 2AS-4 Solving missing number problems using inverses (Please refer to Moor Nook's Mental & Written Calculation Policies)
5. Exploring Shape	8	Identify and describe properties of 2D and 3D shapes 2G-1
6. Reasoning with Measures	8	Use £ and p symbols; find combinations of coins for a given total; solve simple addition and subtraction problems using money (Please refer to Moor Nook's Mental & Written Calculation Policies)
7. Discovering Equivalence 8. Reasoning with Fractions	11	Recognise, find and name 1/3, ¼, 2/4, ¾ of a length, shape, set of objects or quantity Write simple fractions and recognise equivalence of 2/4 and ½
9. Solving Number Problems	12	Calculate and write mathematical statements for multiplication and division 2MD-1 (Repeated addition) Show commutativity of multiplication (and non-commutativity of division) Solve simple problems using multiplication and division 2MD-2 (Please refer to Moor Nook's Mental & Written Calculation Policies)
10. Investigating Statistics	6	Interpret and construct simple pictograms, tally charts, block diagrams and tables Count, sort, total and compare categorical data
11. Visualising Shape	8	Identify 2D shapes on the surface of 3D shapes 2G-1
12. Exploring Change	7	Sequence time intervals Know number of minutes in an hour and hours in a day Tell/show the time to the nearest 5 minutes
13. Proportional Reasoning	7	Recall and use 2, 5 and 10 times tables (Be able to count in 2s, 5s & 10s and calculate the product) Calculate and write mathematical statements for multiplication and division Show commutativity of multiplication (and non-commutativity of division) (Please refer to Moor Nook's Mental & Written Calculation Policies)
14. Describing Position	5	Describe position, direction and movement using mathematical language, distinguishing between straight line movement and rotation

15. Measuring and Estimating	8	Choose and use suitable units; compare and order lengths, masses and capacities

Total = 117 hours ~ 24 weeks with 5 hours teaching per week

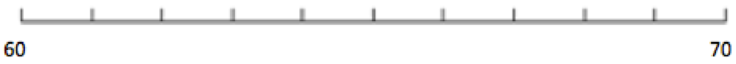
Year 2	Unit 1: Investigating Number Systems	
8 learning hours	This unit introduces the number systems and structures that we use at different levels of the curriculum. At KS1 children are working on the place value system of base 10 with the introduction of Roman Numerals as an example of an alternative system in KS2. Negative numbers and non-integers also come in at this stage and progress into KS3. At KS3 and KS4 we start to look at other ways of representing numbers, including standard form, inequality notation and so on.	
Prior Learning	**Core Learning**	**Extension Learning**
➢ read and write numbers from 1 to 20 in numerals and words. ➢ identify and represent numbers using objects and pictorial representations including the number line, and use the language of: equal to, more than, less than (fewer), most, least	➢ **read and write numbers to at least 100 in numerals and in words** ➢ **recognise the place value of each digit in a two-digit number (tens, ones)** ➢ **identify, represent and estimate numbers using different representations, including the number line** ➢ **compare and order numbers from 0 up to 100; use <, > and = signs** ➢ **use place value and number facts to solve problems** *Working towards expected standard* • *Demonstrate an understanding of place value, though may still need apparatus to support them (e.g. by stating the difference between 2 numbers i.e. 77 and 33 has a difference of 40 for the tens and 4 for the ones; by writing number statements such as 35<53 and 42>36)* • *Read and write numbers correctly in numerals to 100 (e.g. can read and write 14 and 41 correctly)* *Working at expected standard* • *Partition two-digit numbers into different combinations of tens and ones. This may include using apparatus. (e.g. 23 is the same as 2 tens and 3 ones which is the same as 1 ten and 13 ones)* *Working at greater depth:* • *use multiplication facts to make deductions outside known multiplication facts (e.g. a pupil knows that multiples of 5 have one digit of 0 or 5 and uses this to reason that 18 × 5 cannot be 92 as it is not a multiple* of 5).	➢ read and write numbers up to 1000 in numerals and in words ➢ recognise the place value of each digit in a three-digit number (hundreds, tens, ones) ➢ identify, represent and estimate numbers using different representations ➢ solve number problems and practical problems involving these ideas

Exemplification

1. a) Write this number using numerals: Thirty-eight
   b) Write this number in words: 57

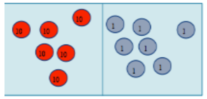
2. What is the value of the digit 7 in the number i) 75 ii) 17

3. a) Represent 63 on this number line



   b) Represent 34 using base 10

   c) State which number is shown here:



4. Write these numbers in order from smallest to largest:
   48, 84, 40, 44, 80, 8

5. Here are some number cards

7 **2** **4** **3**

a) Choose two cards to make the smallest possible 2 digit number
b) Choose two cards to make the largest possible 2 digit number

Vocabulary

ones
tens
digit
number
numerals
hundred
compare
greater than, >
less than, <
equal (to), =
smallest/least
largest/greatest
order
place value
column
most/greatest
least

Representation

Identifying and Representing Numbers

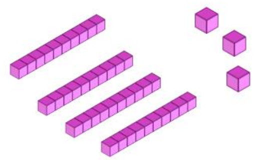
- Represent 2 digit numbers (loose and in column format) using:
  - Individual items e.g. counters, cubes
  - Individual items with 1:1 correspondence e.g. beadstrings
  - Separable to-scale tens and ones e.g. bundles of Straws, sticks of unifix
  - Inseparable to-scale tens and ones e.g. Numicon 10s and 1s, Base 10, Dienes rods
  - Not-to-scale tens and ones e.g. place value counters, tens-ones abacus, money (10p and 1p coins –

Fluency

1. Convert a given representation to a number (verbal or numerals)
   - single digits recap
   - multiples of ten recap (objects and number line)
   - tens and ones (separable e.g. straw bundles, multilink sticks, bead string)
   - tens and ones (inseparable e.g. Numicon, Dienes, base 10)
   - tens and ones (not to scale e.g. place value counters, money)
   - number line position

Probing Questions

Convince me that this is one more than 42



What's the same and what's different?
1, 10, 100

Show me one more than sixty-four

possibly £1 also) ○ Overlapping place value cards ○ Numerals • Develop sense of size of numbers up to 100 using paper strips and paperclips to position e.g. strip represents 0-100, where is 23? What if the strip now represents 0-50?	2. Convert a given number to a stated concrete or visual representation ○ single digits recap ○ multiples of ten recap (objects and number line) ○ tens and ones (separable e.g. straw bundles, multilink sticks, bead string) ○ tens and ones (inseparable e.g. Numicon, Dienes, base 10) ○ tens and ones (not to scale e.g. place value counters, money) ○ number line position	Show me the number 63 in words Show me how we can represent the number 35 using - unifix cubes - the beadstring - the dienes rods - the cuisenaire rods - the numicon - the hundred square - the number line Convince me that 88 comes after 87 What's the same and what's different? 65, 56, 6, 5, 60, 50
Partitioning • Representing numbers using tens and ones apparatus and then separating into different combinations e.g. 28 could be 20 + 8 or 10 + 18 or 25 + 3 and so on	3. Partition a number and state the value of a given digit within a number ○ Numbers 11-19 – partition into tens and ones ○ 2-digit numerals – partition into tens and ones ○ 2-digit numerals – non-standard partitioning ○ Reverse problem to find number from place value information	Convince me that sixty-twelve is the same as seventy-two Convince me that 23 is represented by 2 tens and 3 ones/this apparatus
Reading and Writing Numbers • Use (and make) a washing line to link numbers in words to numbers in numerals to images/representations e.g. numicon • Use matching cards between words, numerals and representations of numbers • Wherever numbers appear in the classroom, represent them in all three ways if possible e.g. on the clock • Where number formation is an issue, use tracing/stencil activities (with pens, in sand, on a tablet) to practice correct formation • Explore the idea of unitisation using double-	4. Convert a number written in words to numerals and vice versa ○ Recap single digits e.g. four or 7 ○ Recap 'Teen' numbers e.g. fourteen or 19 ○ Recap multiples of ten e.g. eighty or 30 ○ Two digits e.g. eighty-four or 45	Show me the number forty-two in symbols What's the same and what's different? eighty, eighty-one, eighteen
	5. Recognise matching numerals, words and representations ○ Matching pairs ○ Matching three or more items	Show me where 62 would be on this blank paper strip that goes from 0-100? Always, Sometimes, Never?

<table>
<tr>
<td>sided counters to see how to use a different colour to represent a set amount of ones (heading towards the use of tens and ones as one of the easiest combinations)</td>
<td>Matching representations without the numerals present</td>
<td>There are 10 numbers with a digit of 3 in them (up to 100).</td>
</tr>
<tr>
<td rowspan="3">Exploring same, more, less
Use apparatus and then visuals and then number cards (abstract) to explore which number is greater when comparing
Compare number by building each using ones and then using ‘movable comparison sticks’ to show if they are equal or which is greater.
Use a pan balance to help develop the concept of equal – this is particularly effective with Numicon, which is weighted so that it balances when of equal value.
Use number cards to explore making different two-digit numbers and finding the smallest/largest</td>
<td>6. Compare two numbers to say which is greater, using >, < or = to notate
Recap: two numbers up to 20
One single-digit and one two-digit
Two two-digit numbers (unrelated)
Two two-digit numbers (similar digits)
Mixture of representations/words/numerals</td>
<td>Show me which is a greater 39 or 93?

Show me a number that could complete 54 < ......

Convince me that 38 < 83

What’s the same and what’s different?
41>32, 76< 85, 50 = 50, 54<45</td>
</tr>
<tr>
<td>7. Order numbers from smallest to largest
Order three numbers:
Recap: numbers up to 20
Two-digit numbers (unrelated)
Two-digit numbers (similar digits)
Order four or more numbers (as above)</td>
<td>What’s the same and what’s different?
49, 50, 51</td>
</tr>
<tr>
<td>8. Solve place value problems
Find a number that lies between two given numbers (2-digits)
Given selection of digits, produce the smallest and largest possible numbers from them
Convert non-standard partitioning to standard e.g. sixty-fourteen
Relate place value to money contexts (£1, 10p, 1p)</td>
<td>Show me a number in between 45 and 76

Always, Sometimes, Never?
There is only one possible answer to this gap question
...... > 99

Always, Sometimes, Never?
If you take a number and reverse its digits, you get a number that is bigger than you started with</td>
</tr>
</table>

Further Extension

1.

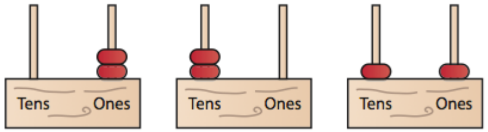
Write all the 2-digit numbers greater than 40 using these digits.

2 4 6 6

How do you know you have them all? Prove it.

2.

If you put 2 beads onto a tens/ones abacus you can make the numbers 2, 20 and 11.



Do the same with 3 beads. How many different numbers can you make?

How many different numbers can you make using 4 beads?

3.

Jo has £2·29.
She only has £1 coins, 10p coins and 1p coins.
How many of each coin does she have?
Can you suggest a different answer?

4.

Here is part of a number square.
What is the largest number on the whole square?

3	6	9	12	15
18	21	24	27	
33	36	39		
48	51	5		
63	66			

Rich and Sophisticated Tasks

Recognise the place value of each digit in a two-digit number (tens, ones)

NRICH: **Snail One Hundred** *** G**
NRICH: **Two-digit Targets** *** P**
NRICH: **6 Beads** **** P**

Compare and order numbers from 0 up to 100; use <, > and = signs

NRICH: **Domino Sequences** *** P**
NRICH: **Domino Number Patterns** **** P**
NRICH: **Next Domino** *** P**
NRICH: **100 Square Jigsaw** *** G**
NRICH: **That Number Square!** *** P I**

Use place value and number facts to solve problems

NRICH: **I Like …** *** G**
NRICH: **Light the Lights** ***** G**
NRICH: **Largest Even** *** G P**
NRICH: **Round the Two Dice** *** P I**

5.

Use < > and = signs to make these number sentences correct.

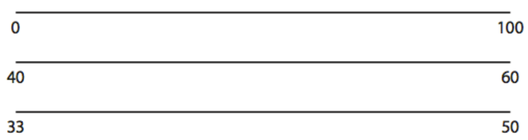
3 tens and 2 ones ☐ 2 tens 12 ones

4 tens and 3 ones ☐ 3 tens 14 ones

5 tens and 4 ones ☐ 4 tens 11 ones

6.

Place 47 on each of these empty number lines.



Misconceptions	Teacher Guidance and Notes
Children confuse 'teen' and 'ty' numbers e.g. fourteen and forty Children may use a 0 placeholder incorrectly, e.g. recording thirty-four as 304. Additionally some children incorrectly read out the ones before the tens e.g reading 23 as either thirty-two or sometimes three (and) twenty or two three rather than twenty-three. Children confuse the meaning of < and >, finding it hard to tell which is which. Children confuse the order of tens and ones and so will think that fifty-eight can be represented as 85 Children think of every number separately e.g. 23 is 23 ones and they cannot see it also as 2 tens and three ones Children also cannot see past standard partitioning e.g. they do not recognise eighty-thirteen as the same as ninety-three Children interpret = as ' the answer is' rather than 'is worth the same as' - make sure you model this carefully and avoid implying that it simply starts the answer!	You need to develop a strong understanding of place value so that children see that they only need to actually learn the numbers 0-9 because all other numbers just use these in different positions. When teaching place value use practical resources to expand on different base representations to emphasise the unitised structure of number ie 31 as 3 ten rods and 1 unit/ ones in Base 10 (the list of suggested apparatus is shown above and the intention is for all children to experience all of these representations) It is important that children develop their number sense here- they should be able to place numbers on a blank number line including where the scale changes. Try taking a blank paper strip as a scale from 0-100 and asking children to place 24 on it. Then change the scale to 1-50 and ask them to do the same thing - they should be developing the ability to change the placement based on the scale.

Key Assessment Checklist
1. I can read and write numerals and words from 10 to 50.
2. I can read and write numerals and words from 50 to 100.
3. I can say the number that is one more or less than a given number to 100.
4. I can understand the value of each digit in a 2-digit number and partition the number as tens and ones
5. I can represent any number up to 100 using: unifix cubes oe; a number line; a hundred square; numicon; cuisenaire rods; Dienes; a tens and ones abacus; a bead string, place value counters
6. I can solve problems using my knowledge of number facts and place value.
7. I can compare two numbers up 100 and use the signs <, > (and =) to show this comparison.
8. I can compare up to three numbers or amounts up to 100, order them and say which is the most and which is the least.

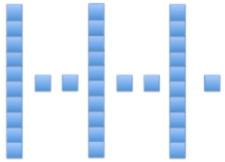
Year 2	Unit 2: Pattern Sniffing	
8 learning hours	This unit explores pattern from the early stages of counting and then counting in 2s, 5s, and 10s up to the more formal study of sequences. This sequence work progresses through linear sequences up to quadratic, other polynomial and geometric for the most able older students. For children in KS1, this unit is heavily linked to the following one in terms of relating counting to reading and writing numbers. Also in this unit children and students begin to study the properties of numbers and to hone their conjecture and justification skills as they explore odd/even numbers, factors, multiples and primes before moving onto indices and their laws.	
Prior Learning	**Core Learning**	**Extension Learning**
➢ count to and across 100, forwards and backwards, beginning with 0 or 1, or from any given number ➢ count, read and write numbers to 100 in numerals; count in multiples of twos, fives and tens	➢ **count in steps of 2, 3, and 5 from 0, and in tens from any number, forward and backward** ➢ **order and arrange combinations of mathematical objects in patterns and sequences** ➢ ***begin to recall*** **and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers** *Working towards expected standard* • *Count in twos, fives and tens from 0 and use counting strategies to solve problems e.g. count the number of chairs in a diagram where the chairs are organised in 7 rows of 5 by counting in 5s* *Working at expected standard* • *recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables to solve simple problems, demonstrating an understanding of commutativity as necessary (e.g. knowing they can make 7 groups of 5 from 35 blocks and writing 35 ÷ 5 = 7; sharing 40 cherries between 10 people and writing 40 ÷ 10 = 4; stating the total value of six 5p coins)* *Working at greater depth:* • *use multiplication facts to make deductions outside known multiplication facts (e.g. a pupil knows that multiples of 5 have one digit of 0 or 5 and uses this to reason that 18 × 5 cannot be 92 as it is not a multiple* of 5).	➢ count from 0 in multiples of 4, 8, 50 and 100; ➢ find 10 or 100 more or less than a given number ➢ recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables

Exemplification

1. a) Write the next three numbers in the number patterns:
   i) 0, 3, 6, 9, 12, 15, ... , ..., ...,
   ii) 0, 5, 10, 15, 20, 25, 30, 35, ..., ..., ...,

b) Count on in 10s from 37: ....

2. Continue this pattern with Dienes rods and ones:



3.
a) Complete the missing numbers in the number sentences:

i) 2 x 8 = ... ii) 5 x .... = 35 iii) 60 ÷ .... = 6 iv) ...... ÷ 2 = 7

b) Which of these numbers are even? Circle them:

13 56 73 84 100

Vocabulary

count from ...
count in ....
forwards/backwards
pattern
number pattern
tens and ones
place value
missing number
times table
multiplication
division
array
row column
groups
fact family
odd
even

Representation

Counting:

- Counting aloud using marked number lines, number tracks, 100 squares and counting sticks to prompt (Give children some apparatus to use alongside that which you are using with the group e.g. their own number track or their hundred square)
- Using a counting stick to start counting at a new value (forwards or backwards)
- Represent counting in 2s, 3s 5s and 10s using repeated addition with:
  - Numicon pieces
  - Counters on a blank track
  - Counters in groups of 2, 5 or 10
  - Bead string
  - Placing counter on/Colouring in 100-square

Fluency

1. Count from 0 in steps of 2, 3 and 5
   - work out the steps using repeated addition
   - work out some steps using doubling skills
   - count from 0 up to 10^{th} multiple of 2, 3 and 5 with concrete/visual aid
   - count from 0 up to 10^{th} multiple of 2, 3 and 5 without concrete/visual aid
   - count from 0 beyond 10^{th} multiple of 2, 3 and 5
2. Count in 10s
   - from 0 (forwards)
   - from 100 (backwards)
   - beyond 100 from a multiple of 10 (forwards or

Probing Questions

Show me
...what comes next 25, 30, 35, ...
...what comes next 20, 30, 40, ...
...what comes next 42, 52, 62, ...

What's the same and what's different?
33,36,39,41,45

Show me how we count in 10s on a hundred square

Convince me that 31 is ten more than 21

Convince me that 55 comes next 35, 40,

○ Money (2ps, 5ps and 10ps) • Represent counting in 10s (from any number) using a hundred square to discover the link between jumping a row and counting on (or back) ten • Use a counting stick to represent the first ten multiples of 2, 3, 5 and 10 – explore which values can be found by doubling.	backwards) ○ forwards from a number that is not a multiple of 10 (using equipment) ○ backwards from a number that is not a multiple of 10 (using equipment) ○ forwards from a number that is not a multiple of 10 (up to 100 only) ○ forwards from a number that is not a multiple of 10 (up to 100 only)	45, 50, ... What's the same and what's different? 3, 10, 13, 23? Always, Sometimes, Never? If I find 10 more than a number, it will end with the same digit as the number I started with.
Pattern • Building patterns from shapes, number equipment (Numicon, dienes etc) and other objects (e.g. animals, pencils) and beginning to describe the rule or pattern • Finding the next item in a practical pattern • Given a rule, building a pattern of objects	3. Build patterns and sequences ○ continue a two-object pattern ○ continue a three-object pattern ○ continue a more complex object pattern ○ begin to describe the rule of a pattern ○ arrange objects to produce a pattern ○ arrange objects to produce a pattern that follows a given rule	Show me a pattern using counting bears Show me what comes next in the pattern (blocks/numicon/other equipment) Show me how you can arrange the numicon pieces into a pattern Convince me that this is a pattern
Times Table Facts • Represent the 2 times table using a counting stick for the first ten multiples (see above) • Represent a times table multiplication calculation using repeated addition in multiple ways: e.g. 2 x 6 as: ○ 6 groups of 2 objects ○ 6 numicon 2s ○ 6 lots of 2-rods (Cuisenaire) ○ **as an array** made of 6 rows of 2 counters/dots to begin to deduce the related facts i.e. 2 x 6 = 12; 6 x 2 = 12; 12 ÷ 2 = 6 and 12 ÷ 6 = 2 (more on this in Unit 9)	4. Find and begin to recall times table multiplication facts (2s) ○ By representing the calculation concretely to deduce the answer ○ By representing the calculation visually to deduce the answer ○ By relating the calculation to another known calculation and counting on/back or doubling etc ○ By beginning to recall key facts	Show me how to represent 2 x 5 What's the same and what's different? 22,24,26,27

Odds and Evens • Use Numicon to explore odd and even numbers by building a number and then sorting out into representations with 'a bit that sticks out' and those that don't. • (You can do a similar thing with cubes i.e. which numbers of cubes can be arranged into an array with 2 columns and no leftovers?)	5. Recognise odd and even numbers ○ Say if a number is odd or even using apparatus ○ Say if an object is odd or even using the numerals ○ Find an odd or even number of a given size e.g. greater than 30 or between 50 and 60	Show me an odd number bigger than 20 Show me an even number between 40 and 50 Convince me that 53 is an odd number What's the same and what's different? 1,3,5,8 What's the same and what's different? 2,12,24,36? Always, Sometimes, Never? Even numbers always end in 0,2,4,6,8. Even numbers can always be shared. Odd numbers always end in 1,3,5,7,9.
Times Table Facts • Represent the 5 and 10 times tables using a counting stick for the first ten multiples (see above) • Represent a times table multiplication calculation using repeated addition in multiple ways: e.g. 5 x 6 as: ○ 6 groups of 5 objects ○ 6 numicon 5s ○ 6 lots of 5-rods (Cuisenaire) ○ **as an array** made of 6 rows of 5 counters/dots to begin to deduce the related facts i.e. 5 x 6 = 30; 6 x 5 = 30; 30 ÷ 5 = 6 and 30 ÷ 6 = 5 (more on this in Unit 9)	6. Find and begin to recall times table division facts (10s, 5s) ○ By representing the calculation concretely to deduce the answer ○ By representing the calculation visually to deduce the answer ○ By relating the calculation to another known calculation and counting on/back or doubling etc ○ By beginning to recall key facts	Convince me that the answer to 4 x 5 is an even number What's the same and what's different? 25,30,36,40

Further Extension

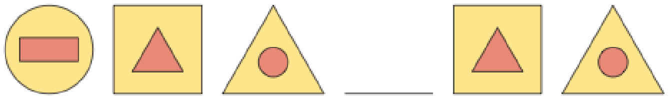
1.

Captain Conjecture says, 'When I count in tens from any number the units digit stays the same.'
Do you agree?

Explain your reasoning.

2.

Fill in the missing shape to complete the pattern.



If the pattern continued what would the tenth shape be?

Explain your reasoning.

3.

Amy thinks of a number. Her number:

- is an even number
- is between 20 and 25
- has two different digits.

What is her number?

Explain your reasoning.

4.

Think of an even number that is more than 30 and less than 50. And another.
Can you find them all? How many are there?

Explain your reasoning.

Rich and Sophisticated Tasks

Count in steps of 2, 3, and 5 from 0, and in tens from any number, forward or backward

NRICH: Buzzy Bee * P
NRICH: Five Steps to 50 * I

Odd and Even Numbers
NRICH: **How Odd ****
NRICH: **Even and Odd ***

Patterns
NRICH: **Poly Plug Pattern ***

Misconceptions	Teacher Guidance and Notes
Pupils forget to include 0 when counting When counting in 10s, children forget what to do after 'ninety-something' as they bridge 100 Children change the rule in a pattern so that it doesn't flow throughout - they also interpret a pattern as 'pretty' rather than a sequence with a rule.... Children think that any number with a 3 in it will be odd e.g. 34. Additionally they believe that any number ending in 3 is a multiple of 3!	• This unit focuses on counting and the beginnings of the use of repeated addition to underpin multiplication. Note that multiplication is fully explored in Unit 9 and 13 and therefore you should avoid delving too deep into commutativity etc at this stage to find fact families. Instead concentrate on representing the 'counting in 2s' process as repeated addition of 2 and on saying this as '2 multiplied by ... is ....'. The other facts will follow later for most children. • Define an even number as a number that is an exact multiple of 2 rather than 'one ending in 2, 4, 6, 8, 0' so that children are encouraged to explore what the number properties actually are to decide. Allow them to 'discover' the shortcuts of using the numerals rather than telling them the rule.

Key Assessment Checklist

1. I can count in steps of 2s, and 5s from 0. I can talk about the number patterns.
2. I can count in steps of 3s from 0.
3. I can count forwards and backwards in 10s from any given number.
4. I can recognise odd numbers.
5. I can recognise even numbers.
6. I can use numicon to make mathematical patterns.
7. I can use mathematical resources to generate sequences.
8. I can begin to recall multiplication and division facts for 2s, 5s and 10s.

Year 2	Unit 3: Solving Calculation Problems	
11 learning hours	This unit explores the concepts of addition and subtraction at KS1 building to wider arithmetic skills including multiplication at KS2. It is strongly recommended that teachers plan this unit for KS1/KS2 with direct reference to the calculation policy! At KS3 students are developing calculation into its more general sense to explore order of operations, exact calculation with surds and standard form (which have been introduced in Inv Number Systems briefly) as well developing their skills in generalising calculation to algebraic formulae.	
Prior Learning	**Core Learning**	**Learning Leads to....**
• represent and use number bonds and related subtraction facts within 20 • add and subtract one-digit and two-digit numbers to 20, including zero • read, write and interpret mathematical statements involving addition (+), subtraction (-) and equals (=) signs	• **recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100** • **add and subtract numbers using concrete objects, pictorial representations, and mentally, including:** **- a two-digit number and ones** **- a two-digit number and tens** **- two two-digit numbers** **- adding three one-digit numbers** • **show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot** *Working towards* • *use number bonds and related subtraction facts within 20 (e.g. 18 = 9 + ?; 15 = 6 + ?)* • *add and subtract a 2-digit number and ones and a 2-digit number and tens when no regrouping is required (e.g. 23+5; 46+20), demonstrate their method using concrete apparatus or pictorial representations* *Working at expected standard* • *add 2 two-digit numbers within 100 (e.g. 48 + 35) and demonstrate their method using concrete apparatus or pictorial representations* • *use estimation to check that their answers to a calculation are reasonable (e.g. knowing that 48 + 35 will be less than 100)* • *subtract mentally a two-digit number from another two-digit number when there is no regrouping required (e.g. 74 – 33)* *Working at greater depth:* • *reason about addition (e.g. pupil can reason that the sum of 3 odd numbers will always be odd)* • *work out mental calculations where regrouping is required (e.g. 52 – 27 or 91-73)*	➢ add and subtract numbers mentally, including: - a three-digit number and ones - a three-digit number and tens - a three-digit number and hundreds ➢ add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction estimate the answer to a calculation and use inverse operations to check answers

Exemplification

1. Find the value of the ★ in each statement:

a) ★+ 5 = 13 b) 16 = ★+★ c) 18 - 7 = ★ d) 15 = 19 - ★

e) Use the cards to complete the number facts:

30 **90** **60**

☐ + ☐ = ☐

☐ – ☐ = ☐

2. Calculate (using objects and images to support as required):

a) 78 + 7 b) 56 – 3 c) 63 - 20 d) 48 + 50 e) 35 + 28 f) 67 – 25 g) 3 + 8 + 4 h) 6 + 4 + 9

3. True or False?

a) 7 + 6 has the same answer as 6 + 7 b) 7 – 6 has the same answer as 6 – 7

Vocabulary

add	how many more?
and	take (away)
more	leave
make	how many left?
sum	less
total	fewer
altogether	difference
score	between
double	equals
one more	is the same as
two (ten) more	minus
plus	number sentence
equals	order
hundred	(commutative)
ten	
one	
exchange	
column	

Representation

Addition

- Representing numbers using tens and ones equipment then combining and finding the total value (aggregation) (exchanging ten 1s for one 10 as required when bridging)
  Progression of tens and ones equipment:
  - **separable**: bundles of straws or sticks of multilink cubes
  - **inseparable**: Dienes rods or Base 10 or Numicon 10s and1s
  - **not to scale**: place value counters or money
- Representing addition as counting or jumping on (augmentation) using:
  - a number track
  - a bead string (reading the answer using 10s and 1s)

Fluency

1. Add a two digit number and ones
   - 2-digit number (not ending in 9) + 1
   - 2-digit number (ending in 9) + 1
   - 2-digit number (not ending in 8 or 9) + 2
   - 2-digit number (ending in 8 or 9) + 2
   - 2-digit number + 3, 4, 5,....,(not crossing next ten) e.g. 34 + 3
   - 2-digit number + 3, 4, 5,....,(crossing next ten) e.g. 34 + 9
   - 2-digit number + 3, 4, 5, ... (making exact multiple of 10) e.g. 34 + 6
   - 2-digit number + 0
2. Add a two digit number and tens
   - 2-digit multiple of 10 + 10 e.g. 30 + 10
   - 2-digit number +10
   - 2-digit multiple of 10 + 20 e.g. 30 + 20
   - 2-digit number + 20

Probing Questions

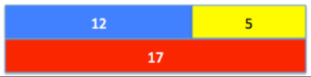
Show me two numbers with a sum of 20
Show me two numbers with a sum of 17

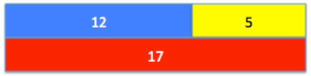
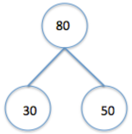
Always, Sometimes, Never?
A two-digit number add a one-digit number gives a two-digit number

Convince me that if I add ten to a number I will get the same digit in my 1s column as a I started with

Always, Sometimes, Never?

○ a hundred square (just a number track split into rows – jumping in 10s and 1s) ○ a marked number line (jumping in 10s and 1s) ○ an unmarked number line • Representing addition word problems using the bar model	• 2-digit multiple of 10 + 30, 40, 50, ... (answer ≤ 100) • 2-digit number + 30, 40, 50, ... (answer ≤ 100)	Addition makes a number larger.
	3. Add a two-digit number and a two-digit number (answer ≤ 100) • two-digit + two-digit (not crossing a ten) • two-digit + two-digit (crossing a ten) • two-digit + two-digit (answer a multiple of ten)	Show me two numbers with a sum of 50 Show me two numbers that are easy to add Show me two numbers that are hard to add Always, Sometimes, Never? A two-digit number add a two-digit number gives a two-digit number
Subtraction • Representing first number using tens and ones equipment then removing or taking away the second number and finding the resulting value (paritioning) (exchanging one 10 for ten 1s as required when bridging) Progression of tens and ones equipment: ○ **separable**: bundles of straws or sticks of multilink cubes ○ **inseparable**: Dienes rods or Base 10 or Numicon 10s and1s ○ **not to scale**: place value counters or money • Representing subtraction as counting or jumping back (reduction) using: ○ a number track ○ a bead string (reading the answer using 10s and 1s) ○ a hundred square (just a number track split into rows – jumping in 10s and 1s) ○ a marked number line (jumping in 10s and 1s) ○ an unmarked number line • Representing subtraction as a comparative difference between two sets of objects using: ○ counters ○ beadstrings ○ Numicon tens and ones (and other	4. Subtract ones from a two-digit number • 2-digit number (not ending in 0) - 1 • 2-digit number (ending in 0) - 1 • 2-digit number (not ending 0 or 1) - 2 • 2-digit number (ending in 0 or 1) - 2 • 2-digit number - 3, 4, 5,....,(not crossing next ten) e.g. 34 - 3 • 2-digit number - 3, 4, 5,....,(crossing next ten) e.g. 34 - 9 • 2-digit number - 3, 4, 5, ... (making exact multiple of 10) e.g. 34 - 4 • 2-digit number - 0	Always, Sometimes, Never? Subtraction makes a number smaller Convince me that when I subtract 9 it is the same as subtracting 10 and adding 1.
	5. Subtract tens from a two-digit number • 2-digit multiple of 10 - 10 e.g. 80 - 10 • 2-digit number - 10 • 2-digit multiple of 10 + 20 e.g. 80 - 20 • 2-digit number - 20 • 2-digit multiple of 10 - 30, 40, 50, ... e.g. 70 - 40 • 2-digit number - 30, 40, 50, ... e.g. 73 - 40	Show me two numbers with a difference of 20
	6. Subtract a two-digit number from a two-digit number • two-digit - two-digit (not crossing a ten) • two-digit - two-digit (answer a multiple of ten) • two-digit - two-digit (crossing a ten)	Convince me that the number 60-12 (sixty-twelve) is another way of saying 72 Show me two numbers that are easy to subtract Show me two numbers that are hard to subtract

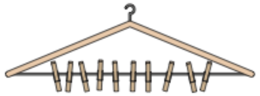
tens and ones equipment) laid out in two lines ○ number lines with both numbers marked and difference found • Representing subtraction word problems using the bar model		Always, Sometimes, Never? A two-digit number subtract a two-digit number gives a two-digit number
Commutativity • Exploring whether a+b gives the same answer as b+a using different equipment (Numicon, Dienes, bead strings, counters) • Exploring whether a – b gives the same answer as b – a using different equipment (discovering that when you try to take a larger number from a smaller one you 'run out' of objects to take away)	7. State whether two addition or subtraction calculations have the same result • addition of single digits • addition of two-digit numbers • subtraction of single digits • subtraction of two-digit numbers • general principle for addition • general principle for subtraction	Convince me that the answer to 45 + 28 will be the same as the answer to 28 + 45 Convince me that the answer to 7 + 9 + 6 will be the same as 6 + 9 + 7 AND 9 + 7 + 6. What's the same and what's different? addition and subtraction
Fact Families • Drawing a bar model to represent the four related calculations E.g. 12 + 5 = 17; 5 + 12 = 17; 17 – 5 = 12 and 17-12 = 5 	8. Find fact family given one addition/subtraction fact • single digit addition fact given e.g. 6 + 3 = 9 (so find 3 + 6 = 9, 9 – 6 = 3 and 9 – 3 = 6) • two-digit addition fact given e.g. 26 + 37 = 63 • single digit subtraction fact given e.g. 8 - 3 = 5 • two-digit subtraction fact given e.g. 43 – 28 = 15	Show me if I know that 7 + 4 = 11, what else I know?
Number Bonds • Exploring the pattern of bonds with the same answer e.g. using two different colour counters to show the different ways that you can make 11 by adding two numbers Double-sided counters can work well for this. • Alternatively use a bead string to see how you can partition a number in different ways e.g. count out 13 beads and find all the different pairs of numbers that add to 13 by splitting up your beads differently. • Using a number line/counting stick to find pairs of numbers with the same total – a string number line is excellent for this as you can hold it up to show pairs of numbers with the same total hanging together	9. Recall number bonds • bonds to 10 • bonds within 10 (i.e. to 9, 8, 7, 6, 5, 4, 3) • bonds to 20 • bonds within 20 (i.e. to 11, 12, 13, 14, 15, 16, 17, 18, 19)	Show me all the pairs of numbers with a sum of 13 What's the same and what's different? 6+4, 5+5, 3+8, 1+9
	10. Add three one-digit numbers • answer < 10 • example where two numbers bond to 10 e.g. 6 + 4 + 7 or 7 + 8 + 3 • example where two numbers bond to 11 e.g. 8 + 3 + 5 • examples where two numbers of the three bond to 12, 13, etc.	Show me three one digit numbers with a sum of 15 Convince me that there are 5 more sums with the same numbers that give the same answer as 4 + 6 + 8

• Using a bar model to represent bonds, for example,  • Using a part-part-whole model to represent number facts: for example,  • Using tens place value counters or other 'tens' objects to represent facts such as 20 + 70 = 90 and make the link to 2 + 7 = 9	11. Find related number facts • addition of multiples of ten e.g. 30 + 40 using recall knowledge of 3+4 • subtraction of multiples of ten e.g. 70–50 using knowledge of 7-5 • tens + single digits e.g. 30 + 7 = 37 using recall knowledge of 0+7 • tens – single digits e.g. 40 – 6 using recall knowledge of 10-6	Show me if I know that 7 + 4 = 11, what else I know? Convince me that 80 - 50 = 30 What's the same and what's different? 30, 70, 20, 80 What's the same and what's different? 90-50, 80-40, 100-60, 70-40

Further Extension	Rich and Sophisticated Tasks
1. Find different possibilities. $\square + \square = 50$ $50 - \square = \square$ 2. Complete the calculations. $30 + 40 + \square = 100$ $40 + \square + 20 = 100$ $36 + 44 + \square = 100$ $36 + 54 + \square = 100$ $47 + \square + 20 = 100$ $47 + \square + 30 = 100$	Recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100 **NRICH: Number Round Up *** G P** **NRICH: 4 Dom *** G P** **NRICH: Strike it Out * G** Add and subtract numbers using concrete objects, pictorial representations, and mentally, including: • a two-digit number and ones • a two-digit number and tens • two two-digit numbers • adding three one-digit numbers **NRICH: Cuisenaire Environment * G** **NRICH: Jumping Squares ** G** **NRICH: Number Balance ** I** **NRICH: Unit Differences * P I** **NRICH: Dicey Addition * G**

3.

If each peg on the coat hanger has a value of 10, find three ways to partition the pegs to make the number sentences complete.



□ + □ + □ = □

□ + □ + □ = □

□ + □ + □ = □

What is the total of each addition sentence?
Will the total always be the same?
Explain your reasoning.

4.

Fill in the missing numbers. What do you notice?

27	
15	?

12	15
?	

37	
15	?

23	14
?	

13	14
?	

57	
15	?

5.

Captain Conjecture says,
'An odd number + an odd number + an odd number = an even number'.
Is this sometimes, always or never true?

Explain your reasoning.

Concrete resources might help pupils to explain their reasoning.

Misconceptions	Teacher Guidance and Notes
Children struggle to interpret whether to add or subtract from the language used. Children can find 'How many more/less?' particularly troublesome as it relates to ordinal values of numbers and relationships. Children struggle to add two digit numbers when their place value understanding is weak. If they do not read a number like '52' as 5 tens and 2 ones then they struggle to combine the ones and the tens from two numbers appropriately. Children often do not see difference as a representation of subtraction because take away is emphasised so much. They need to see subtraction represented in this way also to challenge this. When subtracting, children will sometimes subtract the larger number from the smaller initially. They may also fail to exchange a ten for 10 ones when necessary and simply find the difference between the digits in the column e.g. 43 - 27 - they may find the difference between the 3 and the 7 in the ones column as 4 rather than exchanging a ten for 10 ones to get 13 - 7. When exchanging does happen, children sometimes forget to 'remove' the 10 and so end up with ten too many in the answer. When counting on, children may start counting the first extra number using the start number itself, rather than counting the next number (thus they end up with one less than the real answer. The same is true for subtraction in reverse. Look out for weak counting in general, which will hold this unit back a great deal. Children forget to include 0 when counting and using number lines – they often see the distance between 0 and 1 as being different to that between other whole numbers (wrongly!). When working with number facts and bonds, children sometime realise there is a connection e.g. 3 + 4 = 7 but then incorrectly rearrange this to make a false second fact e.g. 4 + 7 = 3. This is particularly true with subtraction facts, where children struggle to place the numbers in a correct order.	• The aim of this unit for these children is to explore how we can more efficiently add up two numbers beyond the known methods of counting all or counting on. Therefore, many of the techniques at Stage 2 rely on some understanding of place value and so this must be secure first. You should make reference to the calculation policy specifically when teaching this unit as the range of equipment and the language are modelled there. • The Representing section shows the different ways we can carry out addition – in general the place value approaches are leading towards (eventually!) written column methods while the counting on/number line approaches are leading towards mental methods. Both are important but they are not interchangeable so think carefully about whether you are focusing on written or mental calculation to decide which representation is best in a given lesson/skill/concept. Both need to be covered in the unit but not necessarily at the same time! • When learning about addition that bridges 10, children need to literally and practically exchange 10 ones for a ten so that they can see how this happens when we come to the written method. More advanced children can carry out this procedure with apparatus such as place value counters where the value of the counter is not obvious from its appearance. Similarly, when learning about subtraction that bridges 10, children need to physically exchange a ten for 10 ones to enable them to carry out the subtraction of the 1s. They can eventually move on to doing this with more abstract equipment such as place value counters. • To begin to embed the written routines of the calculation policy, it is advised that children work in pairs with one child manipulating the equipment and saying what they are doing aloud while the other child records the calculation using the column method so that they learn that the column method is just a written representation of the practical process (rather than a 'different' method). • It is important to let children explore whether order matters when we add and subtract - this is a great opportunity for some early investigation. Does 44 + 27 give you the same answer as 27 + 44? etc. This knowledge saves a lot of time then in recalling number bonds and facts as only half need to be learnt! • The pitch of this unit is numbers up to 100, but of course these ideas an be extended beyond 100 for those children who are confident working with in this area. • Children need to see and use a variety of question types during this work

The equals sign is not always correctly interpreted as 'has the same value as' by children, who may see it as 'the answer is'.	including: oral questions 'thirty-four add forty-eight', written questions using symbols '71 –34', simple 'real' problems such as shopping and abstract problems such as finding as many pairs of numbers with a sum of 41 as possible and spot the pattern. • Try to model the wide range of language used to signify addition and subtraction – see vocabulary list above. The children ultimately need to be able to recognise that a problem is an addition problem from the language (and same for subtraction). • Use 'sum' only to mean an addition calculation – use the word 'calculations' to mean mixed operation computations • Challenge issues with the use of the = sign by looking at examples where the question is on the right e.g. ? = 4 + 8 as well as balance problems in Further Extension e.g. 3 + 4 = ? + 2

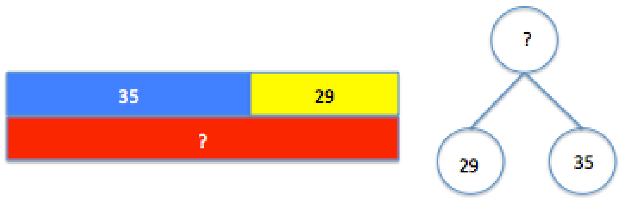
Key Assessment Checklist

1. I can quickly recall addition and subtraction facts to 20 and use them to derive facts to 100
2. I can mentally add and subtract ones to and from a two-digit number
3. I can add and subtract tens to and from a two-digit number
4. I can add and subtract two two-digit numbers that do not bridge ten with equipment.
5. **I can add any two two-digit numbers (using apparatus and recording)**
6. **I can subtract any two two-digit numbers (using apparatus and recording)**
7. I can add three one digit numbers
8. I can say whether the order matters in a statement

Year 2	Unit 4: Generalising Arithmetic
10 learning hours	This unit is focused on developing fluency in the manipulation of number. At primary level this is focused on arithmetic itself and the methods for four operations particularly; however, this is naturally generalised to thinking about rules of arithmetic more widely at secondary level i.e. algebra. These aspects have been paired together intentionally to help teachers describe algebra as simply a generalisation of number. It is expected that teachers will go back to arithmetic to help students see where the 'rules' of algebra come from. Note that the greyed out content is covered elsewhere and hence is not required content here.

Prior Learning	Core Learning	Learning Leads to...
➢ solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as 7 = ☐ – 9 ➢ read, write and interpret mathematical statements involving addition (+), subtraction (–) and equals (=) signs	➢ **Solve problems with addition and subtraction:** • **using concrete objects and pictorial representations, including those involving numbers, quantities and measures** • **applying their increasing knowledge of mental and written methods** ➢ **recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems.** *Working at expected standard* • *recognise the inverse relationships between addition and subtraction and use this to check calculations and work out missing number problems (e.g. Δ – 14 = 28).* • *use estimation to check that their answers to a calculation are reasonable (e.g. knowing that 48 + 35 will be less than 100)* *Working at greater depth:* • *solve more complex missing number problems (e.g. 14 + – 3 = 17; 14 + Δ = 15 + 27).*	➢ solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction

Exemplification	Vocabulary	
1. a) There are 24 children in Class A and 28 children in class B. How many children is this altogether? b) There are 53 people on a bus. Some people get off at the bus stop. There are now only 29 people left. How many people got off at the bus stop? 2. a) Joey thinks of number. He adds 14 and the answer is 41. What is Joey's number? b) Amy has some string. She cuts off a piece that is 23cm long. Amy is left with only 55cm of string. Amy says "I must have started with 22cm string". Do you agree with Amy?	add addition plus more increase altogether total sum subtract subtraction take away leaves / left	represent apparatus tens and ones number line bar model part-part-whole inverse opposite fact family commutative missing number solve

Representation	Fluency	Probing Questions
Addition • Representing numbers using tens and ones equipment then combining and finding the total value (aggregation) (exchanging ten 1s for one 10 as required when bridging) Progression of tens and ones equipment: ○ **separable**: bundles of straws or sticks of multilink cubes ○ **inseparable**: Dienes rods or Base 10 or Numicon 10s and1s ○ **not to scale**: place value counters or money • Representing addition as counting or jumping on (augmentation) using: ○ a number track ○ a bead string (reading the answer using 10s and 1s) ○ a hundred square (just a number track split into rows – jumping in 10s and 1s) ○ a marked number line (jumping in 10s and 1s) ○ an unmarked number line	1. Add two two-digit numbers together (recap) • two-digit add one-digit • two-digit add two-digit (no exchange) • two-digit add two-digit (exchanging from 1s to 10s)	Show me how you can add 34 and 47 together using equipment Convince me that 39 + 45 = 84
Representing Addition Problems • Representing addition word problems using the bar model or a part-part-whole model 	2. Recognise, represent and solve a simple addition problem (2-digit numbers) • simple one-step word problem with addition trigger word and given structure for representation (e.g. blank bar model or blank part-part-whole) • simple one-step word problem with addition trigger word (without scaffolded structure) • one-step addition word problem with subtle reference to addition • two-step problems requiring two additions	Show me two numbers that sum to 83 Show me three numbers that sum to 100 Always, Sometimes, Never? Addition is commutative i.e. the order doesn't matter
Subtraction • Representing first number using tens and ones equipment then removing or taking away the second number and finding the resulting value (partitioning) (exchanging one 10 for ten 1s as required when bridging) Progression of tens and ones equipment:	3. Subtract a two-digit number from a two-digit number (recap) • two-digit subtract one-digit • two-digit subtract two-digit (no exchange) • two-digit subtract two-digit (exchanging from 10s to 1s)	Show me how we can find 76 - 48 using equipment Always, Sometimes, Never? Subtraction is commutative i.e. the order doesn't matter

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separable: bundles of straws or sticks of multilink cubes
inseparable: Dienes rods or Base 10 or Numicon 10s and1s
not to scale: place value counters or money

Representing subtraction as counting or jumping back (reduction) using:

a number track
a bead string (reading the answer using 10s and 1s)
a hundred square (just a number track split into rows – jumping in 10s and 1s)
a marked number line (jumping in 10s and 1s)
an unmarked number line

Representing subtraction as a comparative difference between two sets of objects using:

counters
beadstrings
Numicon tens and ones (and other tens and ones equipment) laid out in two lines
number lines with both numbers marked and difference found

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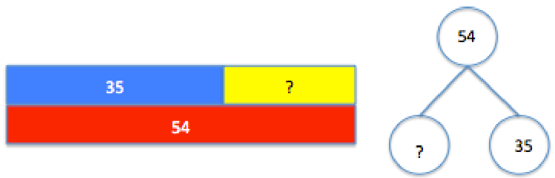
Representing Subtraction Problems:
Representing subtraction word problems using the bar model or a part-part-whole model



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4. Recognise, represent and solve a simple subtraction problem (2-digit numbers)

simple one-step word problem with subtraction trigger word and given structure for representation (e.g. blank bar model or blank part-part-whole)
simple one-step word problem with subtraction trigger word (without scaffolded structure)
one-step subtraction word problem with subtle reference to subtraction
two-step problems requiring two subtractions

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Show me two numbers that have a difference of 27

Convince me that if I have 38 pencils and I take away 13, there are 25 left.
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2-Step Problems
Representing problems using the bar model
Example:
There are 37 people on a bus. 17 people get on the bus at the next stop.
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5. Recognise, represent and solve two-step problems combining addition and subtraction

simple problems where operation is clear (e.g. people getting on and off a bus or items going in and out of a bag) and structure for

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Convince me that you need to use addition and subtraction to solve this problem 'There were 19 people on the train then 14 got off and 21 got on. How many are left?'
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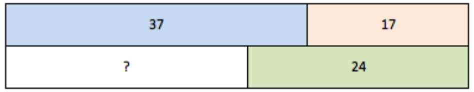
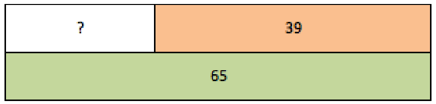
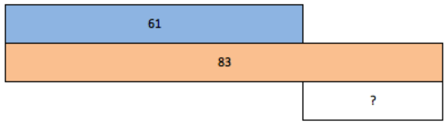
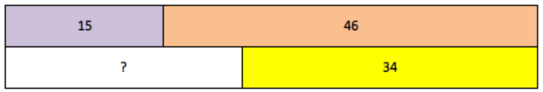
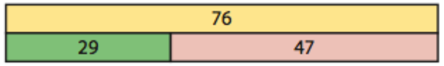
Then 24 people get off the bus. How many are left?  	representation provided (e.g. blank bar model or part-part-whole model) • simple problems where operation is clear (e.g. people getting on and off a bus or items going in and out of a bag) without a structure for representation provided • more complex examples involving subtle reference to addition and subtraction	What's the same and what's different? Addition and Subtraction
Checking Calculations • Reversing a calculation to check an answer by adding or subtracting to see if this gives the original value • Rounding numbers to nearest 10 to estimate the value of the calculation first	6. Check a calculation using inverse operations • recap: state the fact family for a calculation e.g. 63 + 24 • estimate the answer to a calculation by rounding • check an addition calculation • check a subtraction calculation • check a two-step calculation	Show me the fact family for 73 + 19 Always, Sometimes, Never? If I know one number fact, I automatically know 3 others Always, Sometimes, Never? You check an addition using a subtraction and a subtraction using an addition.
Missing Number Problems • Representing using a bar model Example 1 $\blacksquare + 39 = 65$  Example 2: $61 = 83 - \blacksquare$  	7. Solve an missing number addition problem using a subtraction • second number missing e.g. $23 + \blacksquare = 45$ • first number missing e.g. $\blacksquare + 39 = 65$ • answer given first, second number missing e.g. $61 = 23 + \blacksquare$ • answer given first, first number missing e.g. $43 = \blacksquare + 29$	Always, Sometimes, Never? If I know that ? + ■ = 50, then 50 – ■ = ?
	8. Solve a missing number subtraction problem using an addition or a subtraction • first number missing e.g. $\blacksquare - 39 = 25$ • second number missing e.g. $63 - \blacksquare = 45$ • answer given first, first number missing e.g. $43 = \blacksquare - 29$ • answer given first, second number missing e.g. $61 = 83 - \blacksquare$	What's the same and what's different? $\blacksquare - 17; 17 - \blacksquare$ Always, Sometimes, Never? You find a missing number in a subtraction by adding.
	9. Recognise, represent and solve a missing number problem • addition problems e.g. Martha has some	Always, Sometimes, Never? Word problems can be represented with a bar model

	sweets. Her aunt gives her 18 more sweets, so now she has 42. How many did she start with? • subtraction problems (smaller amount missing) e.g. Ella bakes 40 cakes. She sells some to her neighbours. She is left with 11 cakes. How many did Ella sell? • subtraction problems (larger amount missing) e.g. Some football teams enter a competition. 14 football teams drop out, so there are now only 52 football teams left. How many football teams entered at first? • mixture of addition/subtraction problems where children select appropriate operation • 2-step problems e.g. There are some people on a bus. 17 more people get on. Then 24 people get off. There are now 30 people on the bus. How many people were there to start with?	
Combination Missing Number Problems • Representing balancing problems e.g. $15 + 46 = \blacksquare + 34$ using scales, real or virtual (could use Numicon as weighted) • Using bar models to represent these problems 	10. (Ext) Solve combination missing number problems • abstract, all addition e.g. $23 + 41 = 72 - \blacksquare$ • abstract, all subtraction e.g. $71 - \blacksquare = 52 - 39$ • abstract, one of each operation e.g. $34 + \blacksquare = 19 + 51$ • worded problems (and hence requiring representation) e.g. Jane is given 15 sweets by her mum and 32 sweets by her uncle. Jane now has the same number of sweets as Paul, who started with 65 but gave some to his sister. How many sweets did Paul give to his friend? • problems where only a digit of a number is missing e.g. $52 - \blacksquare 3 = 29$	What's the same and what's different? $23 + 41 = 72 - \blacksquare$ $72 - \blacksquare = 23 + 41$ $23 + 41 = \blacksquare - 72$

Further Extension

1.



Using the bar model complete the four number sentences.

□ + □ =

□ + □ =

□ - □ =

□ - □ =

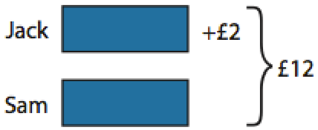
2.

Dan needs 80 g of sugar for his recipe. There are 45 g left in the bag. How much more does he need to get?

The temperature was 26 degrees in the morning and 11 degrees colder in the evening. What was the temperature in the evening?

A tub contains 24 coins. Saj takes 5 coins. Joss takes 10 coins. How many coins are left in the tub?

3.
Together Jack and Sam have £12.
Jack has £2 more than Sam.
How much money does Sam have?
A bar model can be very helpful in solving these types of problems.



£12 – £2 = £10
£10 ÷ 2 = £5
Sam has £5

Rich and Sophisticated Tasks

Solve problems with addition and subtraction:

- using concrete objects and pictorial representations, including those involving numbers, quantities and measures
- applying their increasing knowledge of mental and written methods

NRICH: Getting the Balance * I**
NRICH: Noah ** P
NRICH: Eggs in Baskets ** P
NRICH: The Brown Family * G P**
NRICH: Birthday Cakes ** P
NRICH: Sitting Round the Party Tables * P I
NRICH: Cuisenaire Counting * G P**
NRICH: Two Spinners * I
NRICH: Heads and Feet ** P
NRICH: Double or Halve? * G

Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and missing number problems

NRICH: The Add and Take-away Path * I
NRICH: Secret Number ** G
NRICH: How Many? * G P
NRICH: What Was in the Box? * G P
NRICH: Doing and Undoing * I

Misconceptions	Teacher Guidance and Notes
Children do not use place value when adding and subtracting - signs of this can be them counting repeatedly from 0 or failing to use models that group tens differently. They may need to use a wider range of representations to develop this idea more strongly - some children can do this when the model 'looks' like 10 but not when the visual link has gone. Children confuse the order of their facts e.g. they know that 16 + 5 = 21 and so they say that 21 + 5 = 16. This is because children do not always understand that addition can be done in any order but that subtraction is not commutative. A significant challenge for children in this unit is making the decision about whether to add or subtract the numbers that they have been given. This relies on a solid language understanding of the many variants of words used to imply add and subtract, which may be weak.	• In this unit pupils build on their skills of addition and subtraction as developed in Unit 3 to apply them to problems and wider contexts. They also explore the inverse of addition and subtraction, using this to check calculations and to solve missing number problems • However, this unit is also an opportunity to consolidate the skills and fluency of addition and subtraction (particularly if children did not easily master these operations with larger numbers in Unit 3). • The use of manipulatives and visual representations is stipulated in the objectives of this unit and so these should be available throughout – the representation section gives more guidance on what could/should be used and how • It is critical that children learn how to turn a word problem into a number problem and make the decision about whether it is asking them to add or subtract (or possibly both if it is a 2-step problem). • The bar model and part-part-whole models are strongly recommended as ways to represent a word problem to help decide which calculation to use. • Language here is key – try to model different terms that imply add/subtract to broaden children's vocabulary • Try to avoid implying that it is not possible to subtract a larger number from a smaller number as this may cause problems when introducing negative numbers later on.

Key Assessment Checklist

1. I can add a two-digit number to a two-digit number using manipulatives, pictures and my head
2. I can subtract a two-digit number from another two-digit number using manipulatives, pictures and my head
3. I can represent and solve addition word problems
4. I can represent and solve subtraction word problems
5. I can check a calculation using the inverse
6. I can use the inverse of addition and subtraction to find missing numbers.

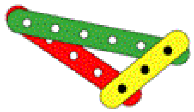
Year 2	Unit 5: Exploring Shape
8 learning hours	In this unit children and students explore the properties of shapes, both 2D and 3D. At KS1 this is focused on common shape names and basic features of vertices, sides etc. but this then develops to classifying quadrilaterals and triangles in KS2. Alongside this focus children begin to explore angle and turn in KS2 and develop this to more formal angle rules through Stages 5, 6, 7, 8. Older students begin to explore the field of trigonometry, encountering first Pythagoras' Theorem, then RA-triangle trig before finally looking a the sine rule and cosine rule.

Prior Learning	Core Learning	Learning Leads to...
➢ recognise and name common 2-D and 3-D shapes, including: - 2-D shapes [for example, rectangles (including squares), circles and triangles] - 3-D shapes [for example, cuboids (including cubes), pyramids and spheres]	➢ **identify and describe the properties of 2-D shapes, including the number of sides and line symmetry in a vertical line** ➢ **identify and describe the properties of 3-D shapes, including the number of edges, vertices and faces** ➢ **compare and sort common 2-D and 3-D shapes and everyday objects** *Working towards* • *recognise and name triangles, rectangles, squares, circles, cuboids, cubes, pyramids and spheres from a group of shapes or from pictures of the shapes* *Working at expected standard* • *describe the properties of 2-D and 3-D shapes (e.g. the pupil describes a triangle: it has 3 sides, 3 vertices and 1 line of symmetry; the pupil describes a pyramid: it has 8 edges, 5 faces, 4 of which are triangles and one is a square)* *Working at greater depth* • *describe similarities and differences of shape properties (e.g. finds 2 different 2-D shapes that only have one line of symmetry; that a cube and a cuboid have the same number of edges, faces and vertices but can describe what is different about them).*	➢ identify horizontal and vertical lines and pairs of perpendicular and parallel lines ➢ recognise angles as a property of shape or a description of a turn ➢ identify right angles, recognise that two right angles make a half-turn, three make three quarters of a turn and four a complete turn; identify whether angles are greater than or less than a right angle

Exemplification	Vocabulary	
1. a) Complete this statement: A hexagon has .... edges and .... vertices b) I am thinking of a 2D shape. It has 4 sides. What could my shape be? Give two possible answers. c) Draw a triangle without a line of symmetry 2. a) Complete this statement: A cuboid has .... faces, and ....vertices b) I am thinking of a 3D shape. It has a circular base. What could my shape be? Give two possible answers. c) True of False: a pyramid has 5 faces	2D circle triangle square rectangle parallelogram rhombus quadrilateral pentagon	3D cube cuboid sphere pyramid ...-based (pyramid) prism (.........) prism cylinder

3. Look at these shapes: Rectangle Pentagon Cone Triangular Prism Square Triangle Circle Cuboid Which of these shapes a) are quadrilaterals? b) have only straight sides/edges? c) are prisms?	hexagon octagon decagon straight sides vertices (line of) symmetry	cone hemisphere faces edges vertices (vertex) base sort classify regular/irregular

Representation	Fluency	Probing Questions
2D Shapes: Sides and Vertices • Matching the names of shapes to images of the shapes and to solid versions. [NRich version of shape cards] • Counting (and marking off) the vertices of a 2D shape • Using shapes to make pictures – mosaic pieces, sticky paper shapes, 'fuzzy felts'. Can they make an animal with a circular head, a triangular body etc… Ask the children to make 'ruler' pictures –use a ruler to draw a series of intersecting lines and to colour in each enclosed shape, talking about the properties of these shapes and naming them as they do so.	1. Describe the sides and vertices of 2D shapes (from diagrams or names) • sketch a named 2D shape • state the number of vertices of a 2D shape • state the number of sides of a 2D shape • use number of sides/vertices to identify polygons i.e. to say if a shape is a quadrilateral or not or is a pentagon or not • identify whether the sides are straight or curved • identify whether any of the sides are the same length • given a description of the sides/vertices of a shape, suggest its name or sketch the shape	Show me a shape with three vertices Show me a shape with four sides Show me a pentagon Convince me that this is a quadrilateral Always, Sometimes, Never? Shapes have the same number of sides as vertices
Symmetry • Folding paper versions of 2D shapes to discover lines of symmetry (and test whether lines really do give symmetry) • Using mirrors to explore possible symmetry in shapes • Making butterflies using paint and folding • Making symmetrical patterns by folding paper with paint on in one or two directions. • Completing a symmetrical image e.g. take photo of child's face- fold in half and stick to a piece of paper so that the child then paints the other side to match - model how to measure features for a more accurate finish. • Exploring symmetry in art, play, photos etc.	2. Describe the symmetry of 2D shapes • identify lines of symmetry on drawn shapes (vertical) • identify lines of symmetry on drawn shapes (non-vertical) • identify all the lines of symmetry on a drawn shape • identify shapes that do not have any lines of symmetry • say whether a line dividing the shape into two is a line of symmetry or not • complete a shape given half of it and a vertical mirror line	Show me a shape with a line of symmetry Show me a shape with more than 1 line of symmetry Convince me that this shape has a line of symmetry Convince me that this shape does not have any lines of symmetry Always, Sometimes, Never? 2D shapes have 1 line of symmetry

to develop a more secure comprehension. • Exploring shapes made on grids to see how to colour in more squares to create a symmetrical shape		
Properties of 2D Shapes • Folding paper in half in different ways. If they make one fold on a square, what shapes can they make? Can they make one fold and make a four-sided shape that isn't a rectangle? • Playing with tangrams to explore the properties of a square. **Nrich** version • Using shapes to make patterns and describing these using mathematical language • Using geoboards and elastic bands to make shapes • Making shapes out of identical triangle pieces and naming these • Using the Polygon ITP to explore shapes with ICT • Using plastic geostrips to investigate triangles or quadrilaterals with sides of different lengths 	3. Compare 2D shapes • by number of sides (including ordering 3 or more) • by number of vertices (including ordering 3 or more) • by type of sides e.g. curved or straight • by length of sides i.e. sides that are equal • by lines of symmetry • by whether they meet a definition e.g. quadrilateral or triangle	Show me a shape where all the sides are equal length Convince me that this is a rectangle What's the same and what's different? triangle, rectangle, pentagon, circle Always, Sometimes, Never? A square is a type of rectangle
Identifying 2D Shapes • Playing peekaboo with shapes, trying to guess the shape as more and more is revealed • Playing 'Guess My Shape' using 20 questions format (can be done nicely using a shape fan so that all children put forward a guess after each new fact is revealed)	4. Identify/draw (sketch) a 2D shape given its properties • given a property, suggest a possible shape e.g. four sides or all sides are equal length • given a property, suggest all possible shapes e.g. four sides or all sides are equal length • given several properties, pinpoint the exact shape e.g. three sides, one line of symmetry	Show me all the shapes with no vertices Show me all the shapes with a right angle

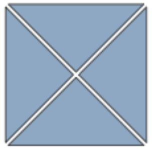
3D Shapes • Exploring and handling 3D shapes using mathematical models as well as everyday objects, especially packaging, to help count the faces, edges and vertices. • Going on a shape hunt to find specific shapes in real life e.g. cuboids • Printing with 3D shapes to explore the shapes of the faces. Which shapes have square faces? Did you print with any shapes with circular faces? What happens when you print with a sphere? A cylinder? • Pulling 3D shapes apart (e.g. packets) to see the 2D shapes that they are made from • Making models of shapes using plasticine or using construction materials (e.g. blocks, duplo, multi-link etc) • Making models using straws and balls of modelling clay or equivalent. Then exploring how many straws and connecting balls you need to make a cube? A pyramid?	5. Describe the faces, edges and vertices of 3D shapes • sketch or find a named 3D shape • state the number of faces of a 3D shape • identify whether the faces are flat or curved • identify whether any of the faces are the same • name the shapes of each face of a 3D shape (not curved faces) • state/count the number of vertices of a 3D shape • state/count the number of edges of a 3D shape • given a description of the faces/vertices/edges of a shape, suggest its name	Show me a sphere Show me a shape with six faces Show me all the shapes that are 3D Convince me that this shape will roll Always, Sometimes, Never? A pyramid has one square face
	6. Compare 3D shapes • by number of faces • by shapes of faces • by number of vertices • by whether faces are curved or straight • by whether they meet a category definition e.g. pyramid or prism	Convince me that a cube and a cuboid have the same number of faces, edges and vertices Convince me that a cuboid is a prism
Identifying 3D Shapes • Playing peekaboo with shapes, trying to guess the shape as more and more is revealed • Playing 'Guess My Shape' using 20 questions format (can be done nicely using a shape fan so that all children put forward a guess after each new fact is revealed)	7. Identify a 3D shape given its properties • given a property, suggest a possible shape e.g. five faces or 8 vertices • given a property, suggest all possible shapes e.g. five faces or 8 vertices • given several properties, pinpoint the exact shape e.g. four faces, all of them are triangles or five faces and five vertices	Show me all the shapes with 12 edges
Sorting Shapes • Finding all the shapes that match a given rule e.g. a shape with 6 vertices or a shape with all sides the same length • Organising a set of shapes into groups of the child's own choosing and explaining how they have organised them • Making a shape family tree • Finding all the shapes that match a given	8. Sort and classify 2D and/or 3D shapes using given criteria • sort sets of shapes into 2 groups e.g. 2D/3D or no curved edges/has curved edges • sort sets of shapes into more complex groups e.g. triangles; quadrilaterals; shapes with 5 more sides; others • sort sets of shapes using 2 criteria e.g. 2D/3D AND no curved edges/curved edges	What's the same and what's different? cube; cuboid; square; sphere What's the same and what's different? circle; sphere; cylinder; cube

rule e.g. a shape with 6 vertices or a shape with all sides the same length • Positioning shapes in hoops (which could overlap to begin to introduce Venn diagrams etc)	• extension: sort sets of shapes into structures such as tables, Venn diagrams or Carroll diagrams	

Further Extension

1.

Cut a square piece of paper as shown. Rearrange the pieces to make different shapes. What different shapes can you make?



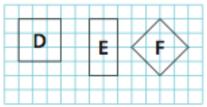
Describe the properties of the shapes you make.

Can you make some shapes which have at least one line of symmetry?

2.

Captain Conjecture says, 'All of these shapes are rectangles because they have four sides.'

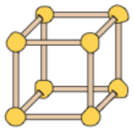
Do you agree?



Explain your reasoning.

3.

Jack has made a cube using 12 sticks and 8 balls of modelling clay.



What shape could he make with:

6 sticks and 4 balls of clay?

4 long sticks, 8 short sticks 8 balls of clay?

Rich and Sophisticated Tasks

Identify and describe the properties of 2-D shapes, including the number of sides and line symmetry in a vertical line

NRICH: Shapely Lines * I
NRICH: Chain of Changes ** P
NRICH: Colouring Triangles ** P I
NRICH: Exploded Squares * P
NRICH: Complete the Square * G**
NRICH: Let's Investigate Triangles * P
NRICH: Poly Plug Rectangles * G I
NRICH: Square It * G
NRICH: Inside Triangles * G P**

Identify and describe the properties of 3-D shapes, including the number of edges, vertices and faces

NRICH: Building with Solid Shapes * I
NRICH: Skeleton Shapes ** P I
NRICH: Rolling That Cube * I

Compare and sort common 2-D and 3-D shapes and everyday objects

NRICH: Matching Triangles * G
NRICH: Data Shapes * P
NRICH: Cubes Cut into Four Pieces *** P

Other

NRICH: Take a ….. Geoboard
NRICH: Properties of Shapes KS1
NRICH: Stringy Quads
NRICH: Let us reflect

4.

Fill in the missing shape to complete the pattern.

If the pattern continued what would the tenth shape be?

Explain your reasoning.

Misconceptions

As in Stage 1, a key misconception occurs where children relate the orientation of a shape to its definition. For example, they fail to recognise a square that has been rotated or a cone whose circular face is not at the base of the page/on the surface. In fact, children can infer that part of the definition of a shape is its colour etc. if this is how they always see that particular shape presented so ensure these presentations are varied to avoid inadvertently implying a property that is not intended!

Some children believe that the terms corner and vertex are not interchangeable - however, they are (and apply to both 2D and 3D shapes). Distinguishing between these can help separate 2D and 3D shapes, however.

Children miscount vertices and sides sometimes because they forget where they have started counting.

Some children are surprised that there is more than one shape with, for example, four sides and they therefore assume that it must be a square because they know a square has four sides.

Relatedly, they do not recognise 'quadrilateral' as a category of shape that other shapes fit into; instead they see it a distinct shape.
In a similar way, the inclusive definition of rectangle, for example, which includes squares, is not always clear to children who believe they are distinct.

Children can sometimes identify lines of symmetry that are not present because the area and shape of the two pieces they create are the same, although they are not correctly oriented to give reflection symmetry. E.g. they believe a rectangle has a (two) diagonal line(s) of symmetry because it creates two seemingly identical shapes when you cut it down the diagonal.
Children find symmetry lines particularly hard to spot when they are not vertical to the page.

Teacher Guidance and Notes

- In this Stage we are trying to develop children's use of mathematical language to describe shapes and their properties more technically.
- There is no specific range of shapes that should be covered, however it is recommended that those highlighted in the vocabulary box be used.
- You need to ensure that children are beginning to be able to use category words for shapes also, such as quadrilateral or pyramid, and that they recognise that these names cover multiple shapes.
- Be aware that most of the standard sets of shapes that you can buy or print represent larger polygons in their regular form. Try to use some examples of irregular pentagons, hexagons etc so that children do not assume that all pentagons have equal length sides and 5 lines of symmetry for example. This belief may stop them recognising a shape as a pentagon, even when it is!
- Don't be frightened of introducing a wider range of shapes - many children of this age can handle a significant amount of new vocabulary such as hemisphere or cylinder or octagon. You may even begin to be able to look at types of triangle with the most able to see the difference between an equilateral and the others. You may NEED to do this to show the children some shapes WITHOUT symmetry .....
- Consider making each child a shape fan (like a number fan!) that they can use for show-me activities on the carpet or for investigating shape features.
- Be aware that we are ultimately moving to the idea that a rectangle is the family name and the family divides into squares and oblongs - if you can prepare the way for this it will help!
- When trying to teach children the meaning of a property e.g. has a line of symmetry or is a triangle, it can be very helpful to spend time looking at non-examples to show why they do not meet the criterion. This helps children to begin to understand what qualifies and what does not. For example, ask "why is this not a quadrilateral?/prism?/3D shape?/symmetrical shape?"

Key Assessment Checklist
1. I can recognise and name 2D and 3D shapes
2. I can identify the number of sides and vertices of a 2D shape
3. I can recognise if a 2D shape has a vertical line of symmetry and say/show where this is
4. I can identify the number of faces, edges and vertices on a 3D shape.
5. I can describe the faces of a 3D shape
6. I can compare 2D shapes
7. I can compare 3D shapes
8. I can sort 2D and 3D shapes into groups using given criteria.

Year 2	Unit 6 : Reasoning with Measures
8 learning hours	This unit focuses on mensuration and particularly the concepts of perimeter, area and volume. Primary children are also working on money concepts at this stage, while older secondary students develop mensuration into volume and surface area of challenging shapes, applying Pythagoras' Theorem and trigonometry also in combination with these problems. Note the focus on reasoning within this unit: it is common for children to complete routine problems involving mensuration but this unit is about the developing a secure conceptual understanding of these ideas that they can apply to a wide range of problems and contexts. The opportunity to use and build on earlier number work is built into this unit and it is expected that children apply their arithmetic skills, for example, in these problems.

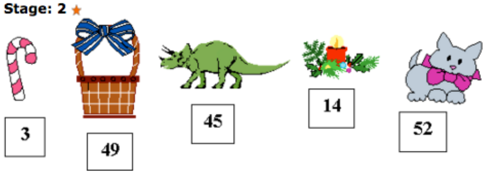
Prior Learning	Core Learning	Learning Leads to...
➢ recognise and know the value of different denominations of coins and notes	➢ **recognise and use symbols for pounds (£) and pence (p); combine amounts to make a particular value** ➢ **find different combinations of coins that equal the same amounts of money** ➢ **solve simple problems in a practical context involving addition and subtraction of money of the same unit, including giving change** *Working at expected standard* • *use different coins to make the same amount (e.g. pupil uses coins to make 50p in different ways; pupil can work out how many £2 coins are needed to exchange for a £20 note)*	➢ add and subtract amounts of money to give change, using both £ and p in practical contexts

Exemplification	Vocabulary	
1. a) Write these amounts using numbers and the £ and p signs (i) Four pounds (ii) Sixty-four pence (iii) Two pounds and thirty pence b) Write 150p using the £ symbol 2. a) Find three coins that make 60p. b) Find a different set of three coins that make 60p. 3. Shaheen buys an apple for 15p and a banana for 26p. She pays using a 50p coin. Find the right coins for Shaheen's change.	money coin note pound £ pence p total amount price	cost altogether pay shop bill change subtract less take away difference

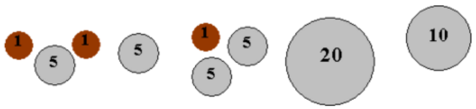
Representation	Fluency	Probing Questions
Money • handling coins to learn their values and features e.g. colour, shapes • making amounts out of given coins • finding different coins that give the same total (and trying to get all the possible combinations) • finding total of coins and sharing into piggy banks – see this [Coin matching computer activity.] (from Nationwide)	1. Recognise and use 1p, 2p and 5p coins • identify the value given the coin • identify the coin given the value • identify and record the value of a combination of 1ps and 2ps • identify and record the value of a combination of 1ps, 2ps and 5ps • find coins (1ps, 2ps, 5ps) to make a given amount e.g. 12p	Show me all the ways I can make 9p using coins Convince me that I can make any amount out of 1p, 2p and 5p coins
	2. Recognise and use 10p, 20p and 50p coins • identify the value given the coin • identify the coin given the value • identify and record the value of a combination of 10ps and 20ps • identify and record the value of a combination of 10ps, 20ps and 50ps • find coins (10ps, 20ps, 50ps) to make a given amount (<100) e.g. 90p • find coins (10ps, 20ps, 50ps) to make a given amount (>100) e.g. 150p	Show me where 50p lies on this paper strip from £0 to £1 (£0 to £2 etc) Show me all the ways I can make 24p using coins Convince me that there is only one way to make 20p using 3 coins Convince me that you cannot make 58p out of 3 coins
	3. Recognise and use £1 and £2 coins and notes • identify the value given the coin • identify the coin given the value • identify and record the value of a combination of £1s and £2s • identify and record the value of a combination of £1s, £2s, £5 notes and £10 notes. • find coins (£1/£2) to make a given amount e.g. £9 • find coins and notes to make a given amount e.g. £9, £23	Always, Sometimes, Never? Every amount can be made from 1, 2 or 3 coins.
	4. Use all coins and notes simultaneously • identify and record the value of a combination of coins • identify and record the value of a combination of coins and/or notes	Show me which amounts can be made using only 2 coins

	• find coins and/or notes to make a given amount • find multiple combinations of coins or notes to make a given amount	
Understanding Equivalence • Finding different coins that match £1 to explore the idea of 100 pence = 1 pound • Replacing £1 coins with 100 place value counters to help convert between pounds and pence	5. Understand the equivalence of £ and pence • know that £1 is the same 100p • say/write a whole amount given in pounds in pence e.g. £2 or £5 • say/write a decimal amount given in pounds in pence e.g. £1.50 or £1.20 • say/write an amount given in pence in pounds e.g. 250p (ext: 85p) • ext: say how many pence are equivalent to amounts in pounds <1 e.g. £0.50 or £0.35	Show me how many pence are worth the same as £2 What's the same and what's different? £ ; pence ; p ; pounds ; dollars
Finding Total Amount • Using addition representations e.g. tens and ones materials or bar model to represent the process	6. Find the total of some items • two items, in whole pounds, no bridging • two items, in whole pounds, bridging • two items, in pence, no bridging (<100) • two items, in pence, bridging (<100) • three or more items, in pence (<100) • two items, in pence, >100 • ext: two items in pounds (not whole number) • ext: mixed units e.g. £1.50 + 65p	Convince me that 20p is not enough money to buy three 7p sweets
Shopping and Change • Playing shops to practise finding the right coins, working out the total, working out the change and giving correct coins for change • Exploring which items can be bought using exact money with coins and items provided. For example (from N:Rich)	7. Choose appropriate coins to pay for a purchase • exact amount • estimate amount and give appropriate coin	Convince me that a 50p coin will be enough to pay for two items costing 21p and 18p.
	8. Find the change for a purchase • one item, whole pounds e.g. £5-£2 • one item, pence, no bridging e.g. 50p – 30p • one item, pence, bridging e.g. 70p - 59p • one item, pence and pounds e.g. £1 – 85p • two items, all pence • two items, pence and pounds given the change and amount paid, say how much the item(s) cost	Show me how to give change when I buy things for 12p and 5p and give a 20p What's the same and what's different? Change; subtraction; addition; total cost Always, Sometimes, Never? The shopkeeper will be able to give the right change.

Stage: 2 ★



Vera is shopping at a market with these coins in her purse. Which things could she give exactly the right amount for?

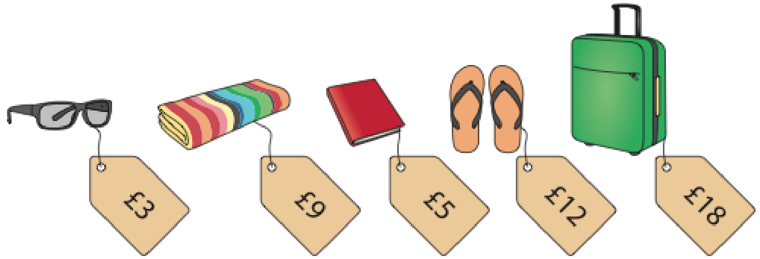


- Practising giving change using coins or using a computer simulation e.g. Change White Elephant from Crickweb

Always, Sometimes, Never?
There is more than one way to give any amount of change.

Further Extension	Rich and Sophisticated Tasks
1. Holly uses a £1 coin to buy a pack of stickers. Here is the change she was given.  How much did the pack of stickers cost? 2. I spend £2 on a drink and sandwich. The sandwich costs 80p more than the drink. How much does the sandwich cost? 3. Grace uses a £1 coin to buy a can of drink which costs 80p. She is given three coins in change. What coins could she have been given? 4. Grace uses a £2 coin to buy a can of drink which costs 85p. She is given four coins in change. Find all the possible combinations of coins she could have been given.	Recognise and use the symbols for pounds (£) and pence (p); combine amounts to make a particular value **NRICH: Five Coins ** P I** Find different combinations of coins that equal the same amounts of money **NRICH: Money Bags ** P** Solve simple problems in a practical context involving addition and subtraction of money of the same unit, including giving change **NRICH: The Puzzling Sweet Shop ** P**

5.
Sid says, 'I have bought 2 items for my holiday.
One item cost £9 more than the other. I spent over £15.'
What two items did Sid buy?
The ______ and the ______ .



Make up your own problems using the holiday items.

6.
Look at these coins. How could you make up the same total amount using just one type of coin?

7.
Sam says I can make 97p using just four coins. Is he correct?

Explain your reasoning.

Misconceptions	Teacher Guidance and Notes
Children may find it hard to see that an amount over 100p can be written in pence or in pounds e.g. 160p is the same as £1.60 Some children may write p as 9 Weak arithmetic skills can cause issues in selecting appropriate coins and notes.	• This unit is focused on developing confidence and fluency with money in real settings. • Therefore, it is crucial that children have the opportunity to work in context for much of this unit. • The pitch of the work is at the level of children's arithmetic skills i.e. 2-digit amounts in pence and whole numbers of pounds.

Sometimes children treat all coins of the same colour as the same value i.e. they see 1p and 2p coins as the same etc. Some children struggle to grasp the concept of not having right money and needing change.	However, it also expected that children will understand the equivalence of £1 and 100p and use this to find change in very simple decimal situations such as a cost of £1.50 and someone paying £2 or £5. This is achievable due to the focus on using the actual coins, rather than seeing this abstractly at this stage. • A critical learning point is the ability to record, using correct terminology. Children can show creativity in finding all the ways of making eg 11p (11 x 1p, 2p + 9 x 1p, etc) but will probably record using '+' only (eg 5p + 2p + 2p + 2p = 11p). • To support this work you can use coins to represent arithmetic problems/calculations to help children cement them as a legitimate model/representation for number. • The idea of giving change is not always easy for pupils and may require some work. It can link to subtraction on a number line if the 'jumps' are restricted to coin amounts.

Key Assessment Checklist

1. I can make small amounts of money in different ways using 1p and 2p coins and record the result (eg as 7p = 2p + 2p + 2p + 1p)
2. I can make small amounts of money in different ways using 1p, 2p, 5p and 10p coins and record the result (eg as 17p = 10p + 5p + 2p)
3. I can make larger amounts of money in different ways using any coins and record the result using p or £ (eg as £1.50 = £1 + 50p)
4. I can add up and record the total cost of 2 or 3 items in pence up to 99p.
5. I can select appropriate coins to pay for one or more items in a pretend shop
6. I can add up and record the total cost of 2 or 3 items where the total is over £1 .
7. I can use counting on or formal subtraction to work out change.
8. I can select appropriate coins to give change for an item in a pretend shop

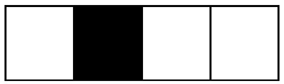
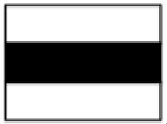
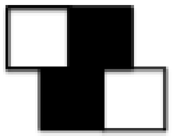
<table>
<tr><th>Year 2</th><th colspan="2">Unit 7/8 : Discovering Equivalence and Reasoning with Fractions</th></tr>
<tr><td>11 learning hours</td><td colspan="2">This unit is a combination of two units that are separated in older year groups to allow teachers extra time to master the concepts.
The unit explores the concepts of fractions (decimals and percentages) as ways of representing non-whole quantities and proportions.
For the youngest children, the work is focused on fractions and developing security in recognising and naming them.
At KS2 this then builds to looking at families of fractions and decimals and percentages.
At secondary level this is extended to more complex percentage work and equivalence with recurring decimals and surds.</td></tr>
<tr><th>Prior Learning</th><th>Core Learning</th><th>Learning Leads to...</th></tr>
<tr><td>➢ recognise, find and name a half as one of two equal parts of an object, shape or quantity
➢ recognise, find and name a quarter as one of four equal parts of an object, shape or quantity
➢ identify and represent numbers using objects and pictorial representations including the number line, and use the language of: equal to, more than, less than (fewer), most, least</td><td>➢ recognise, find, name and write fractions 1/3, 1/4, 2/4 and 3/4 of a length, shape, set of objects or quantity

➢ write simple fractions for example, 1/2 of 6 = 3 and recognise the equivalence of 2/4 and ½

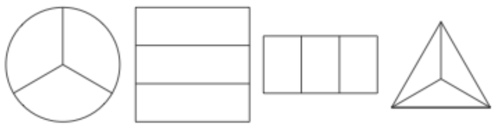
➢ identify, represent and estimate numbers using different representations, including the number line

Working at expected standard
• identify $\frac{1}{3},\frac{1}{4},\frac{1}{2},\frac{2}{4},\frac{3}{4}$ and knows that all parts must be equal parts of the whole

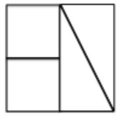
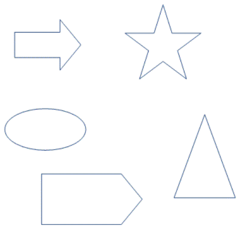
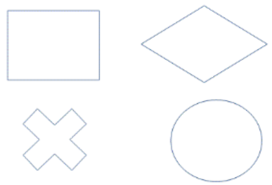
Working at greater depth:
• find and compare fractions of amounts (e.g. 1/4 of £20 = £5 and 1/2 of £8 = £4 so 1/4 of £20 is greater than 1/2 of £8).</td><td>➢ recognise, find and write fractions of a discrete set of objects: unit fractions and non-unit fractions with small denominators
➢ recognise and use fractions as numbers: unit fractions and non-unit fractions with small denominators
➢ compare and order unit fractions, and fractions with the same denominators
➢ recognise and show, using diagrams, equivalent fractions with small denominators
➢ identify, represent and estimate numbers using different representations</td></tr>
<tr><th colspan="2">Exemplification</th><th>Vocabulary</th></tr>
<tr><td colspan="2">1. Name the fractions represented by the black section in each picture:

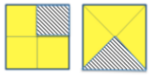
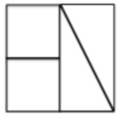
a) 
b) 
c) 
d) </td><td>fraction
half ½
quarter
one quarter ¼
two quarters 2/4
three quarters ¾
one third 1/3

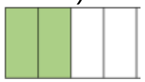
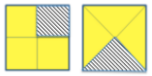
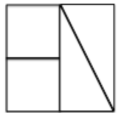
parts
whole
equal
numerator</td></tr>
</table>

2. a) I have 10 counters. Half of them are red. How many are red? b) ¼ of 8 is .... 3. Represent the fraction $\frac{1}{3}$ in in 3 different ways. You can use these diagrams to help you if you wish. 	denominator calculate find represent show out of

Representation	Fluency	Probing Questions
Recognising and Writing Fractions • Exploring ideas of parts and wholes e.g. Kirsty is part Hummingbirds Class, this table of 6 children is part of Hummingbirds Class Filling in the blanks e.g. if the whole is the school building, what could the part be? Or if the part is Mrs Brown, what could the whole be? • Counting the parts in the whole • Counting the parts that are shaded/highlighted/featured • Orally rehearsing … parts out of … parts in the whole and writing this as a fraction	1. Recognise and write fractions of shapes/sets of objects • state the number of parts in the whole (of a shape/set of objects) • know that these parts must be of equal size (and say when they are not) • state the number of parts that are shaded/have a specific property • recognise and say the fraction of a shape that has been shaded when all equal parts are of the same shape (half, a quarter, a third) • recognise and say the fraction of a shape that has been shaded when all equal parts are of the same shape (two quarters, three quarters) • recognise and say the fraction of a shape that has been shaded when the equal parts are of different shapes (half, a quarter, a third, two quarters, three quarters) • write the fraction of a shape that has been shaded $(\frac{1}{2},\frac{1}{4},\frac{2}{4},\frac{3}{4},\frac{1}{3})$ • recognise and say the fraction of a set of objects (2, 3 or 4) that have a given property e.g. fraction of these four cars that are red • write this as a fraction $(\frac{1}{2},\frac{1}{4},\frac{2}{4},\frac{3}{4},\frac{1}{3})$	Convince me that ½ of this shape has been shaded  What's the same and what's different? $1/4, 2/4, 3/4, 4/4$ What's the same and what's different? Part and Whole
Representing Half as Proportions of a Shape/Object • Colouring in 1 part out of 2 in given shapes to find $\frac{1}{2}$ • Folding (and colouring) a range of	2. Find and recognise half of a shape • find half of a shape with two marked equal parts of same shape • find half of a shape by splitting it into two equal parts (in different ways if appropriate) • find half of a shape with four marked equal parts	Show me ½ of these shapes

symmetrical paper shapes in half to find half of the shape • Specifically folding (and colouring) paper strips to show $\frac{1}{2}$, • Finding different ways to fold a post-it to show $\frac{1}{2}$ • Manipulating fraction pieces (bars and/or circles)	• find half of a shape with six, eight or ten marked equal parts • find half of a shape with marked equal parts of different shapes e.g. 	
Finding Fractions of a set of objects • Sharing children out into groups to find $\frac{1}{2}$ of the group • Sharing sets of objects into 2 containers and counting contents of one container to find $\frac{1}{2}$ • Grouping objects (especially drawn objects as these are harder to share) into 2s and counting the number of groups to find $\frac{1}{2}$ • Counting objects to find, for example, the fraction of set of four cubes that are green • Choosing objects so that a given fraction have a property e.g. $\frac{1}{2}$ of the counters are red • Laying out objects equally onto each part of a representation of a fraction e.g. a paper shape or strip and counting the number of objects in the shaded area.	3. Find and recognise ½ of a number of objects • find half of a set of objects (even number) by sharing them into two containers and counting the contents of one container • find half of a set of objects (even number) by grouping them into 2s and counting the number of groups • find half of a set of drawn objects (even number) by sharing them into two groups and counting the number of objects in each group • find half of a set of drawn objects (even number) by grouping them in 2s and counting the number of groups • write a statement to represent each calculation, for example, ½ of 10 = 5 • recognise that when you find half of a set of objects where there are an odd number, you will be left with one object • know that if objects are in two groups with the same number in each group, then each group represents ½	Show me 1/2 of 12 Always, Sometimes, Never? You can find one half of 19 Convince me that sharing and grouping give you the same result Convince me that one half of 20 sweets is bigger than one half of 10 sweets
Representing a Quarter as Proportions of a Shape/Object • Colouring in 1 part out of 4 in given shapes to find $\frac{1}{4}$ • Folding (and colouring) a range of symmetrical paper shapes in half and in half	4. Find and recognise a quarter of a shape • find a quarter of a shape with four marked equal parts of same shape • find a quarter of a shape by splitting it into four equal parts (in different ways if appropriate) • find a quarter of a shape with eight marked equal parts	Show me ¼ of these shapes 

again to find a quarter of the shape • Specifically folding (and colouring) paper strips to show $\frac{1}{4}$ • Finding different ways to fold a post-it to show $\frac{1}{4}$  • Manipulating fraction pieces (bars and/or circles)	• find a quarter of a shape with twelve, sixteen or twenty marked equal parts • find a quarter of a shape with marked equal parts of different shapes e.g. 	Show me ¼ of this square in three different ways
Finding a Quarter of a set of objects • Sharing children out into groups to find $\frac{1}{4}$ of the group • Sharing sets of objects into 4 containers and counting contents of one container to find $\frac{1}{4}$ • Grouping objects (especially drawn objects as these are harder to share) into 4s and counting the number of groups to find $\frac{1}{4}$ • Counting objects to find, for example, the fraction of set of four cubes that are green • Choosing objects so that a given fraction have a property e.g. $\frac{1}{4}$ of the counters are red • Laying out objects equally onto each part of a representation of a fraction e.g. a paper shape or strip and counting the number of objects in the shaded area.	5. Find and recognise ¼ of a number of objects • find a quarter of a set of objects (multiple of 4) by sharing them into four containers and counting the contents of one container • find a quarter of a set of objects (multiple of 4) by grouping them into 4s and counting the number of groups • find a quarter of a set of drawn objects (multiple of 4) by sharing them into four groups and counting the number of objects in each group • find a quarter of a set of drawn objects (multiple of 4) by grouping them in 4s and counting the number of groups • write a statement to represent each calculation, for example, ¼ of 12 = 3 • recognise that sometimes there will be objects leftover when trying to find a a quarter • know that if objects are in four groups with the same number in each group, then each group represents ¼	Show me 1/4 of 12 Show me how can you find a 1/4 of these 20 counters Convince me that half of 10 sweets is the same as 1/4 of 20 sweets
Representing Fractions as Proportions of a Shape/Object • Colouring in 2 or 3 parts out of 4 in given shapes to find $\frac{2}{4}, \frac{3}{4}$ • Folding (and colouring) a range of symmetrical paper shapes in half and in half again to produce quarters, before shading	6. Find and recognise a two or three quarters of a shape • find two quarters of a shape with four marked equal parts of same shape • find three quarters of a shape with four marked equal parts of the same shape • find two or three quarters of a shape by splitting it into four equal parts (in different ways if appropriate) and shading in two or three parts respectively	Show me two quarters? three quarters? Convince me that two quarters is worth the same as one half What's the same and what's different? half, halve, $1/2, 2/4$

two or three of them. • Specifically folding (and colouring) paper strips to show $\frac{2}{4},\frac{3}{4}$ (precursor to the bar model)  • Finding different ways to fold and then colour a post-it to show $\frac{2}{4},\frac{3}{4}$  • Manipulating fraction pieces (bars and/or circles)	• find two or three quarters of a shape with eight marked equal parts • find two or three quarters of a shape with twelve, sixteen or twenty marked equal parts • find two or three quarters of a shape with marked equal parts of different shapes e.g.  • begin to realise/notice that two quarters is the same as a half visually	Always, Sometimes, Never? There is no such thing as $\frac{4}{4}$
Finding Fractions of a set of objects • Sharing children out into groups to find $\frac{2}{4},\frac{3}{4}$ of the group respectively • Sharing sets of objects into 4 containers and counting contents of two or three containers to find $\frac{2}{4},\frac{3}{4}$ respectively • Counting objects to find, for example, the fraction of set of four cubes that are green • Choosing objects so that a given fraction have a property e.g. $\frac{2}{4}$ of the counters are red • Laying out objects equally onto each part of a representation of a fraction e.g. a paper shape or strip and counting the number of objects in the shaded area.	7. Find and recognise $^2/_4$ or $^3/_4$ of a number of objects • find two or three quarters of a set of objects (multiple of 4) by sharing them into four containers and counting the contents of two or three containers • find two or three quarters of a set of drawn objects (multiple of 4) by sharing them into four groups and counting the number of objects in two or three groups • find a two or three quarter of a set of objects (multiple of 4) by grouping them in 4s in an array and counting the number of objects in the first two or three columns • write a statement to represent each calculation, for example, $2/4$ of 12 = 6 or ¾ of 8 = 6 • realise/notice that $2/4$ of a set of objects give the same result as ½ of the set of objects • know that if objects are in four groups with the same number in each group, then two groups represents 2/4 and three groups represents ¾	Show me how you can find 2/4 of the 20 counters Show me how you can find 3/4 of the 20 counters What's the same and what's different? sharing; grouping Always, Sometimes, Never? $3/4$ of an amount is larger than $1/2$ of an amount
Representing a Third as Proportions of a Shape/Object • Colouring in 1 part out of 3 in given shapes to find $\frac{1}{3}$	8. Find a third of a shape • find a third of a shape with three marked equal parts of same shape • find a third of a shape by splitting it into three equal	True or False? 1/3 is less than 1/4 Convince me that there is a such a fraction

• Folding (and colouring) a range of symmetrical paper shapes into three • Specifically folding (and colouring) paper strips to show $\frac{1}{3}$ (precursor to the bar model) • Finding different ways to fold a post-it to show $\frac{1}{3}$ • Manipulating fraction pieces (bars and/or circles)	parts (in different ways if appropriate) • find a third of a shape with six marked equal parts • find a third of a shape with nine, twelve, ... marked equal parts • find a third of a shape with marked equal parts of different shapes	as 2/3
Finding a Third of a set of objects • Sharing children out into groups to find $\frac{1}{3}$ of the group • Sharing sets of objects into 3 containers and counting contents of one container to find $\frac{1}{3}$ • Grouping objects (especially drawn objects as these are harder to share) into 3s and counting the number of groups to find $\frac{1}{3}$ • Counting objects to find, for example, the fraction of set of three cubes that are green • Choosing objects so that a given fraction have a property e.g. $\frac{1}{3}$ of the counters are red • Laying out objects equally onto each part of a representation of a fraction e.g. a paper shape or strip and counting the number of objects in the shaded area.	9. Find and recognise $1/3$ of a number of objects • find a third of a set of objects (multiple of 3) by sharing them into three containers and counting the contents of one container • find a third of a set of objects (multiple of 3) by grouping them into 3s and counting the number of groups • find a third of a set of drawn objects (multiple of 3) by sharing them into three groups and counting the number of objects in each group • find a third of a set of drawn objects (multiple of 3) by grouping them in 3s and counting the number of groups • write a statement to represent each calculation, for example, $1/3$ of 6 = 2 • recognise that sometimes there will be objects leftover when trying to find a third • know that if objects are in three groups with the same number in each group, then each group represents 1/3	Show me 1/3 of 12 Show me how you can find 1/3 by sharing Show me how you can find 1/3 by grouping
Fractions of Lengths • Using a bead string to count out the number and share it into 2/3/4 equal groups to find the value of $\frac{1}{2},\frac{1}{3},\frac{1}{4},\frac{2}{4},\frac{3}{4}$ of the number • Using a paper strip alongside to represent a line and folding it into 2/3/4 pieces to find the value of $\frac{1}{2},\frac{1}{3},\frac{1}{4}$ of its length	10. Find fractions of lengths • find half/a quarter/a third of a length on a beadstring by splitting the correct number of beads into 2/3/4 equal groups and counting the size of the first group • find half of a drawn length (shown alongside a ruler) by splitting the line into two equal parts and	What's the same and what's different? $1/4, 2/4, 3/4, 4/4$

• Using Cuisenaire rods to produce lengths that are half/a third/a quarter of another. • Using a counting stick to count up and down in halves, thirds and quarters. Beginning to recognise where the wholes appear in this process.	measuring one part • find a quarter of a drawn length (shown alongside a ruler) by splitting the line into four equal parts and measuring one part • find a third of a drawn length (shown alongside a ruler) by splitting the line into three equal parts and measuring one part • find two quarters/three quarters of a length on a beadstring by splitting the beads into 4 equal groups and counting the size of the first two/three groups combined • find two or three quarters of a drawn length (shown alongside a ruler) by splitting the line into four equal parts and measuring the total length of the first two/three parts	
Solving Fraction Problems • Producing a bar model representation for the fraction in the question and laying objects onto it or colouring it as appropriate	11. Solve problems involving fractions of shapes and sets of discrete objects. • read and find a fraction of a number by choosing own concrete or pictorial aids e.g. 1/3 of 15 • create a set of objects with a given fractional property e.g. set of animals where ½ of them have four legs • find the whole given the fraction e.g. here are 1/3 of my number is 4. What is my number? • word problems involving fractions e.g. Sam's age is a half of his brother's. His brother is 14. How old is Sam? • comparison problems e.g. which is greater? ½ of 12 or ¼ of 20? • comparison word problems e.g. would you rather have ½ of this bar of chocolate or 1/3 of that one? • find fractions of objects that can be subdivided in different ways e.g. bars of chocolate with 12 squares (3 x 4) • solving problems with money e.g. find half of 18p (shown as coins)	Convince me that 1/4 of 12 sweets is less than 1/3 of 12 sweets What's the same and what's different? $1/2, 1/3, 1/4, 2/4$ Always, Sometimes, Never? $2/4$ is the same as $1/2$

Further Extension

1.

Use the pictures to complete the number sentences.

$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	
$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

$\square$ is less than $\square$ $\square < \square$

$\square$ is greater than $\square$ $\square > \square$

$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	
$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

$\frac{3}{\square}$ is greater than $\frac{2}{\square}$ $\frac{3}{\square}$ is less than $\frac{3}{\square}$

2.
a)
Jo bought a bag of 12 cherries.
Jo ate half the number of cherries in the bag.
How many cherries did Jo eat?

b)
Jo bought a bag of cherries.
Jo ate half the number of cherries in the bag.
Jo had 7 cherries left. How many cherries did Jo buy?

c)
Sam bought a bag of 18 cherries.
Sam ate 6 cherries.
What fraction of the bag of cherries did Sam eat?

d)
Sam bought a bag of cherries.
Sam ate 9 cherries and had 3 left over.
What fraction of the bag of cherries did Sam eat?

Rich and Sophisticated Tasks

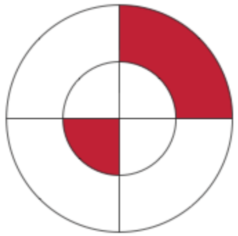
Recognise, find, name and write fractions 1/3, ¼, 2/4 and ¾ of a length, shape, set of objects or quantity

NRICH: Making Longer, Making Shorter ** I
NRICH: A Bowl of Fruit

Other Problems

What fraction is the red part of the whole circle?

Explain your reasoning.



Useful Resources

Fraction Manipulatives - exploring equivalence http://donnayoung.org/math/fraction.htm

Fraction models and support questions - http://www.annery-kiln.eu/gaps-misconceptions/all-images.html

3.

$\frac{1}{3}$ of 3 = 1

$\frac{1}{3}$ of 6 = 2

$\frac{1}{3}$ of 9 = 3

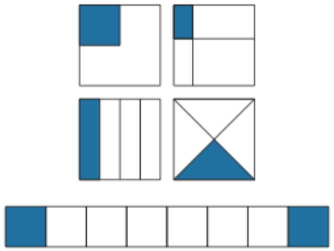
$\frac{1}{3}$ of 12 =

Continue the pattern.

What do you notice?

4.

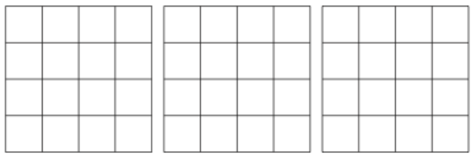
Which of these diagrams have $\frac{1}{4}$ of the whole shaded?



Explain your reasoning.

5.

Colour in $\frac{1}{4}$ of each of these grids in a different way. Try to think of an unusual way.



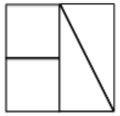
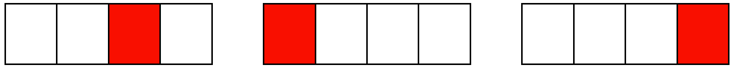
How many squares did you colour each time?

6.

If you count in steps of $\frac{1}{2}$ starting from 0, how many steps will it take to reach:

2, 4 or 6

What do you notice?

Misconceptions	Teacher Guidance and Notes
Some children do not understand that the parts of a whole must be of equal size. They therefore think that you can find a fraction by dividing the shape up into the correct number of pieces, whether or not these are equally sized. For example, to find a third, they may cut a shape into 3 differently sized pieces. Note that it is not essential for each part to be the same shape (only the same size) and some children find this difficult to accept. For example, this diagram does show quarters of the whole although the four parts are not all the same shape.  Similarly, children may become used to certain representations and the position of the shaded piece. They do not always appreciate that the position of the shaded piece can vary. For example, these images all show ¼  Conceptually, some children do not understand the role of the denominator in telling us the number of parts in the whole (or that that the whole is shared into) They therefore often believe that, for example, ½ is smaller than ¼ because 2 is less than 4. Sometimes children read fractions as pieces and do not break down larger pieces into equal parts. They can think of fractions as ratios rather than comparing to the whole. So, for example, ¾ can be seen as 1/3. Children tend to stick to one particular representation of a fraction (often the circular one) and do not recognise other models or images as also being worth the same. Exposure to a wide range of apparatus and pictorial stimuli is required to challenge this. Sometimes when finding 2/4 or ¾, children incorrectly divide twice. For	• This unit precedes the unit on multiplication and division. Therefore, the focus of this unit is on the concepts of fractions as parts of wholes; the process of dividing is less crucial. Note that basic concrete sharing and grouping will have been covered in Stage 1 and so children have some prior knowledge. Teachers should keep the numbers for calculations small to allow children to develop the concepts instead. They should avoid using the division sign. Pupils can then apply these ideas after Unit 9 when they have developed their skills in division further. • In Stage 2, fractions are referred to as proportions of an amount (i.e. ½ of ...). • Be aware of the conceptual difference in referring to a fraction of a shape or object using partitioning into equal pieces (for example, half of this shape has been shaded) and referring to finding a fraction of an amount using calculation. • In Stage 3, children will begin to see fractions as numbers themselves (rather than proportions only) – so leave this development for the next stage. • Ensure children have lots of opportunity to find fractions of wholes concretely and visually. It is useful to start to prompt children to use an array to 'organise' their groups. For example, showing a third of 12 by arranging twelve items into an array with three rows (because we are finding a third) and counting how many columns this creates. This will help make links to division also. • This is a good time to introduce the bar model as a way of representing unit fractions (and then non-unit examples). You can start this with paper rectangles/strips at this stage and model the bar electronically. • As with all work on fractions, it is essential that children understand the importance of dividing the quantity or object into equal parts - so emphasise the use of the same size blocks in the bar model. • Ensure children know that the denominator represents the number of parts in the whole and the numerator represents the number of parts that we are working with e.g. that are shaded/blue/tomatoes etc. • When exploring representations, do not forget the symbols themselves - it is worth exploring that 1/2 means 1 divided by 2 and so on. You can see a lovely example of children exploring fractions as a sharing model on YouTube here: https://www.youtube.com/watch?v=Q-yichde66s&feature=youtu.be • You can make connections to children's skills in doubling when solving reverse problems

example, to find ¾ they may split into 4 equal pieces to find ¼ and then split into 3 pieces once again, rather than combining three separate quarters together. There can sometimes be misunderstanding about identical fractions of different 'wholes' where the whole is not clear, for example with money e.g. half of 50p.	

Key Assessment Checklist

1. I can recognise and say the fraction of a shape that has been shaded or the fraction of a set of objects that has a certain feature
2. I can find a half, a third and a quarter of a shape by splitting it into 2, 3 or 4 equal parts
3. I can find two quarters or three quarters of a shape by splitting it into 4 equal parts and shading 2/3 of them.
4. I can represent a third, a quarter and a half using a range of models and images (including the fraction symbols themselves i.e. $1/2, 1/3, 1/4$)
5. I can represent two quarters and three quarters using a range of models and images (including the fraction symbols themselves i.e. $2/4, 3/4$)
6. I can find a half, third or a quarter of a number of objects or quantity by sharing into 2, 3, 4 equal groups respectively.
7. I can find a half, third or quarter of a number of objects or quantity by grouping into 2s, 3s or 4s respectively.
8. I can find 2/4 or 3/4 of a number of objects or quantity by first finding 1/4.
9. I can show 1/2, 1/3, 1/4 of a shape or length by dividing it into two, three or four equal parts respectively.
10. I can recognise that the fractions 2/4 and 1/2 are equivalent and show this for a given quantity.
11. I can write simple fractions of quantities to show my calculations e.g. 1/2 of 6 =3 to show finding a half of 6 or 1/3 of 15 = 5 to show finding a third of 15.
12. I can solve problems involving fractions of shapes and sets of objects

Year 2	Unit 9 : Solving Number Problems	
12 learning hours	This unit continues pupils' earlier study of arithmetic (and algebra for secondary students). At Key Stage 1 children are working on multiplication (and division in Stage 2) as a way to represented repeated addition and scaling (and repeated subtraction – grouping - and sharing) At Key Stage 2 children are developing skills in applying their arithmetic to more complex problems. At secondary level and in Stage 6, students begin to find unknown values by applying inverse operations. Equations of all types including quadratic and simultaneous are covered in later stages.	
Prior Learning	**Core Learning**	**Learning Leads to...**
➢ solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher	➢ **calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (×), division (÷) and equals (=) signs** ➢ **show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot** ➢ ***begin to recall*** **and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers** *Working towards:* • *recall doubles and halves to 20 (e.g. the pupil know that double 2 is 4, double 5 is 10 and half of 18 is 9)* *Working at expected standard* • *recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables to solve simple problems, demonstrating an understanding of commutativity as necessary (e.g. knowing they can make 7 groups of 5 from 35 blocks and writing 35 ÷ 5 = 7; sharing 40 cherries between 10 people and writing 40 ÷ 10 = 4; stating the total value of six 5p coins) (repeat)* *Working at greater depth:* • *use multiplication facts to make deductions outside known multiplication facts (e.g. a pupil knows that multiples of 5 have one digit of 0 or 5 and uses this to reason that 18 × 5 cannot be 92 as it is not a multiple* of 5). *(repeat)*	➢ write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods

Exemplification

1. a) Write a **multiplication** statement to that this diagram represents:

b) Write a **division** statement that this diagram represents:

c) Calculate 5×10

d) Calculate $20 \div 5$

2. Bob is using the number 4, 2 and 8 to make a fact family.

Put a cross next to the number sentence that is incorrect.

$2 \times 4 = 8$ $\quad$ $4 \times 2 = 8$ $\quad$ $8 \div 2 = 4$ $\quad$ $4 \div 8 = 2$

3. Here are some number cards:

30	**3**	**10**

Show how the cards can be arranged to complete a division and a multiplication:

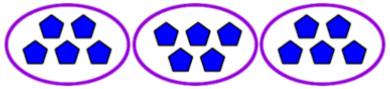
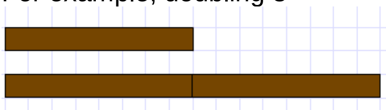
$\square \div \square = \square$

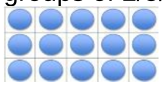
$\square \times \square = \square$

Vocabulary

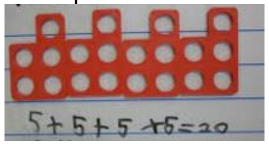
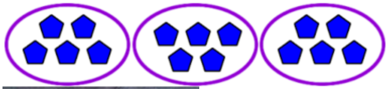
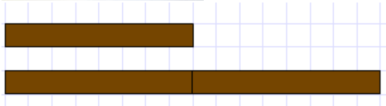
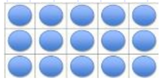
multiply	divide
x	÷
multiplied by	divided by
multiplication	division
times	divided into
lots of	share
groups of	shared between
sets of	group
product	quotient
array	per
row	fair
column	half
double	halve
treble	quarter
twice as	
.... times as	
.... times bigger	
	represent
	problem
statement	fact family
equals	order
=	commutative

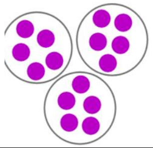
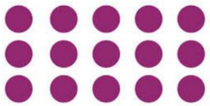
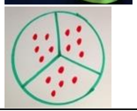
<table>
<tr><th>Representation</th><th>Fluency</th><th>Probing Questions</th></tr>
<tr>
<td>Practical Multiplication
• Arranging objects into equal groups and counting efficiently in 2s, 5s or 10s (to represent repeated addition) – e.g. 3 x 4 would be 4 groups of 3 objects
(e.g. sweets, animals, books, bean bags, counters, cubes and so on arranged into hoops, lily pads, boxes, bags, and so on) e.g. 2 frogs on each lily pad or 5 shapes in each hoop

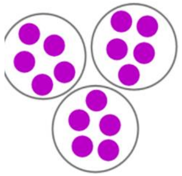
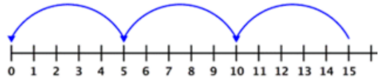
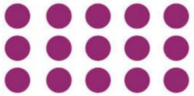
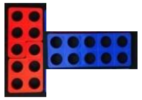
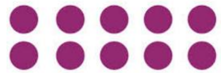

• Using Cuisenaire rods (or straws) to scaling an amount/length by making it twice as big/doubling it/making it ten times bigger and so on.
For example, doubling 8
</td>
<td>1. Solve practical problems involving multiplication e.g. Roy has 3 buckets with 5 crabs in each. How many crabs does he have altogether?
• represent the problem concretely or visually using groups or scaling (as implied by the question)
• find the total by counting (or efficient counting e.g. in 5s)</td>
<td>What's the same and what's different?
5 10ps, 10 5ps, 10 x 5, 5 x 10, 5 lots of 10, 10 lots of 5

What's the same and what's different?
3 groups of 4 and 4 groups of 3</td>
</tr>
<tr>
<td>Abstract Multiplication
• Saying the multiplication that has been made. For example, for 2 × 4 we would expect to see groups of 2 shown four times and children to practise reading and saying the multiplication as they are building it '4 lots of 2' and '4 groups of 2'.
• Using an array (with help) to structure the groups of 2/5/10 more formally e.g. 5 x 3

• Counting efficiently in 2s, 5s or 10s along columns of the array (pointing at the column being counted)</td>
<td>2. Calculate (abstract) mathematical statements for multiplication e.g. 5 x 4
• represent the statement concretely or visually using groups
• represent the statement using a concrete or visual array
• represent the statement concretely or visually using scaling
• find the result by counting (or counting efficiently)
• record the result at the end of the number sentence</td>
<td>Show me the array for 4 x 5

What's the same and what's different?
2 x 6, 12 x 1, 3 x 4, 4 x 2, 6 x 2, 1 x 12</td>
</tr>
</table>

Simple Word Problems • Representing a problem using Numicon, groups, beadstrings, arrays, or a number line Examples:        • Matching different representations together to find equivalent pairs	3. Recognise and solve word problems (out of context) involving multiplication using number sentences e.g. What is twice as many as 8? • choose an appropriate representation • solve the problem • write a number sentence to represent the problem	Convince me that when I multiply an odd number by 2, I get an even answer Always, Sometimes, Never? You can double any number Always, Sometimes, Never? When you multiply an odd number by an odd number you get an odd answer.
Practical Division **(Sharing or Grouping depending on the problem's language)** • Sharing out objects equally into groups E.g. for 15 ÷ 3 there would be 15 items counted and then shared into three piles (divided into 3) and the number in each pile counted.  • Sharing objects more abstractly into groups and linking this to fractions using circles or	4. Solve practical problems involving division e.g. Emma has 12 sweets. She shared the sweets between 4 party bags. How many sweets does she put in each bag? • represent the problem concretely using grouping or sharing (as described by the question) • find the result by counting the number of groups or number in each group as appropriate	Show me 12 shared between 2 people Show me 12 grouped in 2s. What's the same and what's different? Sharing and Grouping What's the same and what's different? 20 shared between 5; 5 shared between 20

bar model templates  • Grouping objects into 2s, 3s and so on before counting the number of groups produced e.g. 15 ÷ 5 or 15 grouped into 5s 		
Abstract Division • Sharing out objects into equally sized groups • Grouping objects and counting the number of groups • Using an array to organise groups e.g. 15 ÷ 5 is shown by taking 15 objects and arranging them in groups (columns) of size 3 to see how many columns this produces 	5. Calculate (abstract) mathematical statements for division e.g. 25 ÷ 5 • represent the statement concretely or visually by sharing • find the result by counting the number in each group • represent the statement concretely or visually by grouping • find the result by counting the number of groups • represent the statement concretely or visually using an array (organised grouping) • find the result by counting the number of columns (groups) • record the result at the end of the number sentence	Show me how you can represent 15 ÷ 5 Convince me that 12 divided by 3 and 12 divided into 3s have the same answer
Simple Word Problems • Representing a problem using Numicon, groups, bead strings, arrays, or a number line Examples:  	6. Recognise and solve word problems (out of context) involving division using number sentences e.g. What is 25 shared between 5? • choose an appropriate representation • solve the problem • write a number sentence to represent the problem	What's the same and what's different? 10 ÷ 5 and 5 ÷ 10 Always, Sometimes, Never? When you divide an even number by an even number you get an even answer.

    • Matching different representations to find equivalent pairs		
Commutativity • Using an array to show that, for example, 5 x 2 is the same as 2 x 5  • Representing $12 \div 2$ using grouping and then trying to do the same for $2 \div 12$ to show why these calculations do not have the same result	7. State /show whether two multiplication or division calculations have the same result. • multiplication of two specific numbers • division of two specific numbers • general principle for multiplication • general principle for division • use the word 'commutative' for multiplication	Convince me that multiplication is commutative Always, Sometimes, Never? You get a different answer if you divide in the other order
Representations → Calculations • Matching pairs of calculations and representations (practically or as images) [Video Example] from NCETM.	8. Given a representation, suggest a calculation that it represents • groups • scaling • array	Show me how you can represent 10 x 4 in as many ways as possible Convince me that this array represents $10 \div 2$ and 5×2  How do these calculations relate? Show me two different calculations that this image could represent: 

Fact Families • Given an array, saying the multiplication it represents • Given an array, saying the division it represents • Making an array to show 3×5 and then representing $15 \div 5$ with an array to discover that you get the same array. Writing the calculations that these represent and then showing 5×3 as well as $15 \div 3$.	9. Find the fact family for a given multiplication or division • given a multiplication, find the answer and then the other three related calculations • given a division, find the answer and then the other three related calculations • from a representation e.g. an array	Convince me that if 4 x 2 = 8 then 8 ÷ 2 must be 4 Always, Sometimes, Never? If you know that a x b = c then you can make another three true number sentences about these numbers
Multiplication and Division • Representing simple problem using trigger words of 'lots of/groups of' or 'times bigger' or 'shared between' or 'shared into groups of' to decide what type of representation	10. Ext: Solve a mixture of multiplication and division problems • know key trigger words for multiplication and division • recognise whether problem is multiplication or division • represent the problem concretely or visually • solve the problem • record the problem using a number sentence	Show me a number that is 5 times bigger than 6 Always, Sometimes, Never? You cannot divide an even number by an odd number.
Multiplication and Division Facts • Using a counting stick to represent counting in 2s, 5s, 10s etc. from 0.  • Chanting these in order AND pointing to specific multiples in a random order to recall them • Removing some labels as time progresses to develop the recall of key facts	11. Begin to recall times table multiplication facts (2s, 5s and 10s) ○ By representing the calculation concretely to deduce the answer. ○ By representing the calculation visually to deduce the answer. ○ By relating the calculation to another known calculation and counting on/back or doubling etc. ○ By beginning to recall key facts.	What's the same and what's different? 15, 40, 12, 16, 35, 30, 18, 20
	12. Begin to recall times table division facts (2s, 5s and 10s) ○ By representing the calculation concretely to deduce the answer. ○ By representing the calculation visually to deduce the answer. ○ By relating the calculation to another known calculation and counting on/back or doubling etc. ○ By beginning to recall key facts.	What's the same and what's different? $30 \div 10; 20 \div 5; 8 \div 4;\ 15 \div 5; 6 \div 2; 40 \div 10$ Convince me that $35 \div 5 = 7$ in 3 different ways

Further Extension

1.
Which has the most biscuits:
4 packets of biscuits with 5 in each packet, or
3 packets of biscuits with 10 in each packet?

Explain your reasoning.

2.

Find different ways to find the answer to 12 × 4.

Children are expected to use their 2, 5 and 10 times tables to answer this question.

3.
Write these addition sentences as multiplication sentences.

10 + 10 + 10 + 5 + 5 =
2 + 2 + 2 + 4 =
2 + 2 + 4 + 4 =
5 + 5 + 5 + 2 + 3=

4.
True or false?

5 × 4 = 4 × 5
5 × 4 = 10 × 2
5 × 4 = 2 × 10
Explain your reasoning.
What do you notice?

5.

Rich and Sophisticated Tasks

Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (×), division (÷) and equals (=) signs

NRICH: Ordering Cards * G
NRICH: Which Symbol? * P
NRICH: I'm Eight * I

http://www.bbc.co.uk/learningzone/clips/problem-solving-how-many-chairs-are-needed/1799.html

Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers

NRICH: Odd Times Even * I**
NRICH: Two Numbers Under the Microscope ** I
NRICH: Even and Odd * I
NRICH: Ring a Ring of Numbers * G
NRICH: More Numbers in the Ring * G P**
NRICH: How Odd ** I
NRICH: Clapping Times * G I
NRICH: Double or Halve? * G
NRICH: Always, Sometimes or Never? * P

Two friends want to buy some marbles and then share them out equally between them.

They could buy a bag of 13 marbles, a bag of 14 marbles or a bag of 19 marbles.

What size bag should they buy so that they can share them equally?

What other numbers of marbles could be shared equally?

Explain your reasoning.

Misconceptions	Teacher Guidance and Notes
Children may assume that, since multiplication is commutative, division is commutative and can be done in any order! They may write sentences such as 6 ÷ 2 = 12 due to this. Children struggle with the range of language for both multiplication and division - they may incorrectly read 3 x 4 as 3 lots of 4 rather than 3 multiplied by 4 (which is actually therefore 4 lots of 3) and so on. This can lead to errors in representing the array correctly (which we want to avoid to enable them to 'see' commutativity and to later represent more complex multiplication like 13 x 4 in a consistent way). Similarly, they may find it hard to understand what operation they need to use from a word problem because there are so many ways to imply a multiplication or division. Children may not see how an array can be used to support division, only multiplication.	• Children will have encountered multiplication and division verbally and practically in Stage 1 but this is their first encounter in Stage 2, and their first in using the notation themselves. • They will work on these skills again in Unit 13. Therefore, the focus at this stage is in the processes of building/drawing representations and using these to complete number sentences. It is advised that you use explicit real life examples to introduce each operation but that you focus on getting children strong and quick at representing a given calculation. The opportunity to apply this to more complex contexts and word problems will then come in Unit 13. • Initially it is important that children can use and recognise a wide range of representations of multiplication and division calculations. • Arrays are longer term the most useful of the representations as they will form the basis of our later work on multiplication and division. Therefore, it is important that adequate time has been spent getting the array right and recording the number sentences alongside the array. Children should be given opportunities to build their arrays before moving on to drawing these and then eventually the children will be able to just visualise the array and solve the multiplication/division. • Arrays are also an effective way to explore commutativity – Children to compare the array for 2 x 6 with one for 6 x 2 to see what they notice Also explore, what happens when you change the numbers around within a division. Children to recognise that they do not have enough objects to share out or group. • Ensure that children can confidently recognise the formal symbols for multiplication and division. • When solving multiplication and division statements try to stick to using examples for from the 2, 5 and 10 times table. However, it is possible for the children to solve multiplications from other times tables if you allow for counting in steps and keep numbers small (e.g. 3 x 4)

Key Assessment Checklist
1. I can explain what different multiplication and division number statements mean.
2. I can solve multiplication and division problems using practical equipment to create groups.
3. I can solve multiplication and division problems by scaling and sharing
4. I can solve multiplication and division problems using arrays.
5. I can use the signs x ,÷ ,and = to record a multiplication or division number sentence.
6. I can show that if I multiply my numbers in any order I will get the same answer.
7. I can show that if I divide my numbers in a different order I will get a different answer.
8. I can solve multiplication and division problems by recalling known facts.
9. I can decide which calculation to carry out and pursue this to solve a problem in context.

Year 2	Unit 10: Investigating Statistics
6 learning hours	In this unit children and students explore the collection, representation, analysis and interpretation of data. It covers a range of calculations of central tendency and spread as well as multiple charts and graphs to represent data. As it is the only unit directly exploring statistics, it is critical that children have time to explore the handling data cycle here and to focus sufficient time on interpreting their results.

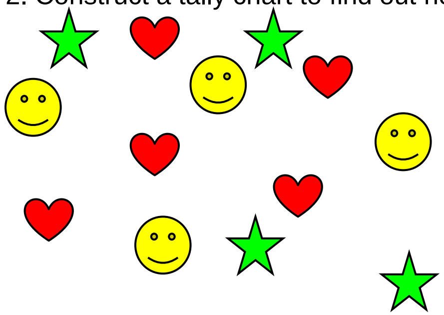
Prior Learning	Core Learning	Learning Leads to…
➢ measure and begin to record the following: - lengths and heights - mass/weight - capacity and volume - time (hours, minutes, seconds)	➢ **interpret and construct simple pictograms, tally charts, block diagrams and simple tables** ➢ **ask and answer simple questions by counting the number of objects in each category and sorting the categories by quantity** ➢ **ask and answer questions about totalling and comparing categorical data**	➢ interpret and present data using bar charts, pictograms and tables ➢ solve one-step and two-step questions [for example, 'How many more?' and 'How many fewer?'] using information presented in scaled bar charts and pictograms and tables

Exemplification

1. a) Some children were asked their favourite flavour ice cream. Make a pictogram for the following information:
Chocolate: 4
Strawberry: 3
Vanilla: 5

b) How many children were asked altogether?

2. Construct a tally chart to find out how many of each type of badge are in the diagram below:



a) Which type of badge are there more of?
b) How many badges are there in total?

3. Make a suitable chart or diagram to represent the following data:

Blue Red Blue Red Yellow Red Red Blue Red Yellow Red Red

Vocabulary

results	block
data	diagram/chart
list	construct
categories	most
tally (chart)	least
frequency	difference
table	total
pictogram	labels
key	axes
symbol	

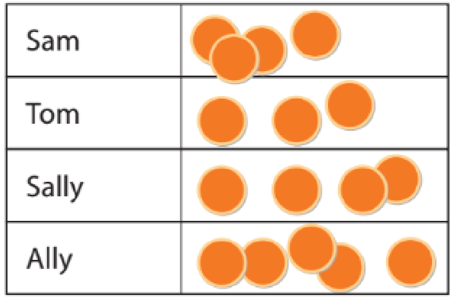
a) Which colour appears the most often? b) Can you put the colours in order by quantity?	

Representation	Fluency	Probing Questions
Tally Charts • Completing a class survey. • Making a human tally chart using children as each marker. • Initially do not group the children so they can see how tricky it is to count the totals. • Then group the children in 5s, with every 5th child then acting as a group collector. • Counting aloud in 5s to find the total of each category to show how this is easier.	1. Construct a tally chart using a template • identify different categories • create tallies for quantities less than 5 • create tallies for quantities between 5 and 10 • create tallies for quantities greater than 10 •	Show me what a tally for 4 looks like. What about 5? 6? 34?
Frequency Tables • Converting human tally chart into a frequency table by finding the total of each row • Practising organising the data into categories also by recording a new category when a child with a new value appears • When using a written list of data, show children how crossing off each item helps us check we haven't counted any twice.	2. Construct a simple frequency table • Complete a template of a table from a list of data by tallying and finding the total • Complete a frequency table from scratch from a list of data ○ identify the different categories from a list of data ○ count number of items in each category ○ create an ordered table	Show me a list of data with three categories Show me the frequency table for this tally chart
Reading/Interpreting Frequency Tables • Using a human frequency table to read values. Getting children who fit into the description of the question to stand up and the others to sit down so they can be counted.	3. Reading values from tables • Reading a given value from a table e.g. the number of children who like Red best • Finding the total of all the categories (total frequency) • Finding a subtotal e.g. the number of children who have 1 or 2 brothers or sisters • Finding a total involving 'more' or 'less' e.g. the number of children who have more than 5 pets • Find the category with the highest (or lowest) frequency • Order the categories from least to greatest frequency	Convince me that there total number of pieces of data is 30 Always, Sometimes, Never? The best way to collect information is to ask your friends
Pictograms • Using counters on a template to make a pictogram practically first	4. Construct a pictogram • Complete a pictogram on a template with a key where each symbol represents 1 item	Show me how you would represent 3 on this pictogram

• Exploring how 1 counter for 1 item is difficult when there are lots of items and so using 1 counter to represent more items (2 or 5) to develop the idea of keys	• Produce own pictogram with a key where each symbol represents 1 item ○ identify the categories ○ choose a suitable symbol to represent items ○ use same symbol in the pictogram ○ space the pictures equally to compare easily ○ include a key • Complete a pictogram on a template with a key where each symbol represents 2 items • Produce own pictogram with a key where each symbol represents 2 items • Complete a pictogram on a template with a key where each symbol represents 5 items • Produce own pictogram with a key where each symbol represents 5 items • Complete/produce a pictogram where each symbol represents 10 items	True or False? You cannot use a symbol to stand for 3 things on a pictogram. Always, Sometimes, Never? A pictogram is the best way to display data
Block Charts • Complete a block diagram template by colouring the blocks • Exploring examples with different sized blocks to discover why it is important that they are all the same size to help compare heights	5. Construct a block chart • colour in blocks on a block diagram template to match a frequency table • create own block diagram where axes given but scale must be added	Always, Sometimes, Never? A block graph shows what people like or don't like
Interpreting pictograms and block charts • Children should practice counting each category and writing them down - useful for finding totals and comparing • Children should also explore how block charts/pictograms are useful for comparing by counting differences • When listing values it is useful to order them for analysis • When comparing categories it is useful to use counters to represent values to easily count difference	6. Organising and comparing • Reading a given value from a table e.g. the number of children who like Red best • Finding the total of all the categories (total frequency) • Finding a subtotal e.g. the number of children who have 1 or 2 brothers or sisters • Finding a total involving 'more' or 'less' e.g. the number of children who have more than 5 pets • Find the category with the highest (or lowest) frequency • Order the categories from least to greatest frequency	Convince me that this pictogram and this block graph come from the same list of data What's the same and what's different? table, block graph, pictogram, tally, list

Further Extension

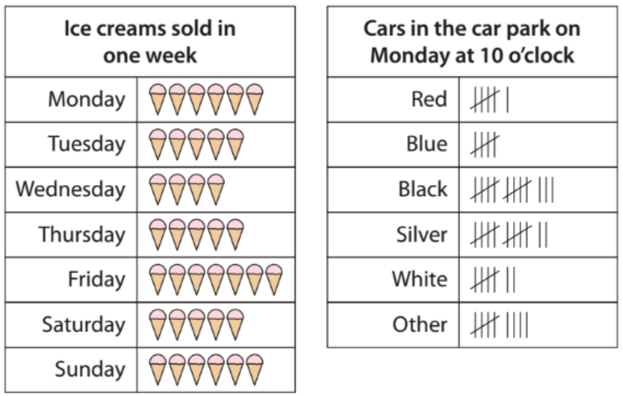
1. Four children played racing games at break time. Each time they won a game they took a counter.



Present the information in a different way to make it clearer and answer the following questions:
Who won the most races?
How many more races did Ally win than Sally?

Does the information answer the question:
Who is the fastest runner?

2.
What's the same? What's different?



Rich and Sophisticated Tasks

Interpret and construct simple pictograms, tally charts, block diagrams and simple tables

NRICH: Sticky Data * G P
NRICH: If the World Were a Village * P I
NRICH: Plants ** P
NRICH: What Shape and Colour?* G
NRICH: Carroll Diagrams * P
NRICH: Ladybird Count * P

Ask and answer simple questions by counting the number of objects in each category and sorting the categories by quantity

NRICH: The Hair Colour Game ** G P
NRICH: Mixed-up Socks ** P I
NRICH: Sort the Street * P
NRICH: Button-up * P
NRICH: Beads and Bags * P

Ask and answer questions about totalling and comparing categorical data

NRICH: In the Playground * I

Misconceptions	Teacher Guidance and Notes
Children may count a piece of data twice or miss some out when completing a tally chart from a list of data - this can be exacerbated if they have collected their own data and recorded it somewhat haphazardly initially. Children sometimes use different symbols for different categories in pictograms, making it hard to compare. They need to use the same symbol consistently throughout the pictogram. Children forget to bundle in 5s when completing a tally chart and may not count in 5s to save time when totalling. Children use limited vocabulary when describing findings e.g. they say that ""category A was the best"" when in fact they mean the most popular or the longest or the heaviest etc. They also avoid using the word 'frequency' and instead use total/amount. Children do not always think about the context of the data when they are reading from it. e.g. they assume that the highest number is the best	• This unit is children's first formal introduction to Statistics • The content of the unit focuses on BOTH representing and interpreting data so ensure you divide your time appropriately to give sufficient focus to reading from graphs and using this to answer questions. • As a minimum, it is recommended that you collect some data from or with the children so that they can relate their tables and charts to this data and make sense of it. • For pictograms, the pitch here is only for keys where the symbol represents, 1, 2, 5 or 10 items. Note that when it represents 5 or 10 items, significant subdivisions of these are not expected to be represented. • One possible approach to this unit is to complete the full data handling cycle to collect, record, represent and then analyse the data. • It is crucial to let children see what happens if you do not organise your data or label your axes and so on so that they realise the importance of doing things properly. • Try to model correct language e.g. frequency rather than amount or total, represents for the key on pictograms, block diagram/chart rather than bar chart

Key Assessment Checklist

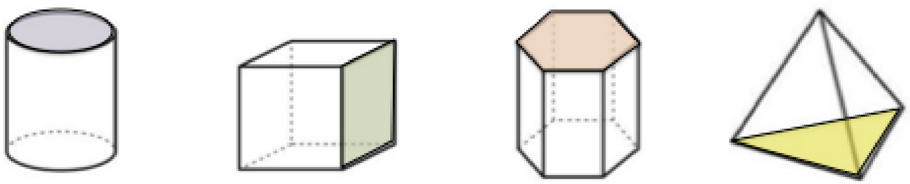
1. I can list the categories of data for a tally chart
2. I can record results in tally chart
3. I can produce a frequency table from a tally chart
4. I can construct a pictogram with one picture per item from a tally chart
5. I can construct a block diagram/chart from a tally chart
6. I can read results from a table, pictogram or block diagram.
7. I can use my readings to answer simple questions about the data
8. I can use my readings to make comparisons with the data

Year 2	Unit 11 : Visualising Shape
8 learning hours	In this unit children focus on exploring shapes practically and visually. There is an emphasis on sketching, constructing and modelling to gain a deeper understanding of the properties of shapes. It is therefore necessary to secure the practical skills at the same time as using them to explore the shapes in questions. At secondary level students are developing their skills in construction and the language/notation of shape up to the understanding, use and proof of circle theorems.

Prior Learning	Core Learning	Learning Leads to...
➢ recognise and name common 2-D and 3-D shapes, including: - 2-D shapes [for example, rectangles (including squares), circles and triangles] - 3-D shapes [for example, cuboids (including cubes), pyramids and spheres]"	➢ **identify 2-D shapes on the surface of 3-D shapes, [for example, a circle on a cylinder and a triangle on a pyramid]** *Working towards* • *recognise and name triangles, rectangles, squares, circles, cuboids, cubes, pyramids and spheres from a group of shapes or from pictures of the shapes (repeat)* *Working at expected standard* • *describe the properties of 2-D and 3-D shapes (e.g. the pupil describes a triangle: it has 3 sides, 3 vertices and 1 line of symmetry; the pupil describes a pyramid: it has 8 edges, 5 faces, 4 of which are triangles and one is a square) (repeat)* *Working at greater depth* • *describe similarities and differences of shape properties (e.g. finds 2 different 2-D shapes that only have one line of symmetry; that a cube and a cuboid have the same number of edges, faces and vertices but can describe what is different about them). (repeat)*	draw 2-D shapes and make 3-D shapes using modelling materials; recognise 3-D shapes in different orientations and describe them

Exemplification

1. a) Here are four shapes. Each shape has one face shaded.
Name the shape of the shaded face.



b) Jake is looking at a face of a 3D shape. The face is a triangle.
What could Jake's shape be? Give two possible answers.

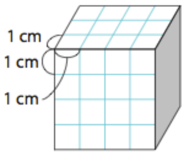
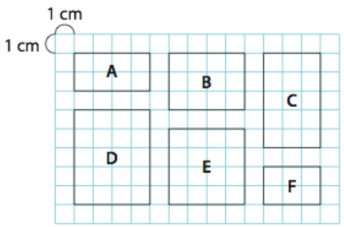
Vocabulary

flat	pyramid
curved	prism
straight	sphere
round	cone
vertex/vertices	cylinder
corner	circle
point	circular
face	triangle
side	triangular
edge	square
end	rectangle
sort	rectangular
surface	pentagon
cube	hexagon
cuboid	octagon

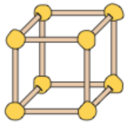
Representation	Fluency	Probing Questions
Sides and Vertices of 2D Shapes • Exploring and handling 2D shapes made of cardboard or plastic to count the sides and vertices • Making 2D shapes using geoboards and elastic bands (or dotty paper if visual rather than concrete preferred). For example, making a shape with 3 sides and 3 vertices. Is it possible to make a shape with 3 sides and 4 vertices?	1. Describe the sides and vertices of 2D shapes (from diagrams or names) • sketch a named 2D shape • state the number of vertices of a 2D shape • state the number of sides of a 2D shape • use number of sides/vertices to identify polygons i.e. to say if a shape is a quadrilateral or not or is a pentagon or not • identify whether the sides are straight or curved • identify whether any of the sides are the same length • given a description of the sides/vertices of a shape, suggest its name or sketch the shape	Always, Sometimes, Never? 2D shapes have the same number of sides as vertices Show me 4 different 2D shapes with 4 sides Always, Sometimes, Never? Shapes with 3 sides are triangles.
Identifying 2D Shapes • Matching the names of shapes to images of the shapes and to solid versions. NRich version of shape cards • Playing 'Guess My Shape' using 20 questions format (can be done nicely using a shape fan so that all children put forward a guess after each new fact is revealed)	2. Identify/draw (sketch) a 2D shape given its properties • given a property, suggest a possible shape e.g. four sides or all sides are equal length • given a property, suggest all possible shapes e.g. four sides or all sides are equal length • given several properties, pinpoint the exact shape e.g. three sides, one line of symmetry	Show me a 2D shape beginning with 'r' Show me a shape with only straight edges Convince me that all rectangles have 4 sides What's the same and what's different? triangle, hexagon, circle
Naming 3D shapes • Going on a shape hunt to find specific shapes in real life e.g. cuboids • Making models of shapes using plasticine or using construction materials (e.g. blocks, duplo, multi-link etc)	3. Recognise and name 3D shapes • cube • cuboid • pyramid (with different bases) • cone • prism (with different cross-sections) • cylinder • sphere (and hemisphere)	Show me a shape that you can see in the room What's the same and what's different? circle, sphere, square True or False? There are 3D shapes that are not cubes, cuboids, pyramids, cones, prisms, cylinders or spheres.

Faces, Edges and Vertices of 3D Shapes • Exploring and handling 3D shapes using mathematical models as well as everyday objects, especially packaging, to help count the faces, edges and vertices. • Printing with 3D shapes to explore the shapes of the faces. Which shapes have square faces? Did you print with any shapes with circular faces? What happens when you print with a sphere? A cylinder? • Pulling 3D shapes apart (e.g. packets) to see the 2D shapes that they are made from • Counting (and marking off) the vertices (edges/faces) of a 3D shape • Making models using straws and balls of modelling clay or equivalent. Then exploring how many straws and connecting balls you need to make a cube? A pyramid?	4. Describe the faces, edges and vertices of cubes and cuboids • sketch or find a cube or cuboid • state the number of faces of a cube or cuboid • identify whether the faces are flat or curved • identify whether any of the faces are the same • name the shapes of each face of a cube or cuboid • state/count the number of vertices of a cube or cuboid • state/count the number of edges of a cube or cuboid	Convince me that cuboids have 6 faces What's the same and what's different? cube, cuboid, rectangle Always, Sometimes, Never? Cuboids have square faces
	5. Describe the faces, edges and vertices of pyramids and cones • sketch or find a pyramid or cone • state the number of faces of a pyramid or cone • identify whether the faces are flat or curved • identify whether any of the faces are the same • name the shapes of each face of a pyramid or cone (not curved faces) • state/count the number of vertices of a pyramid or cone • state/count the number of edges of a pyramid or cone	Always, Sometimes, Never? A pyramid has 5 vertices True or False? A cone is a pyramid
	6. Describe the faces, edges and vertices of prisms and cylinders • sketch or find a prism or cylinder • state the number of faces of a prism or cylinder • identify whether the faces are flat or curved • identify whether any of the faces are the same • name the shapes of each face of a pyramid or cylinder (not curved faces) • state/count the number of vertices of a pyramid • state/count the number of edges of a pyramid or cylinder • given a description of the faces/vertices/edges of	True or False? A shape can be a prism and a pyramid. Convince me that the number of sides on a cross-section of a prism is 2 less than the number of faces.

	a shape, suggest its name	
	7. Describe the faces, edges and vertices of 3D shapes • sketch or find a sphere or hemisphere • given a property, suggest a possible shape e.g. a triangular face • given a property, suggest all possible shapes e.g. square face • given several properties, pinpoint the exact shape e.g. four faces, all of them are triangle	Show me a 3D shape with square faces Show me a 3D shape with some circular faces Show me a 3D shape with some triangular faces
Comparing 3D shapes • Sorting 3D shapes into categories e.g. number of faces using practical table with models of the shapes • Arranging 3D shapes into Venn diagrams and Carroll diagrams • Playing 'Guess My Shape' using 20 Qs format.	8. Compare 3D shapes • by number of faces • by shapes of faces • by number of vertices • by whether faces are curved or straight • by whether they meet a category definition e.g. pyramid or prism	Always, Sometimes, Never? Shapes with square bases are pyramids Convince me that there are at least two different shapes with 5 faces. Show me how you can order 3D shapes by the number of vertices they have

Further Extension	Rich and Sophisticated Tasks
1. We are going to make a box as shown.  Which quadrilaterals shown below do we need? How many of each do we need? 	Identify 2-D shapes on the surface of 3-D shapes [for example, a circle on a cylinder and a triangle on a pyramid] **NRICH: Cubes * I** **NRICH: Shadow Play *** P**

2.
Jack has made a cube using 12 sticks and 8 balls of modelling clay.



What shape could he make with:

6 sticks and 4 balls of clay?

4 long sticks, 8 short sticks 8 balls of clay?

3. This is a triangle – true or false? Explain your answer.

Misconceptions	Teacher Guidance and Notes
Children may confuse 2D and 3D shapes because of the faces on 3D shapes and the complexity of the concept of a 2D shape. Children's understanding of surface can be weak and lead to issues understanding the difference between the shape of a face and the whole shape. Children may interchange prisms and pyramids because of the vocabulary. Children do not always realise that cones are not pyramids and cylinders are not prisms.	• This unit builds on the learning covered in the Exploring Shape unit from earlier in the year • In this unit children focus on describing the faces of 3D shapes in detail (but earlier work on recognising and naming 2D and 3D shapes may need to be recapped alongside). • It is key with this unit to ensure that the classroom environment is rich with examples and that all classroom staff model the technical mathematical language. It is important to encourage children to refine their descriptions so that they too use the mathematical language with ease.

Key Assessment Checklist

1. I can recognise, name and describe the sides and corners of 2D shapes
2. I can count the faces and vertices on 3D shapes
3. I can recognise and name 3D shapes (including cubes, cuboids, pyramids, prisms, cones, cylinders and spheres)
4. I can describe the faces on a cube and a cuboid.
5. I can describe the faces on a range of pyramids and cones.
6. I can describe the faces on a range of prisms and cylinders
7. I can say how many faces a 3D shape has and which shapes
8. I can describe and sort 3D shapes by their faces

Year 2	Unit 12 : Exploring Change
7 learning hours	For primary pupils this unit focuses on the measures elements of time and co-ordinates. There is a progression from sequencing and ordering through telling the time formally to solving problems involving time. The co-ordinate work flows in the secondary students' learning focused on the relationships between co-ordinates. Key objectives include the use of y=mx+c for straight lines, the use of functions and the graphing of more complex functions.

Prior Learning

- ➢ sequence events in chronological order using language [for example, before and after, next, first, today, yesterday, tomorrow, morning, afternoon and evening]
- ➢ recognise and use language relating to dates, including days of the week, weeks, months and years
- ➢ tell the time to the hour and half past the hour and draw the hands on a clock face to show these times

Core Learning

- ➢ **compare and sequence intervals of time**
- ➢ **tell and write the time to five minutes, including quarter past/to the hour and draw the hands on a clock face to show these times**
- ➢ **know the number of minutes in an hour and the number of hours in a day.**

Working at expected standard

- *read the time on the clock to the nearest 15 minutes*

Working at greater depth:

- *read the time on the clock to the nearest 5 minutes*

Learning Leads to...

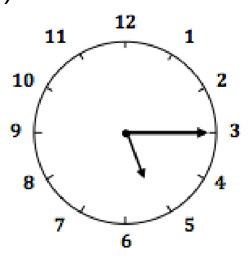
- ➢ tell and write the time from an analogue clock, including using Roman numerals from I to XII, and 12-hour and 24-hour clocks
- ➢ know the number of seconds in a minute and the number of days in each month, year and leap year
- ➢ compare durations of events [for example to calculate the time taken by particular events or tasks]
- ➢ estimate and read time with increasing accuracy to the nearest minute; record and compare time in terms of seconds, minutes and hours; use vocabulary such as o'clock, a.m./p.m., morning, afternoon, noon and midnight

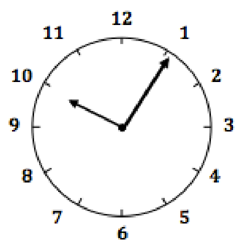
Exemplification

1. Write these times in order from shortest to longest

20 minutes	3 hours	100 minutes	1 hour	15 minutes

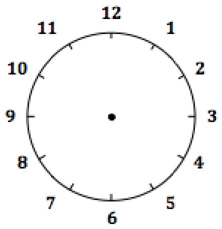
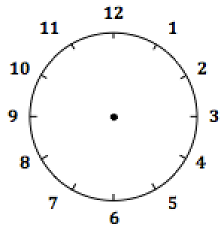
2.
a) Write the time shown on these clocks:





Vocabulary

fortnight	quick, quicker, quickest, quickly
hour	fast, faster, fastest
minute	slow, slower, slowest, slowly
second	old, older, oldest
o'clock	new, newer, newest
half past	takes longer, takes less time
quarter past	how long ago?
quarter to	how long will it be to…?
digital clock	how long will it take to…?
analogue clock	how often?
watch	always
timer	never
now	
soon	
early	

b) Draw hands on these clock faces to show these times: Ten past five  Quarter to eleven  3. a) How many hours are there in one day? b) How many minutes are there in one hour?	late earliest latest	often sometimes usually once, twice, three times

Representation	Fluency	Probing Questions
Units of Time • Counting along with a clock as it ticks a minute to see how many seconds there are in 1 minute • Counting in 5s around a clock face to discover that there are 60 seconds in a minute and 60 minutes in an hour. • Estimating a minute by putting heads down on the desk and sitting up silently when they think 1 minute has gone.	1. Know the relationship between (seconds,) minutes, hours and days • know the number of minutes in an hour • know the number of hours in a day • recognise times greater than an hour and less than an hour • convert a time greater than 60 minutes to hours and minutes • recognise times greater than a day (and less than a day) • begin to have a sense of lengths of times e.g. 3 seconds, 1 minute, 5 minutes.	Show me a time that is shorter than 10 minutes Show me a time that is longer than 1 hour Convince me that 100 minutes is not an hour Convince me there are not 12 hours in a day What's the same and what's different? 12 months, 100 weeks, 1 year
Telling the Time • Labelling a clock with key words and multiples of 5 up to 55 to link to counting in 5s. Then counting round the clock and moving the hands to match e.g. 1:00, 1:05, 1:10, 1:15, ...	2. Tell the time to five minutes using 12-hour format (from an analogue clock) • recap: count in 5s to 60 • read a time from a clock in 12 hour format e.g. 7:25 • move the hands on a clock face to represent a 12-hour time • draw hands on a clock face to represent a 12-hour time	Show me two times which are an hour apart
	3. Tell the time to quarter past (analogue clock)	Show me a time which is later in the day

 • Adding labels to represent past/to and multiples of 5 up to 25 as well as quarters and half. Then counting round the clock and moving the hands to match e.g. one o'clock, five past one, ten past one, quarter past one, twenty past one, twenty-five past one, ...  • Making clocks using paper plates, card sticks and split pins for hands • Using manipulative clocks to show and read times (preferably mini-clocks for each child and a larger one for the teacher) • Making human clocks using arms	• recap: half past the hour • associate half past with ... : 30 • tell the time to quarter past the hour • associate quarter past with .. : 15 • move/draw hands on a clock face to show a time of half past an hour and quarter past an hour	than quarter past eleven Convince me that quarter past three is the same as 3:15
	4. Tell the time to quarter to the hour (analogue clock) • tell the time to quarter to the hour • associate quarter to with ... : 45 • move/draw hands on a clock face to show a time of quarter to an hour	Convince me that quarter to ten is before quarter past ten which is before half past ten What's the same and what's different? quarter to four; 15 minutes past four; quarter past four
	5. Tell the time to the nearest five minutes (analogue clock) • tell the time as a multiple of 5 past the hour (five past, ten past, twenty past and twenty-five past) • tell the time as a multiple of 5 to the hour (five to, ten to, twenty to, twenty-five to) • tell the time from a mixed selection of past and to times (including quarters and halves)	Convince me that quarter past seven is the same as 15 minutes past seven What's the same and what's different? 5:30; twenty past five; quarter past five; 5:20, half past five, 5:15
	6. Show the time to the nearest five minutes (analogue clock) • move the hands on a clock to represent a past time (five, ten, quarter, twenty, twenty-five, half) • move the hands on a clock to represent a to time (five, ten, quarter, twenty, twenty-five) • draw the hands on a clock to represent a given time	Show me two times which are an hour apart on your clock. Show me how you would show five past six on this clock
Time Sequencing • Representing a time (including those in excess of 60 minutes) by counting in 5s and using a clock rotation to convert to hours and minutes. • Using a number line to represent and position times to support ordering	7. Compare and order lengths of time • compare two times in minutes • order three or more times in minutes • compare two times in hours and minutes • order three or more times in hours and minutes • find the interval between two times	Show me how you would order these time intervals by size 15 minutes, 1 hour, 1 day, 2 minutes, 100 minutes Convince me that these intervals are in size order: 5 minutes, half an hour, 1 hour, 90 minutes

- Making timelines to show events in sequence and with more accurate time positioning

Further Extension

1.
Which of these clock faces shows a time between 5 o'clock and 7 o'clock?

2.
Jack says, 'There isn't any point in having a minute hand on a clock because I can still tell the time without it.'

Do you agree with him?

Explain your answer.

Rich and Sophisticated Tasks

Tell and write the time to five minutes, including quarter past/to the hour and draw the hands on a clock face to show these times

NRICH: What's the Time? * P
NRICH: Stop the Clock * G**

Know the number of minutes in an hour and the number of hours in a day

NRICH: Matching Time * G

Misconceptions

Children will encounter a huge range of vocabulary in this topic and may struggle to link the words to the scale of time they represent- for example, when trying to order times they need to have a sense of minutes, hours, days, weeks, months

Teacher Guidance and Notes

- This unit builds on the work in Stage 1 on telling the time to the hour and half hour.
- The expectation is that children can tell the time to 5 minutes (and that

etc. but this may not come naturally. Children may still confuse the hands of the clock and the two scales for hours and minutes. This can cause them to invert a time. For example, they may read the correct time of 'ten past eight' as 'eight past two' or even 'forty past two'. Children may also struggle when the hour hand moves between whole hours to see which hour it is nearest to. The concept of past and to the hour can confuse children in terms of deciding which hour to reference. Children particularly may incorrectly tell 'to' times using the previous hour. For example, they may read the time of 'ten to eleven' incorrectly as 'ten to ten'. Some children may not realise that when the minutes are past the hour, the minute hand must be carefully positioned in relation to how many minutes past the hour it is and not point to the hour. Some pupils may presume that the decimalised number system applies to time and hence assume there are 100 seconds in a minute or 100 minutes in an hour and so on.	those working at greater depth can approximate times between these intervals to the nearest 5 minutes). • Some research indicates that children find it easier to tell the time using 12-hour format first (rather than the fractional language and past/to complexities that we generally commence with). This is particularly true for children with certain specific learning needs. Therefore, it is suggested in the fluency steps above that children commence by telling the time using 12-hour format as an hour and number of minutes up to 55 (despite the direct reference to 12-hour time not appearing in the national curriculum until Stage 3). • More information about this approach can be found in this article on Charlotte's clock • As in Stage 1, try to incorporate work on time into daily routines to build children's knowledge, recall and confidence. • It can be useful to have 2 scales on the clock in different colours: one representing hours and one representing minutes. • Include references to the durations of activities e.g. 5 minutes or an hour to help build children's sense of time (although the calculation of durations is not required until Stage 3)

Key Assessment Checklist

1. I can compare and order intervals of time.
2. I can tell the time to quarter past the hour.
3. I can tell the time to quarter to the hour.
4. I can show the time to the quarter hour on a clock face.
5. I can tell the time to 5 minute intervals.
6. I can show times on a clock face to 5 minutes.
7. I can say the number of minutes and hours in a day.
8. I can use a timeline to order events.

<table>
<tr><th>Year 2</th><th colspan="3">Unit 13: Proportional Reasoning</th></tr>
<tr><td>7 learning hours</td><td colspan="3">In this unit pupils explore proportional relationships, from the operations of multiplication and division on to the concepts of ratio, similarity, direct and inverse proportion.
For primary pupils in Stages 1-3, this is focused on developing skills of division. Stages 4 and 5 revisit the whole of calculation to broaden to all four operations in a range of contexts and combination problems; the emphasis here is really on representing and then solving a problem using their calculation skills, not just calculating alone.
In Stage 6 the real underpinning concepts of proportion and ratio develop.
Secondary pupils begin to formalise their thinking about proportion by finding and applying scale factors, dividing quantities in a given ratio and fully investigating quantities in direct or inverse proportion, including graphically.</td></tr>
<tr><th>Prior Learning</th><th>Core Learning</th><th colspan="2">Learning Leads to...</th></tr>
<tr><td>➢ solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher</td><td>➢ recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables

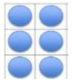
➢ solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts

Working towards:
• recall doubles and halves to 20 (repeat)

Working at expected standard
• recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables to solve simple problems, demonstrating an understanding of commutativity as necessary (e.g. knowing they can make 7 groups of 5 from 35 blocks and writing 35 ÷ 5 = 7; sharing 40 cherries between 10 people and writing 40 ÷ 10 = 4; stating the total value of six 5p coins)

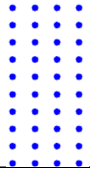
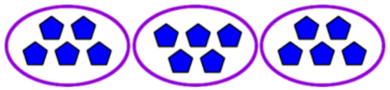
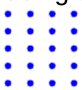
Working at greater depth:
• determine remainders given known facts (e.g. given 15 ÷ 5 = 3 and has a remainder of 0, pupil recognises that 16 ÷ 5 will have a remainder of 1; knowing that 2 × 7 = 14 and 2 × 8 = 16, pupil explains that making pairs of socks from 15 identical socks will give 7 pairs and one sock will be left).
• solve word problems that involve more than one step (e.g. which has the most biscuits, 4 packets of biscuits with 5 in each packet or 3 packets of biscuits with 10 in each packet?).</td><td colspan="2">➢ recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables
➢ write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods
➢ solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems</td></tr>
<tr><th colspan="2">Exemplification</th><th colspan="2">Vocabulary</th></tr>
<tr><td colspan="2">1. Complete the missing numbers:
a) $7 \times 2 = \cdots$ b) $5 \times ... = 20$ c) $60 \div 10 = \cdots$ d) $... \div 2 = 9$</td><td>multiply
x</td><td>column
divide</td></tr>
</table>

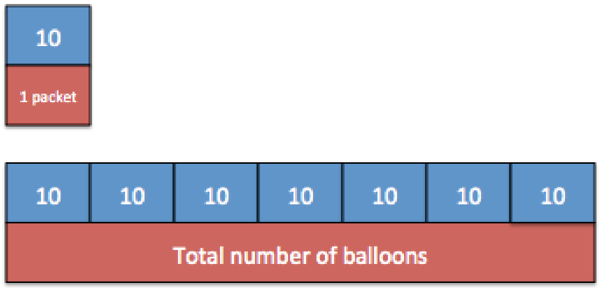
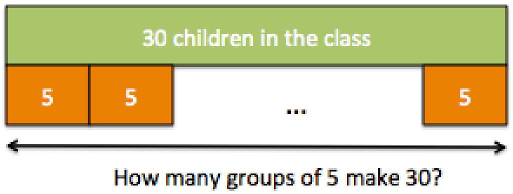
<table>
<tr>
<td colspan="2">2.
a) A teacher puts the children in a class into groups of 5. Altogether there are 30 children in the class.
How many groups will the teacher make?
b) Matt is buying balloons for a party.
The balloons come in packs of 10.
Matt buys 7 packets.
How many balloons will he have for the party?
c) Jessie is calculating 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2.
How can Jessie work this out more quickly?</td>
<td>times
represent
lots of / groups of
multiple of
times as
(big/long/wide/tall as ...)
double
array
row</td>
<td>÷
divided by
division
divided into
dividend
divisor
share
group
halve</td>
</tr>
<tr>
<th>Representation</th>
<th>Fluency</th>
<th colspan="2">Probing Questions</th>
</tr>
<tr>
<td>Multiplication and Division by 2
• Arranging objects into equal groups and counting efficiently in 2s
(e.g. sweets, animals, books, bean bags, counters, cubes and so on arranged into hoops, lily pads, boxes, bags, and so on)
e.g. 2 frogs on each lily pad or 5 shapes in each hoop

• Sharing a set of objects into 2 groups and counting the number in each group
• Grouping a set of objects into 2s and counting the number of groups
• Using an array to represent a multiplication e.g. 2 x 3

• Using an array to represent a division e.g. $10 \div 2$
</td>
<td>1. Recall and use multiplication and division facts for the two times table
• Count in 2s from 0 (forwards and backwards)
• Represent a multiplication by 2 concretely or visually
• Represent a division by 2 concretely or visually
• Complete a missing answer for a multiplication e.g. $7 \times 2 = ■$
• Complete a missing answer for a division e.g. $12 \div 2 = ■$
• Find a missing number in a multiplication statement e.g. $■ \times 2 = 20$
• Find a missing number in a division statement e.g. $■ \div 2 = 18$
• Give the fact family for a multiplication or division by 2</td>
<td colspan="2">What's the same and what's different?
2 x 6, 6 x 2; 24 ÷ 2

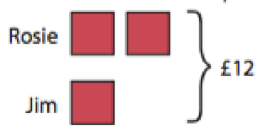
Convince me that if I know 13 x 2 = 26, I also know three other facts.</td>
</tr>
<tr>
<td>Multiplication and Division by 10
• Arranging objects into equal groups and counting efficiently in 10s
(e.g. sweets, animals, books, bean bags, counters, cubes and so on arranged into hoops, lily pads, boxes, bags, and so on)</td>
<td>2. Recall and use multiplication and division facts for the ten times table
• Count in 10s from 0 (forwards and backwards)
• Represent a multiplication by 10 concretely or visually</td>
<td colspan="2">True or False?
A number that divides by 10 will divide by 2

Convince me that the missing number</td>
</tr>
</table>

• Sharing a set of objects into 10 groups and counting the number in each group • Grouping a set of objects into 10s and counting the number of groups • Using an array to structure the groups of 10 more formally e.g. 10 x 3  • Using an array to represent a division e.g. $40 \div 10$ 	• Represent a division by 10 concretely or visually • Complete a missing answer for a multiplication e.g. $7 \times 10 = ■$ • Complete a missing answer for a division e.g. $120 \div 10 = ■$ • Find a missing number in a multiplication statement e.g. $■ \times 10 = 20$ • Find a missing number in a division statement e.g. $■ \div 10 = 9$ • Give the fact family for a multiplication or division by 10	is 7 $■ \times 10 = 70$
Multiplication and Division by 5 • Arranging objects into equal groups and counting efficiently in 5s (e.g. sweets, animals, books, bean bags, counters, cubes and so on arranged into hoops, lily pads, boxes, bags, and so on)  • Sharing a set of objects into 5 groups and counting the number in each group • Grouping a set of objects into 5s and counting the number of groups • Using an array to structure the groups of 5 more formally e.g. 5 x 3  • Using an array to represent a division e.g. $20 \div 5$ 	3. Recall and use multiplication and division facts fort he five times table • Count in 5s from 0 (forwards and backwards) • Represent a multiplication by 5 concretely or visually • Represent a division by 5 concretely or visually • Complete a missing answer for a multiplication e.g. $7 \times 5 = ■$ • Complete a missing answer for a division e.g. $50 \div 10 = ■$ • Find a missing number in a multiplication statement e.g. $■ \times 5 = 20$ • Find a missing number in a division statement e.g. $■ \div 5 = 3$ • Give the fact family for a multiplication or division by 5	What's the same and what's different? Five 2ps, Ten 1ps, 2 x 5, 5 x 2, 1 lot of 10, 2 lots of 5; Two 5ps What's the same and what's different? a number that is a multiple of 5 and a number that is a multiple of 10 Convince me that the missing number is 6 $■ \times 5 = 30$
Multiplication and Division Statements • Using number cards to produce a number sentence to represent a concrete or visual image of multiplication and division • Using number cards and rearranging these to form	4. Create multiplication and division statements • Produce a concrete/visual representation for a given multiplication or division statement • Produce an array for a given multiplication or division statement	Show me how you would write a multiplication sentence for 'the number that is twice as big as 8' Show me a number sentence you can

true statements (to find equivalent calculations)	• Write a multiplication statement for a given concrete/visual representation (e.g. an array) • Write a division statement for a given concrete/visual representation (e.g. an array) • Given number cards and symbols, create correct multiplication and division statements • Say if a given multiplication or division statement is true or false and justify this	make using 5, 3 and 15. Show me four number sentences that can be made using 4, 5 and 20 and the symbols for multiplication, divide and equals.
Solving Problems • Using the bar model to represent multiplication and division problems to help decide what calculation to complete. • For example: Matt is buying balloons for a party. The balloons come in packs of 10. Matt buys 7 packets. How many balloons will he have for the party?  • For example: A teacher puts the children in a class into groups of 5. Altogether there are 30 children in the class. How many groups will the teacher make? 	5. Solve worded multiplication problems • Draw a bar model to represent the word problem • Represent a word problem concretely or visually (e.g. with groups or scaling) • Make or draw an array to represent a multiplication problem • Write a mathematical statement to show the calculation needed • Solve the problem and complete the mathematical sentence to show the answer	Show me a bar model that represents '5 groups of 8' Show me a bar model that represents the length that is 10 times as long as 3cm.
	6. Solve worded division problems • Draw a bar model to represent the word problem • Represent a word problem concretely or visually (e.g. by sharing/grouping) • Make or draw an array to represent a division problem • Write a mathematical statement to show the calculation needed • Solve the problem and complete the mathematical sentence to show the answer	Show me a bar model that represents '35 shared between 5 people' Show me a bar model that represents '16 shared into 2s'
	7. Identify the operation required to solve a multiplication or division problem • know key trigger words for multiplication and division • recognise whether problem is multiplication or division • represent the problem concretely or visually	What's the same and what's different? Shared Between 5 5 Lots of 5 Times as Big as Shared into groups of 5 Divided into 5s Multiplied by 5

	• solve the problem • record the problem using a number sentence	Convince me that $30 \div 5 = 2 \times 3$

Further Extension	Rich and Sophisticated Tasks
1. Sally buys 3 cinema tickets costing £5 each. How much does she spend? Write the multiplication number sentence and calculate the cost. If Sally paid with a £20 note, how much change would she get? 2. Together Rosie and Jim have £12. Rosie has twice as much as Jim. How much does Jim have? *The bar model can be helpful in solving these types of problems.*  $12 \div 3 = 4$ Jim has £4 3. Two friends share 12 sweets equally between them. How many do they each get? Write this as a division number sentence. Make up two more sharing stories like this one. Chocolate biscuits come in packs (groups) of 5. Sally wants to buy 20 biscuits in total. How many packs will she need to buy? Write this as a division number sentence. Make up two more grouping stories like this one. 4.	Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers **NRICH:** Odd Times Even ***** I** **NRICH:** Two Numbers Under the Microscope **** I** **NRICH:** Even and Odd *** I** **NRICH:** Ring a Ring of Numbers *** G** **NRICH:** More Numbers in the Ring ***** G P** **NRICH:** How Odd **** I** **NRICH:** Clapping Times *** G I** **NRICH:** Double or Halve? *** G** **NRICH:** Always, Sometimes or Never? *** P** Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts **NRICH:** Our Numbers *** G** **NRICH:** Are You Well Balanced? ***** G I** **NRICH:** Magic Plant **** P** **NRICH:** The Amazing Splitting Plant ***** P** **NRICH:** The Tomato and the Bean ***** P** **NRICH:** Lots of Lollies ***** P I** **NRICH:** Ip Dip *** I** **NRICH:** Growing Garlic ***** P**

Two friends want to buy some marbles and then share them out equally between them.

They could buy a bag of 13 marbles, a bag of 14 marbles or a bag of 19 marbles.

What size bag should they buy so that they can share them equally?

What other numbers of marbles could be shared equally?

Explain your reasoning.

Misconceptions	Teacher Guidance and Notes
Children sometimes struggle with the range of language for both multiplication and division - they may incorrectly read 3 x 4 as 3 lots of 4 rather than 3 multiplied by 4 (which is actually therefore 4 lots of 3) and so on. This can lead to errors in representing the array correctly (which we want to avoid to enable them to 'see' commutativity and to later represent more complex multiplication like 13 x 4 in a consistent way). Children may not see how an array can be used to support division, only multiplication. They may invert their array accidentally. Children tend to use the 'lots of' representation [repeated addition] of multiplication much more than scaling. Similarly, they may use sharing more than grouping (which makes using an array for division harder). When dividing children sometimes muddle the divisor and the dividend and so try to divide 'the wrong way round'. Children tend to stick to their favourite representations - they may overly rely on, for example, numicon when a bead string or an array could be more helpful. Children can mistake and record the adding and multiplying symbol. Children may not recognise key trigger words for multiplication and division and so use the wrong operation when solving a word problem. Children may make errors if multiplication facts are not secure.	• This unit builds on the earlier work on multiplication and division completed in Unit 9. This previous work focused on the conceptual understanding of the processes of multiplication and division as well as the recording of number sentences. • The focus here is on the recall of the key multiplication tables and the use of these to solve calculations immediately (without working out) as well as on the identification and solution of word and other problems. It is expected that the multiplication and division facts for 2s, 5s and 10s will be a focus of mental maths throughout the academic year and so, by this stage, are more secure. • If children are secure on the earlier work then you can move on to a problem solving focus quickly. However, if you need to revisit the processes and meanings of multiplication and division, there is the opportunity to do so. • It is important to use both the representations of arrays and the bar model within this unit in order to lay the foundations for further use in Stage 3 and beyond. • Note that in Stage 2, we need to formalise what has been learnt in Stage 1 and hence there is greater focus on using correct mathematical language and symbols. • You should in general use examples of problems based within the 2, 5 and 10 multiplication tables, although there may be the opportunity to go beyond this for children working at greater depth. • Commutativity is covered in Unit 9, but is used and presumed here by expecting children to be able to recognise the links between related calculations.

Key Assessment Checklist

1. I can recall and complete multiplication and division facts for the 2 times table.
2. I can recall and complete multiplication and division facts for the 10 times table.

3. I can recall and complete multiplication and division facts for the 5 times table.
4. I can represent and solve multiplication word problems
5. I can represent and solve division word problems
6. I can create multiplication and division statements to represent problems and concrete/visual representations
7. I can represent a problem with practical equipment or a picture to help me decide which calculation to carry out.
8. I can use this to solve a problem in context.

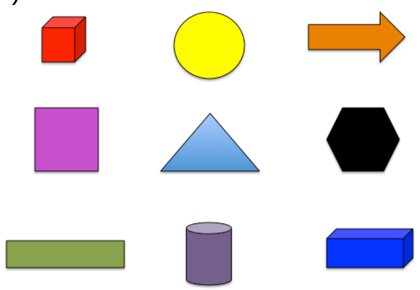
Year 2	Unit 14: Describing Position
5 learning hours	In this unit pupils explore how we can communicate position and movement mathematically. They look at transformations from simple turns to reflection/rotation/enlargement/translations up to similar shapes generated by enlargements, co-ordinate systems and ultimately vectors

Prior Learning	Core Learning	Learning Leads to...
➢ describe position, direction and movement, including whole, half, quarter and three-quarter turns	➢ **use mathematical vocabulary to describe position, direction and movement, including movement in a straight line and distinguishing between rotation as a turn and in terms of right angles for quarter, half and three-quarter turns (clockwise and anti-clockwise)**	➢

Exemplification

1. Look at this picture and complete the sentences:

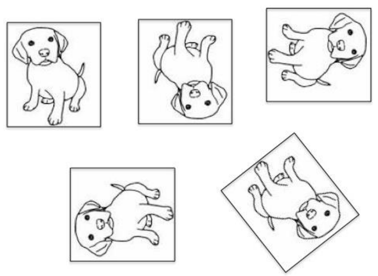
a)



(i) The rectangle is ……… the square
(iii The arrow is …………. the hexagon and ……………. the circle.
(iii) The …………. is between the rectangle and the cuboid.

b) This picture of a dog is rotated three quarters of a turn clockwise.

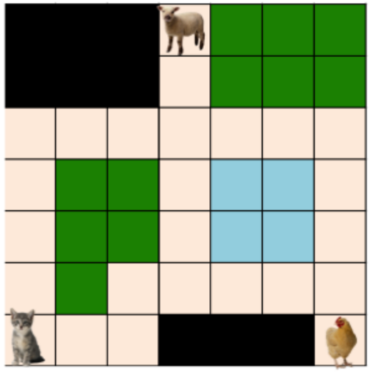
Which of these images show what the picture will look like afterwards?



Vocabulary

where	left
position	right
	forward
above	backwards
below	
next to	turn
beside	half
under	quarter
inside	three quarter
outside	clockwise
	whole
in front of	anticlockwise
on top of	
to the left of	(north)
to the right of	(east)
behind	(south)
between	(west)

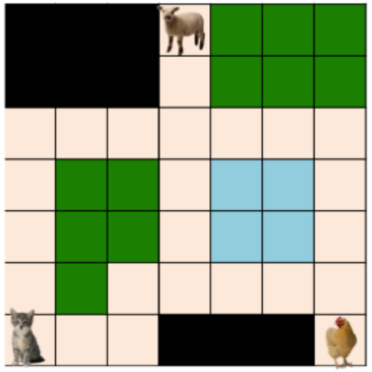
c) Give instructions to get from the chicken to the sheep (you must stick to the paths)

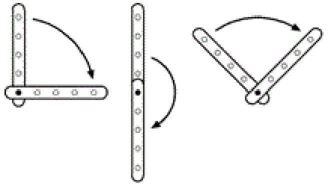


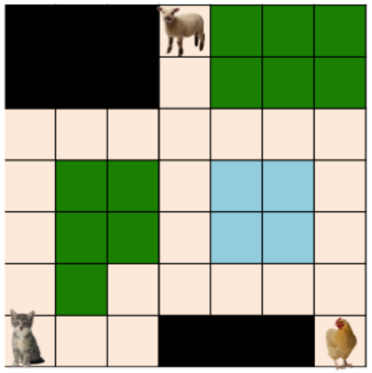
Representation	Fluency	Probing Questions

Position
• Playing hide and seek games where children hide and the others must describe or give clues to where they are hiding. For example: 'inside the cupboard' or 'behind the desk' or 'next to the coats'.
• Playing 'find the shape' games by finding the shape/object from a given picture that is in a given position e.g. above the square. Children can also be the describer and give instructions to others to guess their shape. (The image in Further Extension is useful for this).
• Arranging objects into the right position given instructions about how they should be placed.
	1. Use mathematical language to describe the position of an object
	• find an object given a description of its position
	  ○ above, below, under, on top of
	  ○ in front of, behind
	  ○ next to, beside
	  ○ to the left of, to the right of
	  ○ between
	  ○ inside, outside
	• place an object given a description of its position
	• give a description of a the position of an object
		What's the same and what's different?
		above; left; right; below; next to; under
		
		Show me a diagram where:
		The square lies above the triangle.
		The circle is next to the triangle.
		The square is between the hexagon and the triangle.
		
		Always, Sometimes, Never?
		Something that is to my left is also to your left

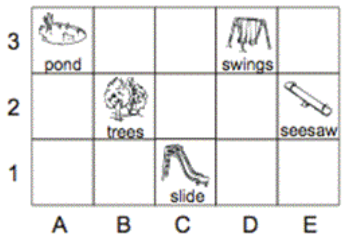
Movement
• Carrying out movements on grids with people or objects to map a route.
• Using a simple grid map and directions involving squares forwards, backwards, left and right to represent journeys
	2. Use mathematical language to describe a movement in a straight line
	• carry out a movement given instructions
	  ○ forwards, backwards
	  ○ to the left, to the right
	• describe a movement
	• producing a set of instructions combining movements in straight lines on a grid e.g. two squares forward and 1 square left
		Show me one way to get from A to B on the diagram
		... a different way
		
		What's the same and what's different?
		up; down; forwards; backwards; left; right



Turn • Turning (people/themselves) through quarter turns and chanting ¼ turn, ½ turn, ¾ turn, whole turn and so on. Changing direction from clockwise to anticlockwise. Using one arm as a point to show the start and end point of each turn. This can also be done with clock hands. • Using geostrips to produce different angles and turns  • Rotating images to explore what they look like in different orientations • Exploring a compass to see the links between compass points and turns.	3. Use mathematical language to describe a turn (direction and size) • recognise and carry out a whole turn • recognise and carry out a half turn (in either direction) • recognise and carry out a quarter turn (in either direction) • recognise and carry out a three quarter turn • use the words clockwise and anticlockwise to describe and carry out turns • understand a diagram showing a turn with an arrow • draw a diagram to show a given turn with an arrow • recognise that the direction does not matter for a half turn • understand that a ¼ turn in one direction has the same effect as a ¾ turn in the other direction • understand that a ¼ turn clockwise is the same as turning right and ¼ turn anticlockwise is the same as turning left • understand that other turns are possible between the quarter turns	Show me ... using your body ... using your geostrips: ... using the clock hands ... using your mini-whiteboard (ie draw) - a quarter turn - a quarter turn clockwise - a quarter turn anticlockwise - a half turn - a half turn clockwise - a whole turn - a three quarter turn anticlockwise Convince me that a half turn clockwise and half turn anti-clockwise will be in the same position Convince me that a quarter-turn clockwise is the same as a three-quarter turn anti-clockwise What's the same and what's different? quarter-turn clockwise; quarter turn anticlockwise; three quarter turn clockwise; three quarter turn anticlockwise

		Always, Sometimes, Never? There are two ways to do any turn
Directions • Following instructions to find the treasure e.g. walk forwards 10 steps. Then turn a quarter turn to the right. • Giving their own instructions to find an item to a partner or teacher • Using a simple map to follow and create instructions for journeys e.g. give instructions to get from the cat to the chicken  • Using Beebots or Scratch to experiment with making other objects follow paths and carry out turns	4. Interpret directions for a simple journey • follow simple directions involving forwards, backwards and turns left and right in a practical situation • follow simple directions involving turns (¼ , ½ , ¾) in a practical situation • follow more complex sets of directions (3 or more steps) in a practical situation • follow directions on a map (i.e. visually rather than practically)	What's the same and what's different? to the right; to the left; clockwise; anticlockwise Convince me that going forwards 4 squares, then turning 90 degrees clockwise and then going forwards 3 squares has the same result as turning 90 degrees clockwise, then going forwards 3 squares, then turning 90 degrees anticlockwise and going forward 4 squares.
	5. Create directions for a simple journey • produce simple directions involving forwards, backwards and turns left and right in a practical situation • produce simple directions involving turns (¼ , ½ , ¾) in a practical situation • produce more complex sets of directions (3 or more steps) in a practical situation • produce directions using only a map (i.e. visually rather than practically)	Show me how you would go round this track using mathematical language

Further Extension	Rich and Sophisticated Tasks
1.	Use mathematical vocabulary to describe position, direction and movement, including movement in a straight line and distinguishing between rotation as a turn and in terms of right angles for quarter, half and three-quarter turns (clockwise and anti-clockwise) **NRICH: Turning Man * I** **NRICH: Walking Round a Triangle * P** **NRICH: Triangle Animals ** P** **NRICH: Cover the Camel * P**

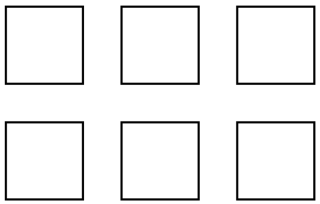


Use the grid to help you complete this table.

trees	B2
slide	
seesaw	
	A3

2. (NRich Task: Coloured Squares)

Use these clues to colour each shape:



- Blue is between green and red
- Orange is below green
- Yellow is to the left of both purple and orange

Misconceptions

Children may still be confusing left and right directions, particularly when starting from a position with a non-standard orientation.

Children may confuse clockwise with anti-clockwise directions or possibly fail to understand that there are two possible directions of turn.
Some children always turn in a clockwise direction, regardless of the instructions.

Some children do not recognise that all the points of a shape move in the same direction and the same distance during a linear movement (translation) and may distort their shape. Similarly they may do this under a rotation.

Teacher Guidance and Notes

- This unit builds on the work of Stage 1 and formalises the use of the language.
- You can relate clockwise and anticlockwise movement to a clock and use this a mainstay of teaching in the unit - children should be able to describe the relationships between the hands and the movement of them in terms of rotations.
- Try to stick at this stage to rotations of multiples of quarter turns- however, if appropriate you can explore the ideas of rotations of less than or more than a quarter turn for example.
- Within this position and direction work, there is a general path of

It is common for children to rotate a shape and end up with a reflection because they do not realise that the shape should look the same when viewed 'the right way up'. Children may not always appreciate that there are a range of ways to describe a possible path between two objects. When turning, some children may lose count of where they started and/or how much (how many quarters) they have turned. They may need to mark the starting point and count as they turn. This can also lead to children believing that right angles must be made from horizontal and vertical lines only.	abstraction moving from gross physical movement of the children themselves, to movement of objects, to movement of virtual objects (using ICT) to movement of abstract objects (on paper). Make sure there are opportunities to follow this progression if needed to secure children's skills in both carrying out and describing these movements. • There is an opportunity to introduce the points of the compass in this unit too.

Key Assessment Checklist

1. I can describe the position of an object using mathematical language
2. I can describe movement in a straight line using mathematical language.
3. I can say if an object has been rotated a quarter turn, a half turn or a whole turn and identify the direction (clockwise and anticlockwise).
4. I can rotate an object, clockwise or anti-clockwise, using quarter, three-quarters and half turns
5. I can follow instructions for a particular path using correct mathematical language.
6. I can give instructions for a particular path using correct mathematical language.

Year 2	Unit 15 : Measuring and Estimating	
8 learning hours	In this unit children and students explore measurement, standard units of measure and conversions between these units. At secondary level this extends to investigating compound measures such as density and pressure.	
Prior Learning	**Core Learning**	**Learning Leads to...**
➢ compare, describe and solve practical problems for: o lengths and heights [for example, long/short, longer/shorter, tall/short, double/half] o mass/weight [for example, heavy/light, heavier than, lighter than] o capacity and volume [for example, full/empty, more than, less than, half, half full, quarter] o time [for example, quicker, slower, earlier, later] ➢ measure and begin to record the following: o lengths and heights o mass/weight o capacity and volume o time (hours, minutes, seconds)	➢ **choose and use appropriate standard units to estimate and measure length/height in any direction (m/cm); mass (kg/g); temperature (°C); capacity (litres/ml) to the nearest appropriate unit, using rulers, scales, thermometers and measuring vessels** ➢ **compare and order lengths, mass, volume/capacity and record the results using >, < and =** *Working at expected standard* • *read scales in divisions of ones, twos fives and tens in a practical situation where all numbers on the scale are given (e.g. pupil reads a temperature from a thermometer or measures capacities using a measuring jug)* *Working at greater depth:* • *read scales in divisions of ones, twos, fives and tens in a practical situation where not all numbers on the scale are given*	➢ measure, compare, add and subtract: lengths (m/cm/mm); mass (kg/g); volume/capacity (l/ml)

Exemplification	Vocabulary	
1. (i) Say which unit would be best for measuring: (a) the length of a pencil (b) the mass of a table (c) the capacity of a bucket (d) the length of the playground (e) the temperature of the moon (ii) Joe is measuring the mass of a box using some scales	unit sensible unit measuring equipment measure estimate length height width	gram(s) litre(s) centimetre(s) kilogram(s) millilitres(s) prefixes: milli, centi, kilo together with metres, grams,

What is the mass of the box?

2. a) Put these lengths in order from smallest to largest

12 cm   12 m   2 cm   2m   212 m

b) Put the signs < or > in to these gaps to make the sentences true
The capacity of a bath …… the capacity of a bucket
The mass of a mouse ….. the mass of a human

distance mass volume capacity temperature metre(s)	litres celsius scales thermometers measuring vessels compare less than greater than

Representation	Fluency	Probing Questions
Measuring Length • Exploring the use of rulers, measuring tapes, tape measures, metre sticks and trundle wheels for measuring lengths • Estimating a length and then checking by measuring • Deciding which is the best piece of equipment for measuring different lengths	1. Measure lengths • measure short length in centimetres using a ruler • measure a longer length in centimetres using a tape measure, measuring tape or metre rule • measure a longer length in metres using a metre rule, tape measure or trundle wheel • record the measurement with the units • know that there are 100cm in a 1 metre • produce a line or other object of a given length using a ruler or tape measure/metre rule.	Show me a piece of equipment you could use to measure: the width of this classroom, the height of a pencil; the length of the field. Show me a line that is 2cm longer than this one Convince me that there will always be more cm in a measurement than m Convince me that 130cm > 1m What's the same and what's different? ruler; tape measure; metre rule; trundle wheel

Measuring Mass • Exploring the use of balance scales, electronic scales, scales with a dial scale, hanging balances for measuring masses • Estimating a mass and then checking by measuring • Deciding which is the best piece of equipment for measuring different masses	2. Measure masses • measure small masses in grams using a set of scales (electronic and with scale) • measure larger masses in kilograms using a set of scales (electronic and with scale) • record the measurement with the units • know that there are 1000g in a 1 kilogram • gather objects together to produce a given mass using a set of scales	Show me an estimate for the weight of this pencil case Show me something that has a mass of approximately a kilogram Show me a unit you could use to measure the mass of this chair Convince me that mass cannot be measured in cm
Measuring Capacity • Exploring the use of measuring cylinders, measuring jugs, litre containers, larger containers and informal measures e.g. cups for measuring capacities • Estimating a capacity and then checking by measuring • Deciding which is the best piece of equipment for measuring different capacities	3. Measure capacities • measure small capacities in millilitres using a set of a measuring jug or measuring cylinder • measure larger capacities in litres using measuring jugs or other 1-litre containers • record the measurement with the units • know that there are 1000ml in a 1 litre • produce an amount of water with a given capacity e.g. 200ml	Show me how you could use this measuring jug to measure the capacity of this cup. Always, Sometimes, Never? You can measure the capacity of a bucket in cups.
Measuring Other Quantities • Exploring the use of thermometers for measuring temperature • Exploring the use of stopwatches for measuring time • Estimating a time and then checking by measuring	4. Measure other quantities • temperatures (in ^{o}C) • time (in seconds and minutes)	Convince me that you cannot use a thermometer to measure the height of a cup Always, Sometimes, Never? You can measure any time in seconds
Scales • Exploring the scales on apparatus already encountered e.g. kitchen scales, thermometers, rulers to see where different numbers sit • Marking a counting stick in 1s, 2s, 5s or 10s and finding where different measurements would go	5. Read and mark scales on equipment correctly • read a measurement from a scale marked in 1s with all numbers shown • mark a measurement on a scale marked in 1s with all numbers shown • read a measurement from a scale marked in 1s but with only periodic numbers shown e.g. every 5 • mark a measurement on a scale marked in 1s but with only periodic numbers shown e.g. every 5 • ext: read or mark on a measurement on a scale marked in 2s or 10s or 5s	Show me where the arrow would point on this scale to show 11. (going up in 1s, 2s, 10s etc) You cannot mark 7 on a scale that goes up in 2s.

Units • Selecting equipment from a table to measure a given measurement and reporting back to the group as to whether the equipment and unit was suitable.	6. Suggest a unit for a given measurement • lengths (widths, heights, lengths and distances) • masses • capacities • temperatures • times/durations • say why a unit of measurement may not be suitable (e.g. too small or too larger)	Show me a piece of equipment you could use to measure: the width of this classroom; the weight of your shoe; how long this lesson lasts; how much water a cup holds. Convince me that estimates can be good even if they aren't the same as the actual measurement What's the same and what's different? cm, m, g, kg, ml, l, C
Comparing and Ordering • Representing a measurement practically to help compare it e.g. 12cm or 15cm? 2 litres or 200 ml? • Positioning measurements on a scale to show which is greater and which is less • Arranging measures on cards or shown practically into order	7. Compare measurements using the signs =, < and > • given two measurements in the same units, say which is greater and show this using < or > • given two measurements in different units that do not overlap e.g. 12cm and 3m, say which is greater and show this using < or > • given a measurement made from a combination of smaller measures (e.g. a set of scales with several weights on it), say which total is greater and show this using < or > (or =) • read two measurements from scales and then compare them • give a measurement that is greater or less than a stated measurement • ext: given two measurements in different units that overlap e.g. 120cm and 1m, say which is greater and show this using < or >	Show me a measurement of length in centimetres that is > half a metre Show me a picture of a thermometer that shows the temperature to be < 15 degrees Celsius. What's the same and what's different? < > = Show me a line that is > 10cm long
	8. Order a set of measurements • order three measurements in the same units • order more than three measurements in the same units • order measurements in mixed units (no overlaps) • ext: order measurements in mixed units (with overlaps)	Always, Sometimes, Never? Measurements in metres are longer than measurements in centimetres. Always, Sometimes, Never? A measuring tool can measure more than one of the following: length, weight, temperature, time.

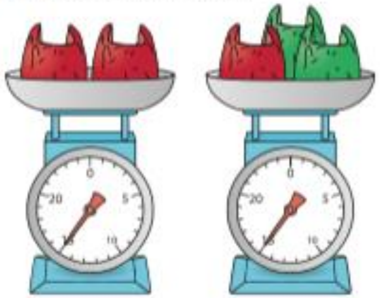
Further Extension

1.

What is the mass of two red bags?

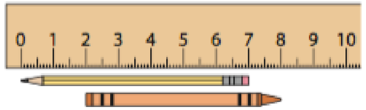
Which is heavier, the red bag or the green bag?

Explain your reasoning.



2.

How long is the crayon?



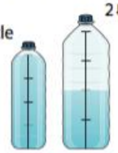
The crayon is ______ cm long.

How much longer is the crayon than the pencil?

3.

Here is a picture of a 1 litre bottle and a 2 litre bottle with some water in them.

What's the same? What's different?



Rich and Sophisticated Tasks

Choose and use appropriate standard units to estimate and measure length/height in any direction (m/cm); mass (kg/g); temperature (°C); capacity (litres/ml) to the nearest appropriate unit, using rulers, scales, thermometers and measuring vessels

NRICH: Discuss and Choose * G
NRICH: Little Man * P

Compare and order lengths, mass, volume/capacity and record the results using >, < and =

NRICH: Order, Order! * P I
NRICH: Compare the Cups * P

Misconceptions	Teacher Guidance and Notes
Children may muddle up what they are measuring and, for example, try to use a ruler to measure a capacity. They may also fail to see the true difference between measuring and estimating; you may also have a child who believes that their estimate was wrong because it was not exactly the same as the final measurement. Children may not always measure from zero e.g. they may align an object to the end of a ruler rather than the 0 mark when measuring length or fail to reset the scales when measuring mass. Some children believe measures using smaller units must be smaller in amount than any using a larger unit e.g. they do not believe that 120cm can be longer than 1m because centimetres are smaller than metres. Many children find reading scales (especially non-linear ones e.g. dials) very difficult. This is particularly exacerbated when the scale does not go up in single units. Children do not always read < and > as less than and greater than, instead they may see it as an arrow, for example.	• In Stage 2, the focus is on developing fluency and confidence with standard units for length, mass, capacity and temperature. • The expectations of the specific units of measurements are shown in the National Curriculum statements above - however, since there is a systematic approach to metric units using the prefixes system, you should be prepared for children to want to move outside of this at times. This is actually beneficial as it helps them to understand that there are a wide range of units to measure mass, say, but that only some of them are really suitable for finding the mass of a car, for example. • Try also to develop a sense of what the most common units look/feel like e.g. how long is a metre roughly? How does 1 kilogram feel? • Be mindful of spending sufficient time on capacity (and to some extent on mass) as this area is often overlooked due to the more demanding nature of the practical work. • Part of this unit involves children reading scales from different equipment – at this level this should be with scales that go up in single units but be prepared to extend to scales going up in 2s, 10s and 5s (or even 100s etc) for those working at greater depth • When making comparisons between amounts, you should use the inequality signs < and >. Encourage children to say less than and greater than as the mathematical translations and encourage them to use this language when they are discussing comparisons. Beware children making comparisons using different units and looking only at the numbers, rather than the units, e.g. saying 2m < 20cm because 2 < 20.

Key Assessment Checklist

1. I can choose and use appropriate standard units to measure length to the nearest unit (using rulers)
2. I can choose and use appropriate standard units to measure mass to the nearest unit (using scales)
3. I can choose, make estimates and use appropriate standard units to measure capacity to the nearest unit using measuring vessels
4. I can choose, make estimates and use appropriate standard units to measure temperature to the nearest unit (using thermometers)
5. I can read scales on measuring equipment correctly.
6. I make estimates using appropriate standard units to measure length, mass, capacity and temperature.
7. I can compare units and record comparisons using inequality symbols.
8. I can order units and record results