Moor Nook CP School

Year 5

Medium Term Plans

February 2021

Overview of Year

Autumn Term	**Number and Algebra**				**Geometry and Measures**	
	1. Investigating Number Systems	2. Pattern Sniffing	3. Solving Calculation Problems	4. Generalising Arithmetic	5. Exploring Shape	6. Reasoning with Measures

Spring Term	**Number and Algebra**			**Statistics**
	7. Discovering Equivalence	8. Reasoning with Fractions	9. Solving Number Problems	10. Investigating Statistics

Summer Term	**Geometry**	**Number and Algebra**		**Geometry and Measures**	
	11. Visualising Shape	12. Exploring Change	13. Proportional Reasoning	14. Describing Position	15. Measuring and Estimating

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Year 5 Overview:		
Unit	**Learning Hours**	**Summary of Key Content**
1. Investigating Number Systems	13	Read, write, compare and order numbers up to 1 000 000; read Roman numerals to 1000; read, write and interpret negative numbers. Round integers to powers of 10, round decimals to 2dp, order decimals to 3dp
2. Pattern Sniffing	11	Count forwards, backwards in steps of powers of 10; multiply and divide numbers mentally Recognise and use square and cube numbers Identify factors and multiples, know and use prime numbers
3. Solving Calculation Problems	12	Add and subtract numbers mentally; Formal addition and subtraction up to 4d; Solve addition and subtraction multi-step problems in context; Formal multiplication up to 4d x 1d. Use rounding to check answers
4. Generalising Arithmetic	10	Multiply and divide whole numbers and decimals by 10, 100, 1000 Formal division up to 4d /1d.
5. Exploring Shape	12	Estimate and compare acute, obtuse and reflex angles Use properties of rectangles to find missing lengths and angles; identify regular polygons
6. Reasoning with Measures	10	Perimeter of rectilinear shapes; area of rectangles; estimate area or irregular shapes; estimate volume.
7. Discovering Equivalence	14	Mixed number and improper fractions; compare and order fractions with multiple denominators; identify and name equivalent fractions; count in hundredths; write decimals as fractions; recognise and use thousandths; understand per cent and % sign; write percentages as fractions over 100; solve problems involving equivalence of simple FDP.
8. Reasoning with Fractions	8	Add and subtract fractions with same denominators or those that are multiples of each other Multiply proper fractions and mixed numbers by integers (supported diagrammatically)
9. Solving Number Problems	8	Recap multiplication and division; Solve problems involving any of the four operations, including problems of factors, multiples and squares and problems involving decimals up to 3dp.
10. Investigating Statistics	6	Line graphs – comparison, sum and difference problems; complete, read and interpret tables
11. Visualising Shape	8	Draw given angles, measure them in degrees; identify 3D shapes from 2D representations
12. Exploring Change	4	Solve problems converting between units of time
13. Proportional Reasoning	4	Recap mental calculations; revisit formal methods for multiplication and division; solve calculation problems for 4 operations.
14. Describing Position	5	Describe position of shape following reflection or translation
15. Measuring and Estimating	8	Solve problems involving four operations and measures; convert between metric units; understand approximate metric-imperial conversions

Year 5	Unit 1: Investigating Number Systems	
13 learning hours	This unit introduces the number systems and structures that we use at different levels of the curriculum. At KS1 children are working on the place value system of base 10 with the introduction of Roman Numerals as an example of an alternative system in KS2. Negative numbers and non-integers also come in at this stage and progress into KS3. At KS3 and KS4 we start to look at other ways of representing numbers, including standard form, inequality notation and so on.	
Prior Learning	**Core Learning**	**Learning Leads to....**
➢ read Roman numerals to 100 (I to C) and know that over time, the numeral system changed to include the concept of zero and place value	➢ **read Roman numerals to 1000 (M) and recognise years written in Roman numerals**	➢ read, write, order and compare numbers up to 10 000 000 and determine the value of each digit
➢ recognise the place value of each digit in a four-digit number (thousands, hundreds, tens, and ones) ➢ identify, represent and estimate numbers using different representations ➢ order and compare numbers beyond 1000	➢ **read, write, order and compare numbers to at least 1 000 000 and determine the value of each digit** ➢ **read, write and interpret negative numbers in context**	➢ use negative numbers in context, and calculate intervals across zero
➢ round any number to the nearest 10, 100 or 1000	➢ **round any number up to 1 000 000 to the nearest 10, 100, 1000, 10 000 and 100 000**	➢ round any whole number to a required degree of accuracy
➢ round decimals with one decimal place to the nearest whole number	➢ **round decimals with two decimal places to the nearest whole number and to one decimal place** ➢ **read, write, order and compare numbers with up to three decimal places**	➢ identify the value of each digit in numbers given to three decimal place (and multiply and divide numbers by 10, 100, 1000)
➢ solve number and practical problems that involve all of the above and with increasingly large positive numbers	➢ **solve number problems and practical problems that involve all of the above**	➢ solve number and practical problems that involve all of the above

Exemplification	Vocabulary
1. a) Write these Arabic numerals as Roman numerals: i) 150 ii) 674 b) Write these Roman numerals as Arabic numerals: i) MMXIV ii) CCIX 2 a) Write this number using numerals: Seven hundred and thirty thousand, six hundred and seventy-two b) Write this number in words: 316 097 3. a) 5°C warmer than -4°C is ………°C b) 8°C colder than 5°C is ………°C 4. Round 316 783 to a) the nearest 100 b) the nearest 10 000 5. Round a) 235.59 to the nearest whole number b) 3.54 to on decimal place 6. Order these numbers from smallest to largest: 0.45 0.405 0.504 0.5 0.54 7. Alesha thinks of a whole number. When rounded to the nearest 10 000, the number is 280 000. What is the smallest possible value of Alesha's number?	Hundreds Thousands Ten Thousands Millions tenths hundredths thousandths Place Value Order Compare Numerals Position Estimate Positive Negative Round Rounding Nearest Decimals Decimal place Integer Roman Numerals 500 D 1000 M

Representation	Fluency	Probing Questions
Roman Numerals • Exploring where Roman Numerals are used in real life, for example in dates and times (clocks) and, particularly in Stage 5, dates such as on gravestones, historical artefacts, TV credits etc. • Use (and make) equivalence cards for roman numeral symbols and either Arabic numerals or word versions or visual representations • Explore this online activity (interactive) to discover the rules of Roman numerals	1. Convert between Roman Numerals and Arabic Numerals up to 1000 ○ recap conversions of numerals up to 100 ○ use symbols for 100, 500 and 1000 ○ use combinations of C, D and M in order e.g. MMCCCX ○ use examples of C, D and M where subtraction is required e.g. MCMIX	What's the same and what's different? MMC, MCM, MMCM What's the same and what's different? Arabic Numerals and Roman Numerals?
	2. Work with years and dates written in Roman numerals ○ Convert a year in Arabic numerals to roman numerals ○ Convert a given year to Roman numerals	Convince me that MMXVI is 2016 in Roman Numerals Find two dates with a difference of XC years.

Representing Whole Numbers and Decimals • Building numbers from place value counters • Exploring the position of numbers on a scale (e.g. on geogebra) • Develop sense of size of large numbers up to 1000 000 using paper strips and paperclips to position e.g. strip represents 0-10 000, where is 2534? What if the strip now represents 0-5000? • Use (and make) word/numeral number cards to help convert between numerals and words • Use number cards to explore making different four (or five or six) digit numbers and finding the smallest/largest	3. Read and write numbers up to 1000 000 in numbers and words ○ recap four-digit numbers e.g. 4536 or 5067 or 7809 ○ five- digit numbers e.g. 45697 ○ five digit numbers that incorporate zeroes e.g. 54 008 or 60870 ○ six digit numbers e.g. 546 789 ○ six digit numbers that incorporate zeroes e.g. 670 080 ○ one million	Show me the number 3 million four hundred and fifty-seven thousand, six hundred and fifty-four in symbols Show me the number 2, 045, 678 in words Show me where 345, 678 would be on this number strip that goes from - 0-1000,000 - 300,000 - 400,000 - 345,000 - 350,000 - 345,000 - 346,000
Partitioning • Using overlapping partitioning cards to construct and deconstruct numbers • Building numbers from base 10 and then splitting equipment into groups to find different ways of partitioning	4. Partition a number into hundred thousands, ten thousands, thousands, hundreds, tens and ones and state the value of a given digit within a number ○ Recap four digit numbers ○ Five digit numbers ○ Six digit numbers ○ Reverse problem to find number from place value information ○ Partition in a non-standard way (i.e. not just HTh, TTh, Th, H, T, U)	Show me three different partitionings of 53 862 True or false? There are an infinite number of numbers with 7 ten thousands
Comparing • Comparing two numbers by constructing, partitioning and analysing place by place.	5. Compare two numbers to say which is greater, using > or < to notate ○ Recap three digit numbers ○ Two numbers of different lengths ○ Two four-digit numbers	Convince me that 46 782 < 47 892

	○ Two five digit-numbers ○ Two six-digit numbers ○ Examples in a mixture of formats (words, numerals, images)	
Ordering • Comparing pairs of numbers at a time to order sets of numbers • Using washing lines as large (unmarked) number lines – giving each child a number and positioning them to help order the set	6. Order numbers from smallest to largest ○ Order three numbers: ▪ Up to three digits ▪ Numbers of different lengths ▪ 4 digit numbers ▪ 5 digit numbers ▪ 6 digit numbers ○ Order four or more numbers (as above) ○ Find a number that lies between two given numbers	Convince me that these numbers are in ascending order: 14 567; 16 714; 56 147; 56 174; 57 000
Rounding • Positioning number on marked (and then unmarked) number line to identify neighbouring rounding options • Using number line to investigate when a number is closer to the lower end than the upper end • If finding the lower option is challenging, then represent a number using partitioned equipment e.g. place value counters or place value cards. Then partition the number and keep the pieces required for rounding to generate the lower rounding option. For example, to round 3467 to the nearest 100 make as 3000 + 400 + 60 + 7 and reject the 60 and the 7 to leave 3000 + 400 = 3400. This is the lower option. Then make the higher option by adding one more 100 i.e. 3500.	7. Round a whole number to the nearest given power of 10 ○ round a number to the nearest 10, 100 or 1000 (recap) ▪ 4 digits ▪ 5 digits ▪ 6 digit ▪ examples with ‘double rounding’ e.g. 485 970 to nearest 1000 ○ round a number to nearest 10 000 (as above) ○ round a number to nearest 100 000 (as above)	Show me a number that rounds to 546,000 when rounded to the nearest 1000 Show me a number that rounds to 567,800 when rounded to the nearest 100 Convince me that both 567,501 and 568499 round to 568000 to the nearest thousand

Negative Numbers • Building a negative number washing line • Using thermometers as a marked number lines to explore negative numbers • Using a counting stick to support counting into negatives • Using a blank number line with 0 marked to help make jumps of 100, 1000 etc into negatives. • Positioning negative numbers on strips of paper	8. Read, write and interpret negative numbers in context ○ mark a positive, zero or negative temperature on a marked scale ○ mark a positive, zero or negative temperature on a blank scale ○ state the highest and lowest temperatures from a set ○ count forwards or backwards involving negative numbers	What's the same and what's different? -5, -50, 50, 5 What's the same and what's different? -6, -5, -2, 4 Always, Sometimes, Never? –36 is greater than –34 Always, Sometimes, Never? There is only one pair of numbers with a sum of 3 and difference of 11
Representing Decimals • Building numbers from place value counters • Using overlapping partitioning cards to construct and deconstruct numbers • Representing decimals using tenth strips and hundredth squares to show why, for example, 32 hundredths is the same as 3 tenths and 2 hundredths. • Exploring the position of numbers on a scale (e.g. on geogebra) • Develop sense of small (decimal) numbers using paper strips and paperclips to position e.g. strip represents 0-1, where is 0.3? 0.03? 0.13? 0.003? What if strip is now 0-0.1?	9. Read and write numbers up to 3 decimal places – partition a number into tenths, hundredths and thousandths ○ state the value of a given digit in a decimal ○ partition a decimal into tenths, hundredths and thousandths ○ record a decimal given in words in numerals (partitioned into tenths, hundredths etc) ○ record a decimal given in words in numerals (combined place values e.g. sixty-four hundredths) ○ write a decimal given in numerals in words (no 0 digits)	Convince me that 0.35 is greater than 0.035 What's the same and what's different? 72.344 and 72.346
Comparing and Ordering – same length • Comparing two numbers by constructing, partitioning and analysing place by place.	10. Compare and order decimals of the same length to say which is greater, using > or < to notate ○ compare two decimals ▪ 1dp ▪ 2dp ▪ 3dp ○ order 3 or more decimals	Show me a number between 0.12 and 0.17. Which of the two numbers is it closer to? How do you know? Always, Sometimes, Never? 0 is greater than 9, so 0.10 is greater than 0.9

Comparing and Ordering • Comparing two numbers by constructing, partitioning and analysing place by place.	11. Compare and order decimals of different/mixed lengths ○ compare two decimals ○ compare and order three decimals ○ compare and order four or more decimals	Show me a possible value for ? in 5.4 < ? < 5.51 Show me how you order these numbers 7.765, 7.675, 6.765, 7.756, 6.776 Convince me that these numbers are in ascending order: 3.41, 3.419, 3.5, 3.507, 3.52
Rounding • Positioning number on marked (and then unmarked) number line to identify neighbouring rounding options • Using number line to investigate when a number is closer to the lower end than the upper end • If finding the lower option is challenging, then represent a number using partitioned equipment e.g. place value counters or place value cards. Then partition the number and keep the pieces required for rounding to generate the lower rounding option.	12. Round a decimal to the nearest whole number ○ one decimal place ▪ identify the two closest integers ▪ determine the closer of the two options ▪ by inspection of the first decimal place ○ two decimal places ▪ identify the two closest integers ▪ determine the closer of the two options ▪ by inspection of the first decimal place	What's the same and what's different? 5.67, 5.69, 5.73, 5.64 Always, Sometimes, Never? 3.5 is closer to 4 than it is to 3
	13. Round a decimal of 2dp to 1dp ○ identify the two closest numbers with one decimal place ○ determine the closer of the two ○ by inspection of the second decimal place	Show me a number that rounds to 2.6 when rounded to 1 decimal place Convince me why might it not be possible to identify the first three places in a long jump competition if measurements were taken in metres to one decimal place

Further Extension

1. Explore 1 million:
   - How large would a stadium need to be to hold one million people?
   - How much would a million grains of rice weigh?

In June 2014 the population of the UK was approximately 64 100 000.

What is the current approximate population of the UK?
Is this number larger or smaller than 64 100 000?
How accurate is this figure in terms of the number of people in the UK at this moment?

2. Using all of the digits from 0 to 9, write down a 10-digit number.

What is the largest number you can write?
What is the smallest number you can write?

Write down the number that is one less than the largest number.
Write down the number that is one more than the smallest number.

Captain Conjecture says, 'Using the digits 0 to 9 we can write any number, no matter how large or small.'

Do you agree?
Explain your reasoning.

3. The temperature at 6 a.m. was recorded each day for one week.

Day	Mon	Tues	Wed	Thurs	Fri	Sat	Sun
Temp (°C)	1	−1	0	3	2	−2	−3

What is the difference in temperature between the coldest day and the warmest day?
At what time of year do you think these temperatures were recorded?
Do you think it might have snowed during the week?

Rich and Sophisticated Tasks

Recognise the place value of each digit
NRICH: Some Games That May Be Nice or Nasty * G
NRICH: Dicey Operations * G
NRICH: The Deca Tree * P
NRICH: Four-digit Targets * P

Round any number to the nearest 10, 100 or 1000
NRICH: Reasoned Rounding * G
NRICH: Round the Four Dice

Decimals
NRICH: Greater Than or Less Than? * I
NRICH: Spiralling Decimals * G**

NRICH: Round the Dice Decimals 2 *

Negatives
NRICH: Tug Harder! * G
NRICH: Sea Level * P I

Misconceptions	Teacher Guidance and Notes
Children find it had to adapt to the code of roman numerals and they try to translate place value concepts directly. Pupils may mispronounce or misread or miswrite larger numbers involving ten thousands and millions. e.g. Three hundred and twenty-seven thousand and four hundred and fifty-six. Similarly they may do this with decimals, saying 'two point two hundred and forty-seven' rather than 'two point two four seven' for 2.247 Children struggle with the different concepts of the magnitude of a number and the sign of a number e.g. they think that e.g. -6 is greater than 3. It is important that they understand that 'greater' means 'higher up the number line' Children do not always fully understand the role of 0 as a place holder and hence struggle to read or write numbers like 20,045 Children 'miss out' ten thousands frequently, jumping straight from thousands to millions in terms of column headings for place value. When rounding, children sometimes want to round up in every case and they do not look carefully at the next number to decide whether to leave the stem alone or whether to round the final digit up. They also sometimes fail to check only the next digit, instead looking at every digit from the end of the number and rounding along in a 'chain reaction' Children confuse the meaning of < and >, finding it hard to tell which is which.	• Note that Roman Numerals are introduced via clocks in Stage 3 and then up to 100 in Stage 4. • When working with Roman Numerals it can be beneficial to ensure a whole school approach is adopted, ie on displays around clock faces. The history will need to be explored to unpick 'the rules'. Note that these are just conventions rather than things that are innate about maths so make this clear to children. • Children need to understand that we are not calculating with Roman Numerals but making connections to real life and how they are represented today. This is just one alternative number system but there are a multitude of others. Good SMSC opportunity. • Help on the rules of Roman Numerals can be found here (NCETM) • When teaching place value use practical resources to expand on different base representations to emphasise the unitised structure of number. This applies equally to large numbers and to decimals – try to sue the same approaches for both to show that these aren't different concepts, just extended ones! • It is important that children develop their number sense here- they should be able to place numbers on a blank number line including where the scale changes. Try taking a blank paper strip as a scale from 0-1000 and asking children to place 200 on it. Then change the scale to 1-500 and ask them to do the same. Similarly with decimals. • When working with negative numbers, try to avoid terminology of 'minus' and stick to 'negative' instead. Therefore, -3 is said as 'negative three' rather than 'minus three'. • Similarly, with decimals, it is important to read each digit separately e.g. 0.12 is said as "nought point one two" and not as "nought point twelve".

Key Assessment Checklist
1. I can read and write numbers beyond 100000
2. I can read Roman Numerals to 1000 and recognise years written in Roman Numerals.
3. I can position and estimate positive and negative numbers on a number line or other representation.
4. I can round any decimal with two decimal places to the nearest integer or 1 decimal place.
5. I can use and apply above number knowledge to solve number problems.
6. I can round any number to the nearest 10, 100, 1000, 10000 or 1000000
7. I can compare numbers with up to three decimal places, using the signs <, > (and =) to show this comparison.
8. I can order decimals with up to 3 decimal places

Year 5	Unit 2: Pattern Sniffing
11 learning hours	This unit explores pattern from the early stages of counting and then counting in 2s, 5s, and 10s up to the more formal study of sequences. This sequence work progresses through linear sequences up to quadratic, other polynomial and geometric for the most able older students. For children in KS1, this unit is heavily linked to the following one in terms of relating counting to reading and writing numbers. Also in this unit children and students begin to study the properties of numbers and to hone their conjecture and justification skills as they explore odd/even numbers, factors, multiples and primes before moving onto indices and their laws.

Prior Learning	Core Learning	Extension Learning
➢ count in multiples of 6, 7, 9, 25 and 1000 ➢ count backwards through 0 to include negative numbers	➢ **count forwards or backwards in steps of powers of 10 for any given number up to 1 000 000**	➢ generate and describe linear number sequences
➢ recognise and use factor pairs and commutativity in mental calculations	➢ **identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers** ➢ **know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers** ➢ **establish whether a number up to 100 is prime and recall prime numbers up to 19** ➢ **recognise and use square numbers and cube numbers, and the notation for squared (^{2}) and cubed (^{3})**	➢ identify common factors, common multiples and prime numbers
➢ recall multiplication and division facts for multiplication tables up to 12 × 12	➢ **multiply and divide numbers mentally drawing upon known facts**	

Exemplification	Vocabulary
1. Find the next two numbers in each pattern: a) 430, 420, 410, ... b) 53 000, 52 000, 51 000, ... 2. a) Find all the factors of 42 b) Give a number that is a common multiple of 6 and 8 c) Find three common factors of 18 and 30 3. True or False: 15 is composite number. Explain your answer 4. Show that 31 is a prime number. 5. Calculate a) 5^3 b) 7 squared 6. Complete one missing numbers in each number sentence: a) 90 x 80 = ... b) 6 x .... = 4800 c) 360 ÷ .... = 40 d) ...... ÷ 80 = 12	prime number composite number factor factor pair multiple common factor common multiple square number cube number squared, ^{2} cubed, ^{3}

Representation	Fluency	Probing Questions
Counting in 10s and 100s: • Representing a start number with place value equipment and modelling counting on 10/100 (or counting back) by addition (or subtraction). • Use a counting stick to represent the start number (either end for counting on or counting back).	1. Count in 10s and 100s • forwards from a multiple of 10 (100) • backwards from a multiple of 10 (100) • forwards from any 2/3 digit number • backwards from any 2/3 digit number • forwards or backwards from a 4/5/6 digit number • backwards into negative numbers	Show me 10 more than 78 654 Show me 100 more than 613 451, Show me 10 less than 67 543 Show me 100 less than 784 299 Convince me that $4090 + 10 \neq 5000$
Counting in 1000s, 10 000s and 100 000s: • Representing a start number with place value equipment and modelling counting on 1000/10000/100000 (or counting back) by addition (or subtraction). • Use a counting stick to represent the start number (either end for counting on or counting back).	2. Count in 1000s (and 10 000s, 100 000s) • forwards from a multiple of 1000 (10 000, 100 000) • backwards from a multiple of 100 (10 000, 100 000) • forwards from any 4 digit number • backwards from any 4 digit number • forwards or backwards from a 5/6 digit number • backwards into negative numbers	Show me 1000 more than 786 513
Factors • Building arrays to show all factor pairs e.g. take 24 counters and arrange as various arrays to show all the different factor pairs (what happens when the number is a square number?) (what happens when the number is a prime number?) (why can you stop trying to find arrays when you pass the half-way point or, better still, the square root?)	3. Define and find factors of a number • by dividing systematically by 1, 2, 3, 4, etc. • by using factor pairs to find all the factors more efficiently • find all factors • examples with only 2 factors (prime)	Show me a factor of 60, xxx Convince me that 8 is a factor of 56 Always, Sometimes, Never? A number has an even number of factors
Common Factors • Using arrays to show common factors (what happens when the numbers are co-prime?) • Practical/Visual Venn diagrams to represent common factors	4. Identify common factors of two numbers • by listing • by using a Venn diagram • where one number is a factor of the other	Show me a common factor of 24 and 40 True or False? Any pair of numbers has a common factor

	• where the only common factor is 1 (defined as coprime)	
Multiples • Use arrays to build representations of multiples of a number (by adding an extra row each time). • Practical/Visual Venn diagrams to represent common factors and multiples	5. Define and find multiples of a number • by listing the 'times table' of the number • by multiplying systematically by 2, 3, 4, etc. • for large numbers (beyond times table)	Show me a multiple of 5, 6, 77 Convince me that 90 is a multiple of 3 What's the same and what's different? 2, 5, 10, 20 What's the same and what's different? factor; multiple Always, Sometimes, Never? A number has an even number of multiples
Common Multiples • Practical/Visual Venn diagrams to represent common multiples	6. Identify common multiples of two numbers • by listing • by using a Venn diagram • where one of the numbers is a multiple of the other • by using the product of the original numbers	Show me a common multiple of 7 and 2. True or False? Multiples of 12 are always multiples of 6
Primes • Practical Sieve of Eratosthenes to remove multiples and leave only prime numbers i.e. which numbers are in no times table except their own?	7. Test to identify prime and composite numbers • define prime numbers and composite numbers and test up to 20 • recall prime numbers to 20 • test numbers beyond 20 (up to 100) by dividing systematically • test more efficiently 20-100 by realising that it is only necessary to test up to the half way point	Show me a prime number < 19 Show me a composite number Convince me that 2 is a prime number Convince me that 1 is not a prime number What's the same and what's different? 1, 3, 7, 11 What's the same and what's different? prime number; composite number Always, Sometimes, Never? Pick a number, multiply by 6, add 1. The

		answer is a prime number. Always, Sometimes, Never? Prime numbers are odd Always, Sometimes, Never? Prime numbers can be a multiple of 4
Square Numbers • Representing square numbers as square arrays (with cubes or as drawn objects). E.g. take 20 cubes – can you arrange these into a square with no gaps? What about 16 cubes? Which is a square number? Why?	8. Find the square of a number • numbers 1 – 10 • numbers beyond 10 • using squared notation e.g. 4^2	Show me a square number Convince me that 225 is a square number. Convince me that 10 is not a square number.
Cube Numbers • Representing cube numbers as cubes (with objects). Which numbers of cubes can you arrange into a cube with no gaps?	9. Find the cube of a number • numbers 1 – 10 • numbers beyond 10 • using cubed notation e.g. 4^3 • compare squared or cubed numbers e.g. 6^2 versus 3^3	Show me a cube number Show me a square number that is a cube number. Always, Sometimes, Never? A number squared is less than the same number cubed.
Times Tables – related facts (multiplication) • Represent a times table multiplication calculation in multiple ways: e.g. 6 x 9 as 9 groups of 6 objects worth one (e.g. place value counters). Then change the representation so it shows 60 x 9 (i.e. exchange the place value counters for '10s'). Continue for 600 x 9 etc.	10. Use times tables to find related multiplication facts • multiple of 10 x number 1-12 e.g. 60 x 3 • multiple of 100 x number 1-12 e.g. 600 x 3 • multiple of 1000 x number 1-12 e.g. 6000 x 3 • multiple of 10 x multiple of 10 e.g. 60 x 30 • multiple of 100 x multiple of 10 e.g. 600 x 30 • multiple of 1000 x multiple of 10 e.g. 6000 x 30 • multiple of 10/100/1000 x multiple of 10/100/100 e.g. 6000 x 3000	What's the same and what's different? 30 x 60; 300 x 6; 300 x 60; 3 x 600; 3 x 6

Times Tables – related facts (division)

- Represent a times table division calculation with unknown answer: e.g. 24÷ 6 = ... as 24 objects grouped into an array (columns of 6).
  Then consider change the representation so it shows 240 ÷ 6 (i.e. exchange the ones for tens). Continue to 2400 ÷ 6 etc.

11. Use times tables to find related division facts (answer a whole number)
    - multiple of 10 ÷ number 1-12 e.g. 240 ÷ 8
    - multiple of 100 ÷ number 1-12 e.g. 2400 ÷ 8
    - multiple of 1000 ÷ number 1-12 e.g. 24 000 ÷ 8
    - multiple of 10 ÷ multiple of 10 e.g. 240 ÷ 80
    - multiple of 100/1000 ÷ multiple of 10 e.g. 24 000 ÷ 80
    - multiple of 100/1000 ÷ multiple of 100/1000 e.g. 24 000 ÷ 8000
    - multiple of 10 000, 100 000, 1000 000 ÷ multiple of 10/100/1000 etc. e.g. 240 000 ÷ 800

What's the same and what's different?

$3600 \div 9$,
$360 \div 9$,
$36 \div 9$,
$360 \div 90$,
$36\,000 \div 9\,000$

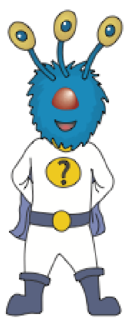
Further Extension

1. Gemma is counting in 100s from 567. Which of these numbers will she say?
1067   6700   5670   7067   5567

2.

Captain Conjecture says, 'Factors come in pairs so all numbers have an even number of factors.'

Do you agree?
Explain your reasoning.



3.
Given that 13 x 5 = 65, how many other facts do you automatically know? How can you organize these systematically?

Rich and Sophisticated Tasks

Recognise and use square numbers and cube numbers, and the notation for squared (²) and cubed (³)
NRICH: Up and Down Staircases *** P**
NRICH: One Wasn't Square ***P**
NRICH: Cycling Squares **** P**
NRICH: Picture a Pyramid … **** P**

Identify multiples and factors, including all factor pairs of a number, and common factors of two numbers
NRICH: Sweets in a Box *** P I**
NRICH: Which Is Quicker? *** P**
NRICH: Multiplication Squares *** P I**
NRICH: Flashing Lights *** P**
NRICH: Abundant Numbers *** I**
NRICH: Factor Track **** G P**
NRICH: Factors and Multiples Game *** G**
NRICH: Pebbles **** I**

4.

8	is a multiple of	4	and a factor of	16
6	is a multiple of	3	and a factor of	
	is a multiple of	5	and a factor of	
	is a multiple of		and a factor of	

Misconceptions	Teacher Guidance and Notes
When counting in powers of 10, pupils struggle when bridging 10, 100 etc e.g. they think that 997 + 100 = 1197 and forget about 1097. Children forget that 1 is a factor of any number and that the number itself is both a factor and a multiple of itself. Children also interchange the meanings of factor and multiple frequently. Pupils think 1 is a prime number. Pupils think 2 is not a prime number Pupils forget to include 0 when counting – they may also struggle to understand its role as neither a positive nor a negative number. When counting in multiples, many children believe that you stop after the 10^{th} or 12^{th} multiple (due to times table practice) – they do not see that multiples are infinite. Some children double a number instead of squaring when they see the notation 7^2, for example.	• This stage focuses on securing counting and times table to the level of finding related facts. Note that the number zero is neither positive nor negative • Counting on should be done from any start number, not just a multiple of the given step. • Encourage children to be systematic when, for example, finding factors or testing a number to see if it is prime. • Develop children's ability to find other facts from a given statement. E.g. If I know that....4 is a factor of 16..... I also know that.... • Work on times table recall over the course of the year where necessary - ensure times table recall is developing to help children 'spot' factors of numbers more easily. • Note that 'Squared' and 'cubed' are special cases of powers. The language 'to the power of' can be used to help get children ready for the next stages of this concept. • Ensure you define a prime number as one with exactly two factors to avoid misconceptions arising from alternatives such as 'can only be divided by one and itself'.

Key Assessment Checklist

1. I can count forwards or backwards in steps of powers of 10 for any given number up to 1 000 000
2. I can count forwards and backwards with positive and negative whole numbers, including through zero
3. I can recognise, calculate and use square numbers and the notation for squared (^{2})
4. I can recognise, calculate and use cube numbers and the notation for cubed (^{3})
5. I can multiply and divide numbers, mentally drawing upon known facts
6. I can identify multiples and factors of numbers
7. I can find all factor pairs of a number and common factors of two numbers
8. I can use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers; I can establish whether number up to 100 is prime and recall prime numbers up to 19

Year 5	Unit 3: Solving Calculation Problems	
12 learning hours	This unit explores the concepts of addition and subtraction at KS1 building to wider arithmetic skills including multiplication at late-KS2. It is strongly recommended that teachers plan this unit for KS1/KS2 with direct reference to the calculation policy! At KS3 students are developing calculation into its more general sense to explore order of operations, exact calculation with surds and standard form (which have been introduced in Inv Number Systems briefly) as well developing their skills in generalising calculation to algebraic formulae. They need to substitute into these formulae and calculate in the correct order to master this strand. The formulae referenced are examples of the types of formula they will need to use, but the conceptual understanding for these formulae will be taught elsewhere in the curriculum.	
Prior Learning	**Core Learning**	**Learning Leads to....**
• add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate	• **add and subtract numbers mentally with increasingly large numbers** • **add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)**	• perform mental calculations, including with mixed operations and large numbers (addition, subtraction and multiplication)
• solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why	• **solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why**	• solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why
• multiply two-digit and three-digit numbers by a one-digit number using formal written layout	• **multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers**	• multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication • multiply one-digit numbers with up to two decimal places by whole numbers
• estimate and use inverse operations to check answers to a calculation	• **use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy**	• use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy

Exemplification	Vocabulary	
1. Calculate mentally a) 4560 + 245 b) 3 000 - 563 2. Calculate a) 36 456 + 29 187 b) 67 264 – 23 509 3. There are 48 087 sports fans in a football stadium. 36 852 of them are home supporters, 7816 are away supporters. How many of them are neutral i.e. neither home nor away supporters?	add and more make sum total altogether score	how many more? take (away) leave how many left? less fewer difference between equals

a) Amy draws a diagram to help answer the problem. Which is the correct diagram?

48 087		
36 852	Neutral Supporters	7 816

Neutral Supporters		
36 852	7 816	48 087

36 852		
7 816	48 087	Neutral Supporters

b) Solve the problem

4. Calculate using a formal method
a) 3758 x 8 b) 452 x 56

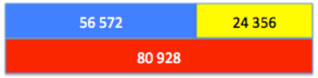
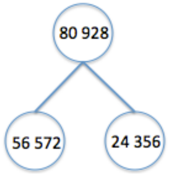
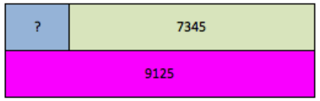
5. Lianne estimates the answer to 63 682 – 19 215 as 45 000
Do you agree with Lianne? Explain your answer

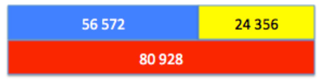
double
one more
two (ten) more
plus
equals
hundred
ten
one
exchange
column digit
columnar
column addition

is the same as
minus
number sentence
order
calculate
column subtraction
estimate
inverse
operation
check
multiply
product
array
grid
long multiplication
expanded method
partition

Representation	Fluency	Probing Questions
Addition - Mental • Representing addition as counting or jumping on (augmentation) using a number line (jumping in 10 000s, 1000s, 100s, 10s and 1s) • Using partitioning jottings	1. Add a five-digit number and ones/tens/hundreds/thousands/ten thousands mentally (up to 100 000) • five-digit number + 10 000 • five-digit number + multiple of 10 000 • five-digit number + multiple of 1000 • five-digit number + multiple of 100 • five-digit number + multiple of 10 • five-digit number + single digit • combinations of the above	Convince me that if I add a multiple of 10 000 to this number, the thousands, hundreds, tens and ones digits will stay the same. Always, Sometimes, Never? Addition makes a number larger
Addition - Written • Representing numbers using place value counters or equivalent [ten	2. Add a five-digit number and a five-digit number • No exchange required e.g. 72 452 + 15 237 • Exchange required from ones to tens e.g. 72 452 + 15	Show me how you could check whether 46 281 + 25 376 = 71 657 using another calculation

thousands, thousands, hundreds, tens and ones] then combining and finding the total value (aggregation) (exchanging ten 1s for one 10 or ten 10s for one 100 or ten 100s for one 1000 or ten 1000s for one 10 000 as required when bridging)	239 • Exchange required from tens to hundreds e.g. 72 452 + 15 287 • Exchange required from hundreds to thousands e.g. 72 452 + 15 3717 • Exchange required from thousands to ten thousands e.g. 72 452 + 19 316 • Multiple exchanges required from both ones to tens and from tens to hundred e.g, 72452 + 19 769 • Examples of adding a four-digit (or fewer) to a 5-digit number	Always, Sometimes, Never? A five digit number add a five digit number gives a ten digit number Always, Sometimes, Never? The sum of three odd numbers is even.
Subtraction – Mental • Representing subtraction as counting or jumping back (reduction) using a number line (jumping in 10 000s, 1000s, 100s, 10s and 1s) • Representing subtraction as a comparative difference between two points on a number line • Using partitioning jottings	3. Subtract ones/tens/hundreds/thousands/ten thousands from a five-digit number mentally • five-digit number - 10 000 • five-digit number - multiple of 10 000 • five-digit number - multiple of 1000 • five-digit number - multiple of 100 • five-digit number - multiple of 10 • five-digit number - single digit • combinations of the above	Always, Sometimes, Never? Subtraction makes a number smaller Always, Sometimes, Never? The difference of two odd numbers is even.
Subtraction - Written • Representing first number using place value counters [ten thousands, thousands, hundreds, tens and ones] then removing or taking away the second number and finding the resulting value (partitioning) (exchanging one 10 for ten 1s or one 100 for ten 10s or one 1000 for ten 100s or one 10 000 for ten 1000s as required when bridging)	4. Subtract a five-digit number from a five-digit number • No exchange required e.g. 65 675 – 23 254 • Exchange required from tens to ones e.g. 65 675 – 23 359 • Exchange required from hundreds to tens e.g. 65 675 – 23 281 • Exchange required from thousands to hundreds e.g. 65 675 – 23 812 • Exchange required from ten thousands to thousands e.g. 65 675 – 28 263 • Multiple exchanges required e.g. 65 675 – 28 886 • Examples of subtracting a four-digit (or fewer) from a 5-digit number	What's the same and what's different? 72285 + 23126; 23126 + 72285; 75411 – 72285; 75411 – 23126 72285 – 23126 72285 + 75411 23126 + 75411 Always, Sometimes, Never? A five digit number subtract a five digit number gives a four digit number
Word Problems • Representing problems using: ○ the bar model	5. Interpret a word problem correctly as an addition or subtraction calculation and solve • represent and solve an addition word problem using a bar model	What's the same and what's different? addition; subtraction Show me as many words as you can that

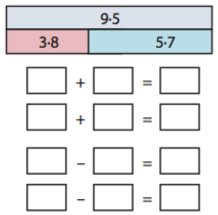
 ○ a part-part-whole model 	• represent and solve a subtraction word problem using a bar model • represent and solve an addition/subtraction word problem using a part-part-whole model • represent and solve an addition/subtraction word problem using a number line • represent and solve a two-step addition and/or subtraction word problem • examples of the above with decimals	signify 'add' in a problem What about 'subtract'?
Missing Number Problems • Using a bar model or part-part-whole model to represent the calculation to decide whether to add or subtract e.g. ? + 7345 = 9125 	6. Solve missing number problems involving addition or subtraction • a+ b = ? • a + ? = b • ? + a = b • a – b = ? • ? – a = b • a - ? = b	What's the same and what's different? • ? – a = b • a - ? = b
Multiplication by a Single Digit • Building and drawing (and even simply describing) arrays to represent multiplication (for a x b build an array of a objects across, copied into b rows) using counters and place value counters for bigger numbers. • Then using place value counters to represent large numbers in arrays e.g. 234 x 5 as 2 hundreds, 3 tens and 4 ones repeated over 5 rows. • Generalising to grid method as an 'undrawn' array. • Making the link between the grid method and an expanded column method and then condensed formal column method	7. Multiply a three-digit number by a single digit using a formal method • no exchange e.g. 231 x 3 • exchange only from ones to tens e.g. 416 x 2 • exchange once only anywhere e.g. 172 x 4 • two exchanges e.g. 347 x 3 • exchange at the end e.g. 243 x 7	What's the same and what's different? 243 x 7 and 247 x 3 Always, Sometimes, Never? A 3-digit number multiplied by single digit equals another 3-digit number.
	8. Multiply a four-digit number by a single digit using a formal method • no exchange e.g. 2131 x 3 • exchange only from ones to tens e.g. 3216 x 2 • exchange once only anywhere e.g. 1272 x 4 • two exchanges e.g. 2317 x 3 • exchange anywhere e.g. 6243 x 7	Convince me that 4157 x 3 cannot equal 12349 Always, Sometimes, Never? A 4-digit number multiplied by a single digit gives a 5-digit number
Multiplying by a 2-digit number • using a grid method to represent the calculation	9. Multiply a three-digit number by a two-digit number using a formal method (recap) • three-digit number multiplied by a multiple of 10 e.g. 286	What's the same and what's different? 453 x 28; 453 x 20 + 453 x 8;

• Making the link between the grid method and an expanded column method and then condensed formal column method • See calculation policy for more detail	x 40 • three-digit number multiplied by a 2-digit number e.g. 286 x 42	400 x 28 + 50 x 28 + 3 x 28 Always, Sometimes, Never? Long multiplication is needed to multiply three digit numbers by two digit numbers
	10. Multiply up to a four-digit number by a two-digit number using a formal method • four-digit number multiplied by a multiple of 10 e.g. 4186 x 30 • four-digit number multiplied by a 2-digit number e.g. 4186 x 34	Convince me that 715 x 79 cannot equal 42075 If I know that 32 x 36 = 1152, I also know that ...... Always, Sometimes, Never? A four digit number multiplied by a two-digit number equals an eight digit number
Estimation • Use place value counters or other place value equipment to represent a number and then round it to the nearest 1000, 100 (or even 10) to allow easy mental addition or subtraction.	11. Estimate the answer to an addition, subtraction or multiplication calculation • addition - numbers close to multiples of 1000, 100, 10 etc. e.g. 5962 + 2135 • subtraction - numbers close to multiples of 1000, 100, 10 etc. e.g. 5962 – 2135 • multiplication- numbers close to multiples of 1000, 100, 10, etc e.g. 5043 x 39.8	Convince me that 78 115 – 20981 is approximately 57 000 Show me how you could estimate the result of 5043 x 39.8
Checking • Use the bar model to represent a problem to explore inverse calculations	12. Find the inverse calculation to an addition or subtraction and use it to check an answer • give fact family for any given addition or subtraction calculation • find inverse (addition) - state checking calculation, estimate, calculate exactly • find inverse (subtraction)- state checking calculation, estimate, calculate exactly • find related calculations using place value knowledge e.g. if 14 + 23 = 37, then 1.4 + 2.3 = 3.7	Show me the four number facts that this bar model shows  Show me the other calculations that you know the answer to if I tell you that 32 348 + 45 417 = 77 765

Further Extension

1.

Write four number facts that this bar diagram shows.



2. (Children should reason rather than calculate here!)

True or False?

- 3999 – 2999 = 4000 – 3000
- 3999 – 2999 = 3000 – 2000
- 2741 – 1263 = 2742 – 1264
- 2741 + 1263 = 2742 + 1264
- 2741 – 1263 = 2731 – 1253
- 2741 – 1263 = 2742 – 1252

Explain your reasoning.

Using this number statement, 5222 – 3111 = 5223 – 3112 write three more pairs of equivalent calculations.

3.

Captain Conjecture says, 'When working with whole numbers, if you add two 2-digit numbers together the answer cannot be a 4-digit number.'

Do you agree?

Explain your reasoning.

Rich and Sophisticated Tasks

Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why

NRICH: Twenty Divided Into Six ** P
NRICH: Reach 100 *** P
NRICH: Maze 100 ** P
NRICH: Six Ten Total ** P I
NRICH: Six Numbered Cubes ** P

Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers

NRICH: All the Digits ** P
NRICH: Trebling * P

4.

Sam and Tom have £67·80 between them.

If Sam has £6·20 more than Tom, how much does Tom have?

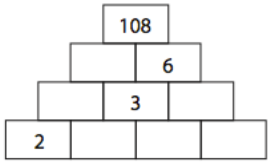
$$\begin{array}{r} 325 \\ +\,247 \\ \hline \\ \hline \end{array}$$

Sam has completed these calculations, but he is incorrect.
Explain the errors he has made.

$$\begin{array}{r} 325 \\ +\,247 \\ \hline 581 \\ \hline \end{array} \qquad \begin{array}{r} 355 \\ -\,247 \\ \hline 112 \\ \hline \end{array}$$

5.

Fill in the missing numbers in this multiplication pyramid.

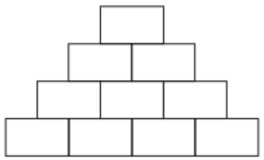


6.

Put the numbers 1, 2, 3 and 4 in the bottom row of this multiplication pyramid in any order you like.

What different numbers can you get on the top of the number pyramid? How can you make the largest number?

Explain your reasoning.



Misconceptions	Teacher Guidance and Notes
Children struggle to interpret whether to add or subtract from the language used. Children can find 'How many more/less?' particularly troublesome as it relates to ordinal values of numbers and relationships. Children struggle to add numbers when their place value understanding is weak. If they do not read a number like '4352' as 4 thousands, 3 hundreds, 5 tens and 2 ones then they struggle to combine the ones, tens, hundreds and thousands from two numbers appropriately. When performing columnar addition, children may forget to include the hundreds, tens or hundreds they have generated from earlier exchanges. They may also fail to exchange them at all and thus end with a two-digit numbers in the 1s column etc. When subtracting, children will sometimes subtract the larger number from the smaller initially. When performing columnar subtraction, children may exchange from the wrong column or fail to exchange altogether (instead just finding the difference between the digits in the column, even where the second one is greater than the first). Children may also fail to correctly record the exchange and thus not reduce the tens, for example, by one so that the answer is 10 too high. Children find calculations where multiple exchanges must be made particularly hard e.g. 4678 + 3945 because the notation becomes unwieldy. Similarly subtractions such as 2304 - 1789 cause issues because of the need to carry out a chain reaction of exchange. In these instances you may need to resort to equipment, even where the child does not need it for 'standard' calculations. Children often do not see difference as a representation of subtraction because take away is emphasised so much. They need to see subtraction represented in this way also to challenge this. Children can struggle to understand why they ‘add a zero’ when multiplying by the tens digit during column multiplication. They also make errors in adding up the results of grid methods. Times tables weaknesses will cause errors in calculations and should be addressed asap to minimise the impact. When doing long multiplication, children sometimes forget to multiply all the parts together - they struggle to record each separate multiplication within one line,	• This unit is focused on revising addition and subtraction of integers and decimals and the development of fluent formal multiplication of integers and decimals. (Division will be covered directly in Unit 5: Generalising Arithmetic). • The pitch of Stage 5 is in adding and subtracting large numbers up to five-digits and in multiplying integers of up to 4 digits by one or two digits. • Simultaneously they should be developing efficiency of mental methods when appropriate. Therefore, encourage children to look at the numbers in a calculation before commencing to decide if they can do it in their head, with jottings or whether they need to use a written method. • Stage 4 and below contain guidance and teaching prompts for the calculation work that precedes this. **Ref the Calculation Policy for Four Operations St1-6** • At this Stage, it is important to introduce a wide range of problems, contexts and situations involving addition and subtraction. The representations of the bar model are particularly crucial and the properties of inverses as applied to solve missing number problems should be directly addressed. • Try to model the wide range of language used to signify addition and subtraction – see vocabulary list above. The children ultimately need to be able to recognise that a problem is an addition problem from the language (and same for subtraction). • Use ‘sum’ only to mean an addition calculation – use the word ‘calculations’ to mean mixed operation computations • Ensure children are secure with column addition and subtraction before teaching long multiplication as this method depends on the ability to use these skills. • Consider teaching an expanded method first as a precursor to long multiplication to see how the different parts are put together in long multiplication. Initially try to minimise the need to exchange and carry numbers across. • Challenge issues with the use of the = sign by looking at examples where the question is on the right e.g. ? = 2514 + 7288 as well as balance problems in Further Extension e.g. 6143 + 2614 = ? + 3271 • Language is critical in this learning process - make sure you use and insist on the correct terminology for place value e.g. 4123+3456 would involve twenty add fifty, not two add five. Also insist on children describing their steps orally e.g. I need to add seven ones and 5 ones which makes twelve ones. So I will exchange 10 of these ones for a ten and regroup

particularly where there are lots of numbers carried across following exchanges. Children often want to write out an expanded multiplication (which is longer!) but don't realise that this isn't proper long multiplication, where the steps need to be compacted. Children forget to put in a place holder of 0 when multiplying by a tens digit. The equals sign is not always correctly interpreted as 'has the same value as' by children, who may see it as 'the answer is'. Some children may use the incorrect operation when checking and fail to realise that they need to use the inverse - this is more pronounced when subtracting. When completing missing number problems and using representations of a problem, children sometimes incorrectly arrange a number sentence e.g. if they are told that a + b = c they incorrectly say that a – b = c etc	(put the ten in the right column).

Key Assessment Checklist

1. I can add and subtract numbers involving three and four digits mentally
2. I can add numbers with up to five-digits using a columnar method
3. I can subtract numbers with up to five-digits using a columnar method
4. I can multiply a three-digit number by a one digit number using a formal method
5. I can multiply a four digit number by a one digit number using a formal method
6. I can multiply a three-digit or four-digit number by a two digit number using an informal method.
7. I can multiply a three-digit or four-digit number by a two digit number using long multiplication
8. I can use rounding to check answers to calculations and problems
9. I can solve multi step addition and subtraction problems choosing the correct operation and using the most appropriate method

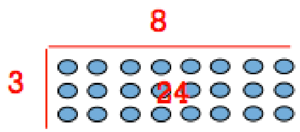
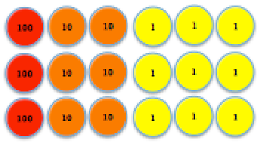
Year 5	Unit 4: Generalising Arithmetic
10 learning hours	This unit is focused on developing fluency in the manipulation of number. At primary level this is focused on arithmetic itself and the methods for four operations particularly; however, this is naturally generalised to thinking about rules of arithmetic more widely at secondary level i.e. algebra. These aspects have been paired together intentionally to help teachers describe algebra as simply a generalisation of number. It is expected that teachers will go back to arithmetic to help students see where the 'rules' of algebra come from. Note that the greyed out content is covered previously and hence is not required content here unless of concern.

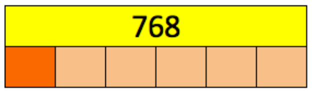
Prior Learning	Core Learning	Learning Leads to...
➢ find the effect of dividing a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as ones, tenths and hundredths	➢ **multiply and divide whole numbers and those involving decimals by 10, 100 and 1000** ➢ **divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context**	➢ multiply and divide numbers by 10, 100 and 1000 giving answers up to three decimal places ➢ divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context

Exemplification	Vocabulary	
1. Calculate a) 23×100 b) 7.6×1000 c) 0.3×10 d) $75 \div 100$ e) $4513 \div 1000$ f) $0.4 \div 10$ 2. a) Calculate i) $616 \div 7$ ii) $7165 \div 5$ b) A company is delivering shopping to people. Each delivery van can carry 8 shopping orders at a time. If there are 283 deliveries to make, how many vans are needed?	multiply decimal decimal place product multiplier place value column multiplication factor	divide grouping array quotient divisor dividend remainder compact short division

Representation	Fluency	Probing Questions
Multiplying Integers by 10, 100 and 1000 • using place value apparatus to build a number and replace each element by one that is ten times bigger [i.e. when multiplying by 10, replace 10s with 100s, 1s with 10s etc] • repeating for 100 times bigger • repeating for 1000 times bigger • using digit cards on a place value	1. Multiply whole numbers by 10, 100 and 1000 • numbers not ending in 0 multiplied by 10 • numbers ending in 0 multiplied by 10 • numbers not ending in 0 multiplied by 100 • numbers ending in 0 multiplied by 100 • numbers not ending in 0 multiplied by 1000 • numbers ending in 0 multiplied by 1000	Convince me that 734 x 100 = 73 400 Show me a number that can be multiplied by 1000 to give 3 million

grid/underlay to multiply by 10, 100 and 1000		
Multiplying Decimals by 10, 100 and 1000 • using place value apparatus to build a number and replace each element by one that is ten times bigger [i.e. when multiplying by 10, replace 0.01s with 0.1s, 0.1s with 1s etc] • repeating for 100 times bigger • repeating for 1000 times bigger • using digit cards on a place value grid/underlay to multiply numbers by 10, 100 and 1000	2. Multiply decimals by 10, 100 and 1000 • decimal with 1dp multiplied by 10 • decimal with 1dp multiplied by 100 • decimal with 1dp multiplied by 1000 • decimal with 2dp multiplied by 10 • decimal with 2dp multiplied by 100 • decimal with 2dp multiplied by 1000 • decimals with 3dp multiplied by 10/100/1000	Show me two numbers that are easy/hard to multiply by 1000 Always, Sometimes, Never? When you multiply a number by 100, you just add two zeroes on the end
Dividing Integers by 10, 100 and 1000 • using place value apparatus to build a number and replace each element by one that is ten times smaller [i.e. when dividing by 10, replace 1000s with 100s, 100s with 10s, 10s with 1s etc] • repeating for 100 times smaller • repeating for 1000 times smaller • using digit cards on a place value grid/underlay to divide by 10, 100 and 1000	3. Divide whole numbers 10, 100 and 1000 (whole number answer) • whole number ending in 0 divided by 10 • whole number ending in 00 divided by 100 • whole number ending in 000 divided by 1000 • whole number ending in more than this number of 0s divided by 10/100/1000	Show me two numbers that are easy/hard to divide by 1000 Convince me that 534 600 ÷ 10 = 53 460
Dividing Integers by 10, 100 and 1000 – decimal answer • using place value apparatus to build a number and replace each element by one that is ten times smaller [i.e. when dividing by 10, replace 1000s with 100s, 100s with 10s, 10s with 1s, 1s with 0.1s etc] – exploring what happens when the number contains ones and hence the answer is a decimal • repeating for 100 times smaller – exploring what happens when the number has ones or tens and hence the answer is a decimal • repeating for 1000 times smaller	4. Divide whole number by 10, 100 and 1000 (decimal answer) • whole number not ending in 0 divided by 10 • whole number not ending in 0 divided by 100 • whole number ending in exactly one 0 divided by 100 • whole number not ending in 0 divided by 1000 • whole number ending in exactly two 0s divided by 1000 • whole number ending in exactly one 0 divided by 1000	Always, Sometimes, Never? If you take a zero off the end of a number, you have divided it by 10. True or False? Any number can be divided by 100

• using digit cards on a place value grid/underlay to divide by 10, 100 and 1000		
Dividing Decimals by 10, 100 and 1000 • using place value apparatus to build a number and replace each element by one that is ten times smaller [i.e. when dividing by 10, replace 10s with 1s, replace 1s with 0.1s, replace 0.1s with 0.01s and so on] • repeating for 100 times smaller • repeating for 1000 times smaller • using digit cards on a place value grid/underlay to divide numbers by 10, 100 and 1000	5. Divide a decimal by 10, 100 and 1000 • decimal with 1dp divided by 10 • decimal with 1dp divided by 100 • ext: decimal with 1dp divided by 1000 • decimal with 2dp divided by 10 • ext: decimal with 2dp divided by 100/1000	Show me ... 45 x 100 ...4.5 x 100 ... 45 ÷ 10 What's the same and what's different? 1234 ÷ 10; 123.4 ÷ 10; 1234 x 10; 123.4 x 10
Dividing • For a calculation p ÷ q, grouping a set of p counters into groups of size q, arranging these groups as an array. For example, for 24 ÷ 3, count out 24 counters and arrange in columns of 3.... then read off the answer of 8 as the number of columns  • Building a number using place value counters and grouping them into groups that are the size of the divisor, before arranging these groups as an array to explore the partitioning approach. For example, 369 ÷ 3  • Repeating the above, but exchanging remaining counters for 10 counters of	6. Divide a 3-digit number by a 1-digit number using a written method • no exchange necessary e.g. 848 ÷ 4 • one exchange from hundreds to tens e.g. 805 ÷ 5 • situation where hundreds digit is less than divisor e.g. 355 ÷ 5 • one exchange from tens to ones e.g. 642 ÷ 3 • two exchanges e.g. 714 ÷ 6	Convince me that 8 is a factor of 912 True or False? Any number can be divided by 5
	7. Divide a 4-digit number by a 1-digit number using a written method (no remainder) • no exchange necessary e.g. 9366 ÷ 3 • first digit is lower than divisor requiring exchange e.g. 2196 ÷ 3 • single exchange e.g. 2376 ÷ 3 or 8476 ÷ 4 • two or more exchanges e.g. 4185 ÷ 5	Always, Sometimes, Never? Division is the inverse of multiplication Convince me that
	8. Divide a 3-digit or 4-digit number by a 1-digit number using a written method (with remainders) • no exchange necessary e.g. 9367 ÷ 3 • first digit is lower than divisor requiring exchange e.g. 2197 ÷ 3 • single exchange e.g. 2378 ÷ 3 or 8479 ÷ 4 • two or more exchanges e.g. 4189 ÷ 5	Show me how you divide 5683 ÷ 4 using place value counters? using a written method? Show me … a division with a remainder ... a division without a remainder

the next size down before continuing to group. For example, 372 ÷ 3  • Recording these methods to arrive at compact division		What's the same and what's different? $125 \div 5$, $98 \div 4$, $145 \div 9$, $126 \div 6$ Always, Sometimes, Never? A calculation involving division will have a remainder
Division Word Problems • Using the bar model to represent a word or other division problem. For example, 768 ÷ 6 	9. Recognise and solve a simple division problem, interpreting any remainders in the context as appropriate. • word problem – sharing language e.g. 282g flour to make 6 cupcakes. How much flour is in each cupcake? • word problem – grouping language e.g. 825 people enter a quiz. There are 5 people in each team. How many teams will there be? • remainder problems – an account has £342 in it. If you spend £6 per day, after how many days will the money run out? • finding unit fractions of an amount e.g. find a sixth of 564 • problems with links to factors (and multiples) e.g. show that 7 is a factor of 441	Convince me that I will need 8 coaches to take 375 children on a trip using coaches that seat 53 children each. Convince me that it will take me 34 days to read my book of 310 pages if I read 9 pages each day

Further Extension	Rich and Sophisticated Tasks
1. Fill in the missing numbers: $\square \div 120 = 117 \div 13 = 10800 \div \square = 234 \div \square$ 2. Sally's book is 92 pages long. If she reads seven pages each day, how long will she take to finish her book?	Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000 **NRICH: Multiply Multiples 1 * P** **NRICH: Multiply Multiples 2 * P** **NRICH: Multiply Multiples 3 * P** Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context **NRICH: Division Rules * P I**

3.
A 5p coin has a thickness of 1·7 mm. Ahmed makes a tower of 5p coins worth 50p.

Write down the calculation you would use to find the height of the tower.

Misconceptions	Teacher Guidance and Notes
Children sometimes add instead of multiplying e.g. they may add on 10 100 or 1000 when multiplying by 10 100 1000 Children struggle to take account of zeroes already held by numbers when multiplying by 10, 100, 1000. Contrastingly, some children simply add zeroes when multiplying by 10, 100 or 100, even when they are working with a decimal Children find division by 10, 100, 1000 challenging where there are insufficient zeroes to give a whole number answer – particularly when there are some zeroes (but not enough) Exchanging causes an issue for some children when using formal division methods – they may forget to carry over any remainder or forget what the remainder actually is. Some children struggle when the first digit of the dividend is less than the divisor because they don't see how to exchange it all (or carry the whole thing over to the next column). They may carry the divisor over, rather than the first digit of the dividend. In division, children get confused when there is a remainder within the calculation and may forget to use it or may put the remainder itself as the answer. Children do not always realise that in some problems, any remainder implies a whole extra unit e.g. how many cars seating 5 people are needed to transport 438 people?	• This unit is focused on formal division but also picks up on multiplication and division by powers of 10. • The pitch of the division is dividends of up to 4-digits and divisors that are single digits only • With the work on multiplying or dividing by 10, 100, 1000, there is no expectation of working with numbers with more than 3dp or of more than 5 digits. • Work on division necessitates strong times table knowledge, so address this once more if required. • Encourage children to use their estimate when calculating so that they can gain a sense of whether their answer is correct. • See the NCETM videos for more guidance on the representation methods shown above. • At all costs, avoid advising children to move digits or the decimal point when looking at multiplying and dividing by 10, 100, 1000 – instead refer back to the place value to establish how to carry out these calculations • Secure division without remainders first, then approach non-exact solutions. Encourage children to think of their remainder in the units of the context of the problem to understand it better.

Key Assessment Checklist
1. I can multiply whole numbers by 10, 100 and 1000
2. I can multiply decimals by 10, 100 and 1000
3. I can divide whole numbers by 10, 100 and 1000
4. I can divide decimals by 10, 100 and 1000
5. I can divide a 3-digit number by a 1-digit number using a written method
6. I can divide a 4-digit number by a 1-digit number using a written method
7. I can divide a 3 or 4-digit number by a 1-digit number when there is a remainder
8. I can solve simple division problems, interpreting any remainder in the context of the problem

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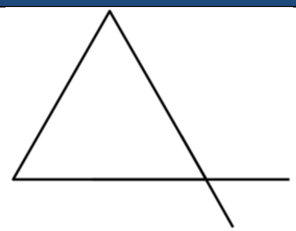
Year 5	Unit 5: Exploring Shape
12 learning hours	In this unit children and students explore the properties of shapes, both 2D and 3D. At KS1 this is focused on common shape names and basic features of vertices, sides etc. but this then develops to classifying quadrilaterals and triangles in KS2. Alongside this focus children begin to explore angle and turn in KS2 and develop this to more formal angle rules through Stages 5, 6, 7, 8. Older students begin to explore the field of trigonometry, encountering first Pythagoras' Theorem, then RA-triangle trig before finally looking a the sine rule and cosine rule.

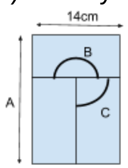
Prior Learning	Core Learning	Learning Leads to...
➢ identify acute and obtuse angles and compare and order angles up to two right angles by size ➢ identify lines of symmetry in 2-D shapes presented in different orientations ➢ compare and classify geometric shapes, including quadrilaterals and triangles, based on their properties and sizes	➢ **know angles are measured in degrees: estimate and compare acute, obtuse and reflex angles** ➢ **use the properties of rectangles to deduce related facts and find missing lengths and angles** ➢ **distinguish between regular and irregular polygons based on reasoning about equal sides and angles.**	➢ recognise angles where they meet at a point, are on a straight line, or are vertically opposite, and find missing angles ➢ compare and classify geometric shapes based on their properties and sizes … and find unknown angles in any triangles, quadrilaterals, and regular polygons ➢ illustrate and name parts of circles, including radius, diameter and circumference and know that the diameter is twice the radius

Exemplification

1. Look at this diagram
a) Label an acute angle, A. Estimate its size in degrees.
b) Label an obtuse angle, B. Estimate its size in degrees
c) Label a reflex angle, C. Estimate its size in degrees.

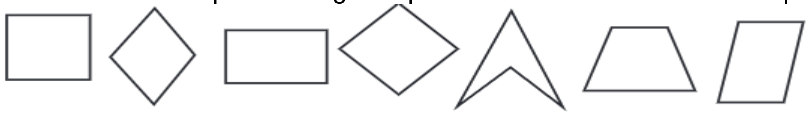


2. Look at this shape that is made from 3 identical rectangles.
a) Calculate the length of side A.
b) What are the sizes of the angles B and C?
c) Identify a reflex angle and mark it on the diagram as D.



Vocabulary

angle	opposite
acute	parallel
right (angle)	symmetry
obtuse	polygon
reflex	regular
degrees	irregular
compare	properties
order	criterion, criteria
estimate	Venn diagram
greater than	Carroll diagram
less than	justify
rectangle	explain
equal	

3. Two of these shapes are regular quadrilaterals – true or false? Explain your answer. 		

Representation	Fluency	Probing Questions
Right Angles • Using a right-angle finder (set square) to identify right angles in real life • Turning quarter turns and calling out the number of degrees e.g. 90 degrees, 180 degrees and so on	1. Recognise the properties of right angles and relate these to degrees • identify right angles in a shape • sketch an example of a right angle • know that a right angle is exactly 90 degrees • know that two right angles or half a turn are exactly 180 degrees • know that three right angles or three quarters of a turn are exactly 270 degrees • know that four right angles or a full turn are exactly 360 degrees • represent a multiple of 90 degrees visually as an angle • name shapes that contain right angles and contain only right angles • draw shapes that contain one or more right angles e.g. a right angled triangle, a quadrilateral containing two right angles, a pentagon using a either one or two right angles	Always, Sometimes, Never? Shapes with all angles as right angles are rectangles Convince me that a turn of three right angles clockwise is equivalent to a turn of 90 degrees anticlockwise.
Acute Angles • Using a right-angle finder to identify angles less than 90 degrees • Making acute angles using paper strips and a paper fastener	2. Recognise the properties of acute angles • identify acute angles in a set of irregular and regular shapes • draw an acute angle • draw a triangle with a single acute angle • draw a shape with 3 acute angles • define an acute angle as angle less than 90 degrees • recognise an acute angle by comparison to a right angle • say whether an angle given in degrees is acute or not	Show me three different acute angles Show me an angle that is greater than this one What’s the difference between an acute angle and a right angle?
Acute Angles – Assigning Degrees • Associating degrees to concrete/visual turns by forming a quarter circle of people with one person at the centre and creating an angle from the centre to the edge using string/metre rules etc. Keeping one ‘line’ fixed, rotate the other	3. Compare, order and begin to estimate acute angles • compare two acute angles (images) and say which is greater • order a set of three (or more) acute angles from smallest to largest • derive the value in degrees of the acute angle half way between zero and a right angle • estimate the size of acute angles very roughly using numbers	Show me an angle of approximately 20 degrees Always, Sometimes, Never? The angles in a triangle are acute.

around the circle, counting aloud in 5s, for example, so that 90 is chanted as a right angle is produced. This will help children associate a number with a particular size/feel of angle. • Using an angle measurer and tracing round the outside while counting in 5s or 10s aloud to replicate the above process at individual/pair level.	close to 0, 90 and 45. • combine ordering and estimating skills	
Obtuse Angles • Using a right-angle finder to identify angles more than 90 degrees (but less than a half turn) • Making obtuse angles using paper strips and a paper fastener	4. Recognise the properties of obtuse angles • recognise an obtuse angle by comparison to a right angle and a straight line (two right angles) • identify an obtuse angle in diagram or shape • sketch an example of an obtuse angle • draw a triangle with a single obtuse angle • name a regular polygon which uses obtuse angles • draw a shape using a single obtuse angle • draw a shape using 2 obtuse angles • define an obtuse angle as angle greater than 90 degrees but less than 180 degrees • say whether an angle given in degrees is obtuse or not	Show me three different obtuse angles Show me an angle that is greater than this one Always, Sometimes, Never? The angles in a triangle are obtuse.
Obtuse Angles – Assigning Degrees • Associating degrees to concrete/visual turns by forming a semicircle of people with one person at the centre and creating an angle from the centre to the edge using string/metre rules etc. Keeping one 'line' fixed, rotate the other around the circle, counting aloud in 10s so that 90 is chanted as a right angle is produced and so on round to 180 after a full turn. This will help children associate a number with a particular size/feel of angle. You can also point out where the change happens from acute to obtuse • Using an angle measurer and tracing round the outside while counting in 10s aloud to replicate the above process at individual/pair level.	5. Compare, order and begin to estimate obtuse angles • compare two obtuse angles (images) and say which is greater • order a set of three (or more) obtuse angles from smallest to largest • derive the value in degrees of the obtuse angle half way between a right angle and two right angles. • estimate the size of obtuse angles very roughly using numbers close to 90, 180 and 135. • combine ordering and estimating skills	Show me an angle of approximately 100 degrees Convince me that this angle is approximately 170 degrees What's the same and what's different? Acute angle; obtuse angle

Reflex Angles • Hunting for reflex angles in the real environment • Making reflex angles using paper strips and a paper fastener	6. Recognise the properties of reflex angles • recognise a reflex angle as being greater than two right angles • draw a reflex angle • given an angle image, mark the part that is reflex (and hence leave the part that is acute/obtuse) • identify a reflex angle in a diagram or shape • define a reflex angle as angle greater than 180 degrees but less than 360 degrees • given an angle in degrees, say whether it is reflex or not	Show me three different reflex angles Show me an angle that is greater than this one Always, Sometimes, Never? Reflex angles always have an acute angle on their 'other side'
Reflex Angles – Assigning Degrees • Associating degrees to concrete/visual turns by forming a circle of people with one person at the centre and creating an angle from the centre to the edge using string/metre rules etc. Keeping one ‘line’ fixed, rotate the other around the circle, counting aloud in 10s so that 90 is chanted as a right angle is produced and so on round to 360 after a full turn. This will help children associate a number with a particular size/feel of angle. • Using an angle measurer and tracing round the outside while counting in 10s aloud to replicate the above process at individual/pair level.	7. Compare, order and begin to estimate reflex angles • compare two reflex angles (images) and say which is greater • order a simple set of reflex angles from smallest to largest • estimate the size of reflex angles very roughly using numbers close to 180, 270 and 360 initially • derive the value in degrees of the reflex angle halfway between a straight line and three right angles (225 degrees) and the reflex angle halfway between three right angles and a full turn (315 degrees) • sketch a reflex angle of 225, 270 and 315 degrees • estimate the size of reflex angles very roughly using relative comparison to 180, 270 and 360 degrees	Show me an angle of approximately 200 degrees Always, Sometimes. Never? The angles in a quadrilateral are reflex.
Mixed Angles • Exploring the angle fact family practically (using arms!) to show different types of angle and act out members of the family  • Sorting angles on cards into categories and ordering these angles from smallest	8. Compare and order mixed angles • order a set of mixed angles in degrees from smallest to largest • order a set of mixed angle images from smallest to largest • recall the values in degrees of each quarter turn (right angle) and their midpoints • estimate the size of acute angles more accurately using relative comparison to 0, 45 and 90 degrees • sketch an acute angle, given its size in degrees by comparison to 0, 45 and 90 degrees • estimate the size of obtuse angles more accurately using relative comparison to 90, 135 and 180 degrees • sketch an obtuse angle, given its size in degrees by	What’s the same and what’s different? acute, obtuse, reflex, right What’s the same and what’s different? 190^o, 220^o, 270^o, 317^o

to largest – including right, straight and full turn as categories as these angles are neither acute, nor obtuse, nor reflex. • Discovering the angle from north of each point on a compass to help derive the midpoint values between each right angle (to support estimation) • Playing the angle estimation game from NRich	comparison to 90, 135 and 180 degrees • estimate the size of reflex angles more accurately using relative comparison to 180, 225, 270, 315 and 360 degrees • sketch a reflex angle, given its size in degrees by comparison to 180, 225, 270, 315, 360 degrees	
Properties of Rectangles • Using geogebra to try to construct a rectangle (and then a square and other quadrilaterals) that will still be a rectangle when one of the vertices is moved [i.e. defining a rectangle sufficiently] • Arranging copies of rectangles together to produce compound shapes and deriving information about their measurements.	9. Describe the properties of rectangles • complete a diagram of a rectangle when given a single vertical, horizontal or diagonal line • define a rectangle as a quadrilateral with four right angles • define a square as quadrilateral with four equal length sides and four right angles (or the regular quadrilateral) • explain why a parallelogram is not a rectangle/rhombus is not a square • given the dimensions of a rectangle, find other related lengths in compound shapes made from it • given the perimeter and one side of a rectangle, derive the other • given the perimeter of a square, derive its side length	Show me a rectangle that is half the size of this one If I know the length and width of a rectangle, I also know ….
Regular and Irregular shapes • Making a regular polygon using equal length sticks etc and then changing it to make it irregular. Exploring what the minimum change can be to make it irregular (and the knock on effects of this change). Can you make an irregular polygon that is still symmetrical? • Sorting regular and irregular polygons practically and using images with hoops and then Venn diagrams using hoops and venn diagrams • Sorting regular and irregular polygons practically and using images using Carroll Diagrams • Exploring which regular polygons will tessellate by themselves. Are there any shapes that tessellate together?	10. Recognise regular and irregular polygons • define a regular polygon • name the regular triangle and regular quadrilateral • recognise regular pentagons, hexagons, octagons • sketch regular polygons • sketch an irregular polygon given the number of sides/vertices • explain why a given polygon is not regular • sort shapes intro groups of regular and irregular polygons (as well as shapes that are not polygons)	Convince me that a house-shape pentagon [isosceles pentagon] is not a regular polygon What's the same and what's different? regular and irregular What's the same and what's different? circle, triangle, quadrilateral, pentagon Always, Sometimes, Never? Polygons are symmetrical Always, Sometimes, Never? There is a 2-sided polygon Always, Sometimes, Never? Shapes with all right angles are regular

Triangles • Making as many different types of triangle as possible on a geoboard (or dotty paper) • Playing 'Definition' where a pupil tries to define a shape and another/the teacher tries to draw one that meets the definition but is not the desired shape. For example, to define an isosceles triangle a pupil might say "it has three sides" so the other pupil/teacher might draw a scalene triangle and then refine this as the definition tightens	11. Begin to classify triangles • recognise whether a triangle is equilateral, isosceles, scalene • define these types of triangle mathematically • recognise whether a triangle is acute-angled or obtuse-angled or right-angled • recognise which type of triangles can occur together e.g. right-angled and isosceles • compare different triangles and their properties	What's the same and what's different? scalene, equilateral, right-angled, isosceles True or False? A triangle can be both isosceles and right-angled.
Quadrilaterals • Playing 'Definition' where a pupil tries to define a shape and another/the teacher tries to draw one that meets the definition but is not the desired shape. For example, to define a square a pupil might say "it has four sides" so the other pupil/teacher might draw a kite, leading the first pupil to refine their definition. • Folding along the diagonals of a square piece of paper. Unfolding and marking all the angles of the same size. Then folding along the lines of symmetry of a square piece of paper. Unfolding and marking all angles of the same size. (Repeat for a rectangle. Repeat for other quadrilaterals. Are there the same number of equal angles each time? Why/not? What if the starting shape was a regular pentagon? Hexagon?)	12. Begin to classify quadrilaterals • recognise whether a quadrilateral is a square, rectangle, rhombus or parallelogram • define these types of quadrilateral mathematically • recognise the types of angles in a quadrilateral, including whether any of them are equal • recognise that some shapes come from multiple categories e.g. a square is a rectangle, a rhombus, a parallelogram, a quadrilateral and a polygon! • compare different quadrilaterals and their properties	Show me which of these is a quadrilateral Convince me that a square is a rectangle What's the same and what's different? kite, parallelogram, rectangle, square Always, Sometimes, Never? Symmetrical shapes are regular Convince me that a rhombus is not a regular polygon

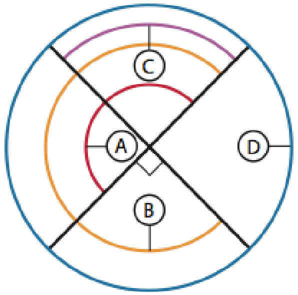
Further Extension

1.
The circle is divided into quarters by the two diameter lines and four angles A, B, C and D are marked.

Are the statements below true or false?

- Angle C is the smallest angle.
- Angle D is the largest angle.
- All the angles are the same size.
- Angle B is a right angle.
- Angle B is an obtuse angle.

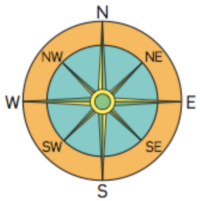
Explain your reasoning.



2.
In the questions, below all of Harry's movement is in a clockwise direction.

If Harry is facing North and turns through 180 degrees, in which direction will he be facing?
If Harry is facing South and turns through 180 degrees, in which direction will he be facing?
What do you notice?

If Harry is facing North and wants to face SW how many degrees must he turn?
From this position how many degrees must he travel through to face North again?

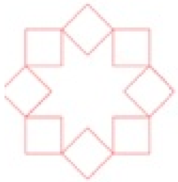


Rich and Sophisticated Tasks

Estimating angles
NRICH: Estimating Angles

Distinguish between regular and irregular polygons based on reasoning about equal sides and angles
NRICH: Egyptian Rope ** P I
NRICH: Bracelets * I

Polygons
Use Logo to draw the design shown below



Sorting Triangles
Draw an example into each position on this grid. Justify any gaps.

	equilateral	isosceles	scalene
acute-angled			
right-angled			
obtuse-angled			

<table>
<tr><td>3.
Which of these statements are correct?
▪ A square is a rectangle.
▪ A rectangle is a square.
▪ A rectangle is a parallelogram.
▪ A rhombus is a parallelogram.

Explain your reasoning.</td><td></td></tr>
<tr><th>Misconceptions</th><th>Teacher Guidance and Notes</th></tr>
<tr><td>The most significant misconception in this work lies around the misconstruing of angle as a measure of distance. For this reason, some children do not recognise equal angles drawn with different length line and similarly may say that an angle is bigger because it has longer lines

Children sometimes confuse reflex, obtuse and acute angles - they also forget about right angles being between acute and obtuse and that 180 degrees separates obtuse and reflex.

When estimating angle sizes, children find it hard to work with a scale centred around 90 and 360 - they cannot quickly find half of 90 or a quarter of it to use their sense of the size in a numeric way. Reflex angles are particularly challenging because they cover an area twice the size of acute and obtuse.

When solving problems using shape properties, children may forget that the symmetry of a shape tells them extra information not shown on the diagram e.g. if you know one side of a rectangle, then you immediately know a second and similarly with angles.

Children do not always understand that polygons are the shape family containing ALL closed shapes made of straight sides - they think that triangles and quadrilaterals are different and therefore that polygons 'begin' with pentagons. Some of this is connected to the language so it can be worth remarking that we can call a triangle a trigon - but we don't!

Children often interpret the meaning of 'irregular' to be 'completely irregular' i.e. that all the sides and angles are different. They don't see irregular as meaning 'just not regular' and so they do not believe that shapes with 5 equal sides and one different length side are irregular.</td><td>Stage 5 is the first time that children have encountered degrees as a unit of measure for angle. Up to Stage 4 children referenced angles only in general terms and in comparison to right angles/quarter turns.
Angle measure and construction in precise terms using a protractor forms part of Unit 11: Visualising Shape but there is a need to introduce the protractor and the idea of the scale of angle measure here to support estimation.
In Stage 4 children have classified angles as acute or obtuse, but this is the first time they will have come across reflex angles.
It is recommended that teachers work in reference to multiples of 90 degrees to help children reason the approximate size of an angle. There is no need to be very precise with estimation at this stage – instead focus on children identifying a reasonable number in terms of reference to 0, 90, 180, 270 and 360. It can be useful to explore the ‘half way’ points between right angles and their numerical values to provide an additional reference point – this can link nicely to compass directions such as NE.
Make reference to the historical and cultural reasons for the focus on 360 in angle work - originally there were thought to be 360 days in a year hence 360 was one full turn/rotation/cycle
Ensure that children gain exposure to angles in their own (isolated) right, but also to angles found in a shape to help build children's angle sense e.g. an equilateral triangle has 60 degree angles so make comparisons to this
When describing reflex angles particularly, make sure children are clear on which 'side' of the angle you are looking at. At this level it is good practice to always mark the angle under scrutiny rather than just referring to the vertex at which it is centred.
Use examples of solving problems using shape facts such as finding</td></tr>
</table>

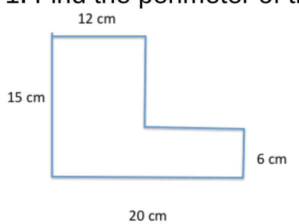
Because of the high language demands, pupils may forget shape names of polygons and how they link to number of sides. Children may label Venn or Carroll diagrams incorrectly (or not at all) and particularly may not allow for all possibilities to be shown in a Carroll diagram.	missing sides and angles using perimeter or symmetry. • In Stage 5 the reference in the national curriculum is to rectangles and regular polygons – however, to successfully bridge from Stage 4 to Stage 6 some elements of looking at the properties of other polygons, especially triangles and quadrilaterals is required, hence their inclusion here and in many other references sources e.g. NCETM • Ensure that children are clear that an irregular polygon is any polygon that does not have both all sides equal and all angles equal. Test this definition on shapes that children perceive as regular but are not e.g. rectangle, isosceles pentagon, rhombus, isosceles triangle. • When defining a shape, be aware that it is not necessary to state all of its properties as some are derived from others. E.g. we define a rhombus as a quadrilateral with four equal length sides but we do not need to specify that opposite angles are equal as this is a fact that occurs because of the situation with the sides. Similarly, a rectangle is defined as a quadrilateral with four right angles and its other properties such as two pairs of equal opposite sides come from this fact.

Key Assessment Checklist

1. I can recognise acute and obtuse angles
2. I can recognise a reflex angle
3. I know the numbers of degrees in a quarter turn, half turn, three quarter turn, full turn
4. I can define an acute, obtuse and reflex angle in terms of degrees
5. I can compare and order angles (acute, right, obtuse, reflex)
6. I can estimate angles in degrees (acute, obtuse)
7. I can estimate angles in degrees (reflex)
8. I can describe the properties of rectangles
9. I can use the properties of rectangles to find missing lengths and angles
10. I can identify regular and irregular polygons and explain my reasoning
11. I can sort regular and irregular polygons using Venn diagrams (3 criteria) and Carroll diagrams (2 criteria)
12. I can compare and begin to classify triangles and quadrilaterals

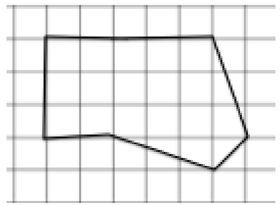
Year 5	Unit 6 : Reasoning with Measures
10 learning hours	This unit focuses on mensuration and particularly the concepts of perimeter, area and volume. Primary children are also working on money concepts at this stage, while older secondary students develop mensuration into volume and surface area of challenging shapes, applying Pythagoras' Theorem and trigonometry also in combination with these problems. Note the focus on reasoning within this unit: it is common for children to complete routine problems involving mensuration but this unit is about the developing a secure conceptual understanding of these ideas that they can apply to a wide range of problems and contexts. The opportunity to use and build on earlier number work is built into this unit and it is expected that children apply their arithmetic skills, for example, in these problems.

Prior Learning	Core Learning	Learning Leads to...
➢ measure and calculate the perimeter of a rectilinear figure (including squares) in centimetres and metres	➢ **measure and calculate the perimeter of composite rectilinear shapes in centimetres and metres**	➢ recognise that shapes with the same areas can have different perimeters and vice versa
➢ find the area of rectilinear shapes by counting squares	➢ **calculate and compare the area of rectangles (including squares), and including using standard units, square centimetres (cm^2) and square metres(m^2) and estimate the area of irregular shapes**	➢ calculate the area of parallelograms and triangles ➢ recognise when it is possible to use formulae for area of shapes
	➢ **estimate volume [for example, using 1 cm^3 blocks to build cuboids (including cubes)] and capacity [for example, using water]**	➢ calculate, estimate and compare volume of cubes and cuboids using standard units, including cubic centimetres (cm^3) and cubic metres (m^3), and extending to other units [for example, mm^3 and km^3] ➢ recognise when it is possible to use formulae for volume of shapes

Exemplification	Vocabulary
1. Find the perimeter of this shape:  	estimate measure calculate perimeter distance sum total

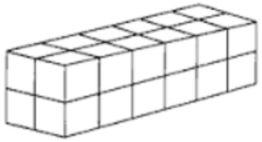
2. Put these shapes in order of area, from smallest to largest. You should explain your answer.

Square of side length 6 cm



(grid is centimetre grid)

3. Find the volume of this cuboid

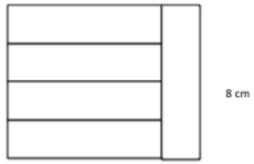
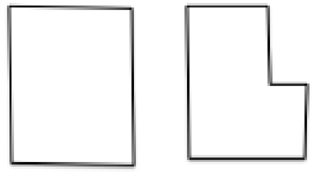
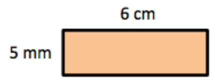


length
cm, m

area
squares
array
row
column
square units (cm2, m2 etc)

volume
capacity
cubic units (cm3, m3 etc)
layers
container
cube
cuboid
litre, ml

Representation	Fluency	Probing Questions
Perimeter • Measuring distance around shapes with string, clinometers, metre rules, trundle wheels • Using geoboards (or isometric [triangular dotty] paper) to produce shapes with a given perimeter • Using compasses to produce triangles with a given perimeter (draw base first then use compasses to produce arcs for two further lengths to make required total and find intersection of these two arcs to be final vertex)	1. Recap: find perimeter of a rectangle • rectangle – shown on squared grid • square – shown on squared grid • rectangle – length and width given • square – length given • rectangle – length given and width described e.g. twice the length • rectangle – mixed units e.g. m and cm • ext: triangles, parallelograms etc where all lengths are given • ext: triangles, parallelograms etc where shape properties help deduce missing lengths e.g. isosceles triangle, parallelogram opposite sides etc.	Show me a rectangle with a perimeter of 12cm Show me how you could find the perimeter of a rectangle of 3cm by 6cm
	2. Find perimeter of a rectilinear composite shape • L-shape – all lengths given • More complex shape e.g. T-shape/E-shape – all lengths given • L-shape – one missing length to be deduced • L-shape – two or more missing lengths to be deduced • More complex shape e.g. T-shape/E-shape – some missing lengths to be deduced	Convince me that cutting a corner (small rectangle) out from a rectangle does not change the perimeter What's the same and what's different?

	• Identical rectangles arranged to form a larger rectangle e.g. five identical rectangles are arranged to produce a shape. Find the perimeter of the shape.  	
	3. Produce a shape with a given perimeter • rectangle, one length given (whole number lengths) • rectangle, one length given (simple decimal length(s)) • square, no length given (whole number lengths) • rectangle, no lengths given (whole number lengths) • L-shape, some lengths given • ext: rectangle (simple decimal lengths) • ext: triangle (making sure it really will form a triangle) • ext: find multiple rectangles with a given perimeter	Show me a shape with a perimeter of 12cm. How many can you find? Always, Sometimes, Never? Longer shapes have larger perimeters
Area • Exploring rectangles and shapes drawn on squared paper to develop efficient counting strategies • Using a given number of squares and arranging into shapes and specifically arrays to suggest possible shapes, specifically rectangles, with a given area	4. Find the area of a rectangle • rectangle – shown on squared grid • square – shown on squared grid • rectangle – length and width given on diagram • square – side length given on diagram • rectangle/square – lengths given in writing, no diagram • rectangle – length given and width described e.g. three times the length • rectangle – mixed units e.g. m and cm	Convince me that the area of a 3cm by 2cm rectangle is 600mm^2. Convince me that the area of this shape is not 30 mm^2  What's the same and what's different? a 4 x 9 rectangle, a 6 x 6 square, a 3 x 12 rectangle and a 5 x 7 rectangle
	5. Produce rectangle with a given area • square, no length given (whole number lengths) • rectangle, one length given (whole number sides) • rectangle, one length given (simple decimal side(s)) • rectangle, no lengths given (whole number sides) • rectangle, no lengths given (simple decimal sides(s)) • ext: find all rectangles with a given area (whole number lengths) • ext: find composite shape with a given area	Show me a rectangle with an area of 2.5 cm^2 Always, Sometimes, Never? The area of a rectangle can be found by calculating the length x width.

	6. Estimate area of an irregular shape • counting squares (all whole) • counting squares (some part-squares) • approximating with a rectangle/square • approximating with two rectangles/squares and finding total	What's the same and what's different? Perimeter and Area Show me an irregular shape with an area of approximately 12 square centimetres
Volume/Capacity • Building cubes and cuboids from a given number of cubes • Deconstructing a cuboid into layers to see how to calculate its volume more efficiently (find the cubes in one layer and multiply by the number of layers) • Estimating the number of cubes in a cuboid (or other 3D shape) and then verifying by counting. Developing systems of improving the estimate. • Filling boxes with 1cm cubes to estimate volumes • Discovering the equivalence between (milli)litres and cm^3 Estimating the capacity of different containers in litres/millilitres by comparison of 1 litre to a 10cm x 10cm x 10cm cube or 1 millilitre to 1cm x 1cm x 1cm cube.	7. Find volume of a cuboid • cuboid shown broken into cubes – counting cubes • cuboid shown broken into cubes - counting cubes efficiently in layers • ext: cuboid given as a whole – constituent cubes imagined/drawn on for efficient counting	Show me a shape with a volume of 6 cm^3 Show me how many more cubes are needed to turn this shape into a cuboid 
	8. Estimate volume of a container in cm3 or m3 • represent one cm^3 • represent one m^3 • represent simple multiples of these volumes • estimate the number of cubic centimetres/metres in a container by approximating it using a cuboid	Show me how you would find the volume of this container Always, Sometimes, Never? A taller glass holds more liquid than a shorter glass.
	9. Estimate capacity of a container in l or ml • know that 1ml occupies $1cm^3$ • know that 1 litre occupies the same volume as a cuboid of 10 x 10 x 10 cm cubes. • estimate the number of ml in a small container • estimate the number of litres in a larger container	Show me how you would find the capacity of this container What's the same and what's different? volume; capacity; area; perimeter; length Always, Sometimes, Never? A cube-shaped box with (internal) sides of 10cm will hold a litre of water

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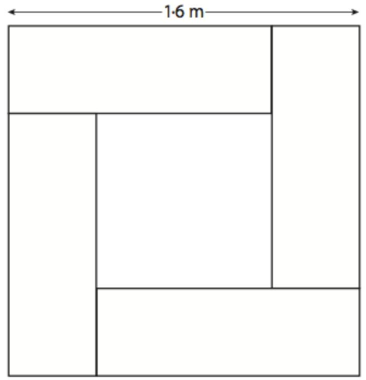
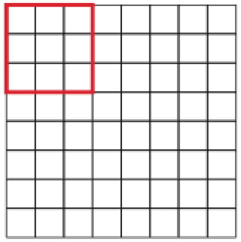
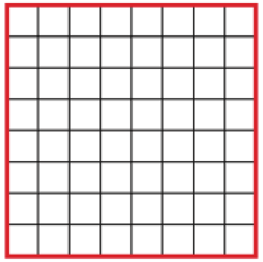
Further Extension	Rich and Sophisticated Tasks
1. The rectangular tiles here are three times as long as they are wide. What is the perimeter of the centre square?  2. Here is a picture of a square drawn on cm^2 paper.  Draw another rectangle with the same perimeter as this square. Do the two rectangles have the same area? Is this always, sometimes or never true of other pairs of rectangles with the same perimeter? Explain your reasoning.	Perimeter **NRICH:** **Area and Perimeter** *** I** **NRICH:** **Through the Window** *** I** Area **NRICH:** **Numerically Equal** **** P** **NRICH:** **Shaping It** *** I** **NRICH:** **Cubes** *** P I** **NRICH:** **Fitted** ***** P** **NRICH:** **Brush Loads** *** P I** **NRICH:** **Making Boxes** **** I** **NRICH:** **Ribbon Squares** ***** P**

3.

Here is a picture of a square drawn on cm² paper.



How many other rectangles are there with the same perimeter as the square, where the sides are a whole number of cm?

Show your workings.

4. Investigate the perimeter of L-shapes made by removing one corner a rectangle measuring 7 squares by 5 squares.
What do you notice?

5. Estimate how many cans of cola would fill a bath!

Misconceptions

When finding the perimeter of a shape drawn on a grid, some children struggle to recall where they started counting. Also, they sometimes count the squares around the outside, rather than the edges and hence at the corners miscount by missing out one of the lengths.

When working on a more abstract perimeter problem, some children only add the given lengths and miss out those that need to be deduced using the properties of the shape. Some children are insecure with shape properties or notation (for example, for equal lengths) and so cannot deduce the missing lengths.

It is common for children to confuse area with perimeter and interchange them during calculation.

If trying out the triangle drawing task from the Representing section above, many children will have difficulty in using compasses (possibly due to motor skills, lack of practice or because the compasses themselves are loose -bad-design)

Teacher Guidance and Notes

- This unit build on earlier work from Stage 4 of finding perimeters of rectangles and squares and finding areas by counting squares. The pitch, therefore, of this unit is on finding perimeters of composite shapes (made of rectangles) and finding areas more efficiently.
- Note, however, that there is no requirement (nor is it particularly beneficial) to introduce formulae here for either perimeter or area. Instead, the focus is on children developing efficient strategies using their addition and multiplication skills respectively and on solving problems. The use of formulae comes in Stage 6.
- Volume and capacity have been introduced previously but here the pitch is on finding volumes by counting cubes (possibly beginning to be efficient and use the number of cubes in a layer) and estimating volume and capacity of containers.
- When working with perimeter of a composite shape, some children will need to be taught to deduce missing lengths on a diagram; others will find it obvious. The key is to realise why it is necessary: keep

Many children lack the real concept of volume and focus on the calculation process rather than what it actually is. Children often think that multilink cubes are 1cm3 (they are 8cm3 each). There is difficulty in understanding the relationship of capacity to volume. It is common to see a failure to state units at all when measuring. Where units are inconsistent, children may still try to calculate the perimeter or area with them without the necessary conversion first.	emphasising the 'journey round the shape' - how long is this (possibly unmarked) piece? The drawing work is vital preparation for future work. • At this stage, try to get children to use rectangles to approximate an irregular shape to help them make an estimate of its area. • Work on volume aims to establish the principle of counting cubes - eg in multilink solids initially (since 1cm cubes are fiddly).

Key Assessment Checklist

1. I can calculate the perimeter of any shape (including composite shapes) by adding up side lengths, including cases where all sides are given directly
2. I can calculate the perimeter of a rectangle where the length and width are given or described; I can calculate the perimeter of a shape using knowledge of shape properties to help me find missing sides
3. I can calculate the perimeter of a composite shape (eg L, E, F shapes) by adding up side lengths, including cases where not all sides are given directly
4. I can produce a rectangle with a given perimeter
5. I can find the area of rectangles and squares, giving the answer in the correct units (mm2, cm2, m2)
6. I can compare the area of rectangles and squares and find a rectangle or square with a given area.
7. I can estimate the area of irregular shapes by counting squares and part squares
8. I can explain that the volume of a cuboid is measured by counting how many unit cubes it takes to fill it (eg with multilink or 1cm cubes)
9. I can estimate and compare the volume of small boxes in cm3 by counting how many (closely packed) 1cm cubes it takes to fill them
10. I can estimate the capacity of different containers using liquids and measuring jugs. I know that capacity in ml is the same as volume in cm3, and that 1 litre is the capacity of a 10 x 10 x 10cm cube

Year 5	Unit 7 : Discovering Equivalence
14 learning hours	This unit explores the concepts of fractions, decimals and percentages as ways of representing non-whole quantities and proportions. For the youngest children, the work is focused on fractions and developing security in recognising and naming them. At KS2 this then builds to looking at families of fractions and decimals and percentages. At secondary level this is extended to more complex % work and equivalence with recurring decimals and surds.

Prior Learning	Core Learning	Learning Leads to...
➢ recognise and show, using diagrams, families of common equivalent fractions	➢ **recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements > 1 as a mixed number [for example, 2/5 + 4/5 = 6/5 = 1 1/5]**	
	➢ **identify, name and write equivalent fractions of a given fraction, represented visually, including tenths and hundredths**	➢ use common factors to simplify fractions; use common multiples to express fractions in the same denomination
	➢ **compare and order fractions whose denominators are all multiples of the same number**	➢ compare and order fractions, including fractions > 1
➢ count up and down in tenths; recognise that tenths arise from dividing an object into 10 equal parts and in dividing one-digit numbers or quantities by 10 ➢ recognise and write decimal equivalents of any number of tenths or hundredths ➢ recognise and write decimal equivalents to 1/4,1/2, ¾	➢ **count up and down in hundredths; recognise that hundredths arise when dividing an object by one hundred and dividing tenths by ten** ➢ **read and write decimal numbers as fractions [for example, 0.71 = 71/100]** ➢ **recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents**	➢ associate a fraction with division and calculate decimal fraction equivalents [for example, 0.375] for a simple fraction [for example, 3/8] ➢ recall and use equivalences between simple fractions, decimals and percentages, including in different contexts
	➢ **recognise the per cent symbol (%) and understand that per cent relates to 'number of parts per hundred'; write percentages as a fraction with denominator 100, and as a decimal** ➢ **solve problems which require knowing percentage and decimal equivalents of 1/2, 1/4, 1/5, 2/5, 4/5 and those fractions with a denominator of a multiple of 10 or 25.**	➢ solve problems involving the calculation of percentages [for example, of measures, and such as 15% of 360] and the use of percentages for comparison

Exemplification

1. a) Write this mixed number as an improper fraction $2\frac{1}{4}$

b) Write this improper fraction as a mixed number $\frac{10}{3}$

2. Complete these equivalent fractions by finding the value of the missing number

a) $\frac{1}{5} = \frac{5}{\blacksquare}$ b) $\frac{3}{7} = \frac{\blacksquare}{21}$ c) $\frac{16}{24} = \frac{6}{\blacksquare}$

3. Which of these fractions is the largest? $\frac{2}{3}, \frac{3}{4}, \frac{5}{8}$ Explain your answer.

4. Write sixteen hundredths as a fraction and a decimal.

5. Write this decimal as a fraction: 0.24

6. Write two hundred and fifty one thousandths as a fraction and a decimal

7. Write 28% as a fraction and a decimal

8. Which of these is the greatest?

34%	$8/25$	0.35	$\frac{3}{10}$

Vocabulary

fraction numerator denominator part whole proper fraction improper fraction vulgar fraction mixed number convert equivalent value simplify equal	compare order denominator numerator common denominator place value decimal hundredth thousandth percentage % parts per hundred

Representation

Mixed Numbers:

- Representing mixed numbers using the bar model or with paper strips
  For example $2\frac{5}{6}$

- Converting an improper fraction by grouping items into groups of the denominator
- Representing mixed numbers as multiple whole circles and parts of circles
  For example, here is $2\frac{5}{6}$

Fluency

1. Convert improper fractions to mixed numbers (and whole numbers)
   - halves e.g. $\frac{3}{2}, \frac{4}{2}, \frac{5}{2}, \ldots$
   - quarters e.g. $\frac{5}{4}, \frac{8}{4}, \frac{13}{4}, \ldots$
   - thirds, fifths and other single digit denominators
   - larger denominators e.g. tenths
   -
2. Convert mixed numbers to improper fractions
   - whole numbers e.g. 4 to thirds and 2 to quarters

Probing Questions

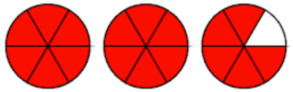
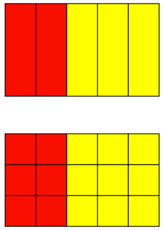
What's the same and what's different?

improper fraction; mixed number; proper fraction; unit fraction; non-unit fraction; vulgar fraction; whole number

Convince me that 13/10 = 1 3/10

Show me as many different representations of 5/4 as you can (use symbols, writing, images and models)

Always, Sometimes, Never?
Improper fractions must be greater than 1

	• mixed numbers with a numerator of 1 e.g. $1\frac{1}{3}, 1\frac{1}{5}, 2\frac{1}{2}, ...$ • mixed numbers with other numerators e.g. $1\frac{2}{3}, 2\frac{3}{5}, 1\frac{9}{10}, ...$ • mixed numbers with larger whole numbers e.g. $4\frac{2}{3}$	Always, Sometimes, Never? Mixed numbers are better than improper fractions
Equivalent Fractions • Folding paper strips vertically (rectangles) to represent a fraction and then folding horizontally to discover equivalent fractions and the proportional link between numerators and denominators For example, for $\frac{2}{5}$ is equivalent to $\frac{6}{15}$ 	3. Recognise equivalent fractions • of shapes divided into the same number of parts (e.g. thirds and thirds) • of shapes divided into a different number of parts (e.g. eighths and quarters) • in numeric form e.g. $\frac{1}{3}$ and $\frac{2}{6}$	What's the same and what's different? $\frac{3}{12}, \frac{25}{100}, \frac{4}{16}, \frac{1}{4}$ What's the same and what's different? $1\frac{8}{10}, 1\frac{4}{5}, \frac{45}{25}, \frac{9}{5}$
	4. Find equivalent fractions • unit fractions, find any equivalent or a list of equivalent fractions • non-unit fractions, find any equivalent or a list of equivalent fractions • given denominator, find numerator of equivalent fraction e.g. $\frac{1}{4} = \frac{\blacksquare}{20}$ • given numerator, find denominator of equivalent fraction e.g. $\frac{2}{5} = \frac{6}{\blacksquare}$	Always, Sometimes, Never? You simplify a fraction by dividing the numerator and denominator by 2. Always, Sometimes, Never? You make an equivalent fraction by multiplying the numerator and denominator by 2. Show me a fraction that is equivalent to ¾ Show me a fraction that is equivalent to 7/10
Comparing Fractions • Representing fractions using the bar model (vertically and horizontally) e.g. $1/6$	5. Compare two fractions with denominators that are multiples of the same number • compare two proper fractions, same denominator • compare two proper fractions, related denominators (i.e. one a multiple of the other) • compare two proper fractions, different denominators but both multiples of the same number • compare one proper and one improper fraction • compare two improper fractions, same denominator • compare two improper fractions, one denominator a multiple	Convince me that 7/12 < 2/3 Show me two fractions where one has a denominator that is a multiple of the other

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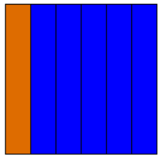
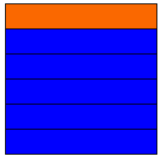
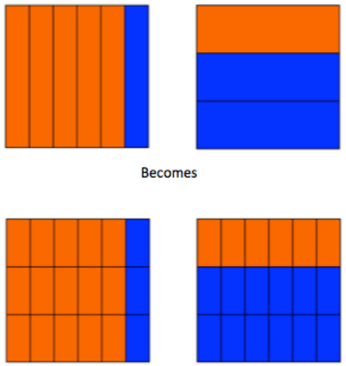


Comparing two fractions with different denominators using an array on the same shape
For example, $\frac{5}{6}$ and $\frac{1}{3}$


$\frac{15}{18}$ and $\frac{6}{18}$
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of the other

compare two improper fractions, both denominators a multiple of the same number
compare two mixed numbers with the same denominator

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6. Order three or more fractions whose denominators are multiples of the same number

order three or more proper fractions with the same denominator
order three or more proper or improper fractions with the same denominator
order three or more proper fractions with denominators that are multiples of the same number
order three or more proper or improper fractions with denominators that are multiples of the same number
order three or more proper fractions, improper fractions or mixed numbers with the same denominator

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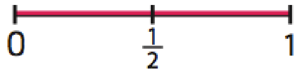
What's the same and what's different?

$\frac{3}{10}, \frac{8}{3}, 3\frac{1}{10}, \frac{25}{100}$

Show me how you order: 3/10, 3/4, 1/5, 3/20

Show me where you would position ¾ and 3/8 on this number line.....
... what about 3/5? 3/6? 3/7?


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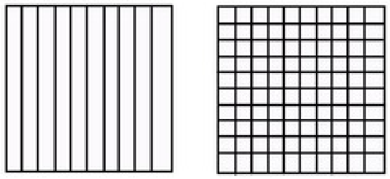
Tenths and Hundredths: Decimals
Use a 100-square grid to represent tenths and hundredths, using one row or column as a tenth

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7. Work with tenths and hundredths

Count up in tenths from any number of tenths e.g. nine-tenths
Count down in tenths from any number tenths e.g. twenty-three-tenths
Count up in hundredths from any number of tenths e.g. eighty-four hundredths
Count down in hundredths from any number tenths e.g. one hundred and twelve hundredths
Count up and down in hundredths from any number of

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Convince me that 1/2 cannot be written as 1.2

Convince me that 11 tenths is the same as 1 1/10
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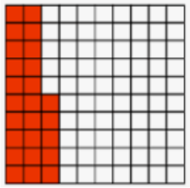
• Reading sections of a whole in different  ways e.g. here is 26/100, 2 tenths and 6 hundredths • Using a place value grid labelled with 1/10 s and 1/100s to represent decimals • Using a 10x10x10 cube to represent one whole and then using each small cube to represent one thousandth	hundredths, reading multiples of ten as tenths e.g. eighty-eight hundredths, eighty-nine hundredths, nine tenths, ...	
	8. Convert fractions with denominator 10 or 100 to decimals • read a fraction aloud e.g. $\frac{7}{10}$ as 'seven tenths' or $\frac{24}{100}$ as 'twenty-four hundredths' • write a proper fraction with denominator 10 as a decimal e.g. $\frac{3}{10}$ as 0.3 (3 in the tenths column) • write a proper fraction with one-digit numerator and denominator 100 as a decimal e.g. $\frac{3}{100}$ as 0.03 (3 in the hundredths column) • write a proper fraction with two-digit numerator and denominator 100 as a decimal e.g. $\frac{23}{100}$ as 0.23 • write a proper fraction with a numerator that is a multiple of 10 and a denominator of 100 as a decimal e.g. $\frac{70}{100}$ as 0.7 (rather than 0.70) • ext: write an improper fraction with a denominator of 10 or 100 as a decimal e.g. $\frac{147}{100}$ as 1.47	What's the same and what's different? 0.1, 3/10, 0.25, ¼ Convince me that 0.3 = 30/100
	9. Convert decimals up to 2dp to fractions • write a decimal with 1 decimal place as a fraction e.g. 0.4 or 0.7 • write a decimal with 2 decimal places as a fraction e.g. 0.71	Convince me that 1.2 = 6/5
	10. Count in and recognise thousandths • count up in thousandths from any number of thousandths • count down in thousandths from any number of thousandths • recognise that ten thousandths makes a hundredth • recognise that one hundred thousandths makes a tenth	Convince me that ten thousandths is equivalent to one hundredth
Percentages • Colouring in a 10x10 square to show tenths and hundredths, as well as the equivalence of, for example, forty hundredths and four tenths. • Using a bar representation (visual or washing line/paper strips) to represent 0-	11. Recognise percentages • know that 100% is a whole • understand that a percentage tells you the number of parts per hundred • given items with one hundred parts, identify the percentage shown or shaded • recognise simple percentages of a shape e.g. 50%, 25%	Always, Sometimes, Never? Percentages are fractions with a denominator of 100

100 and to position percentages correctly, linking this to the position of fractions	• given items with ten parts, identify the percentage shown or shaded	
	12. Convert percentages to fractions and decimals • 50% • 10% and multiples of 10% • 1% and multiples of 1% • ext: percentages greater than 100%	Always, Sometimes, Never? Every percentage can be written as a fraction Always, Sometimes, Never? Every fraction can be written as a percentage
	13. Recall equivalent fractions, decimals and percentages • ½ • ¼ and multiples of ¼ • 1/5 and multiples of 1/5 • 1/10 and multiples of 1/10 • 1/25 and multiples of 1/25	What's the same and what's different? 0.2, 20%, 2/10, 2.1 Convince me that 0.1 = 10%
Problems involving FDP • Use a bar model to represent the problem visually	14. Solve problems involving equivalence of the above fractions, decimals and percentages • compare two proportions where one is a fraction and one is a percentage to say which is greater • compare two proportions where one is a fraction and one is a decimal to say which is greater • compare three or more proportions to put them in order • order sets of simple fractions, decimals and percentages • compare proportions greater than a whole, including mixed numbers with denominator 2, 5, 4, 10, 25	

Further Extension	Rich and Sophisticated Tasks
1. Make each number sentence correct using =, > or <. $\frac{3}{4}$ ○ $\frac{1}{2}$ $\frac{3}{8}$ ○ $\frac{1}{2}$ $\frac{3}{4}$ ○ $\frac{3}{8}$ $1\frac{3}{4}$ ○ $2\frac{1}{2}$ $\frac{3}{2}$ ○ $1\frac{1}{2}$ $3\frac{3}{4}$ ○ $3\frac{3}{8}$ $\frac{2}{4}$ ○ $\frac{1}{2}$ $\frac{2}{5}$ ○ $\frac{4}{10}$ $\frac{2}{5}$ ○ $\frac{5}{10}$	Recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements > 1 as a mixed number (e.g. 2/5 + 4/5 = 6/5 = 1 1/5) **NRICH: Balance of Halves * P** Recognise and show, using diagrams, families of common equivalent fractions **NRICH: Fractional Wall * P** **NRICH: Fractional Triangles * P** **NRICH: Bryony's Triangle * P**

2.
Russell says $\frac{3}{8} > \frac{3}{4}$ because 8 > 4.

Do you agree?

Explain your reasoning.

3.
Choose numbers for each numerator to make this number sentence true.

$\frac{\square}{15} > \frac{\square}{10}$

4.
Which is closer to 1?

$\frac{7}{8}$ or $\frac{23}{24}$

Explain how you know.

5.
Chiz and Caroline each had two sandwiches of the same size.

Chiz ate $1\frac{1}{4}$ of his sandwiches.

Caroline ate $\frac{5}{4}$ of her sandwiches.

Fred said Caroline ate more because 5 is the biggest number.
Tammy said Chiz ate more because she ate a whole sandwich.

Explain why Fred and Tammy are both wrong.

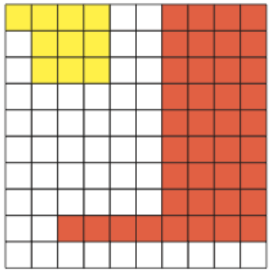
6.
Jack and Jill each go out shopping. Jack spends $\frac{1}{4}$ of his money. Jill spends 20% of her money.

Frank says Jack spent more because $\frac{1}{4}$ is greater than 20%.
Alice says you cannot tell who spent more.

Who do you agree with, Frank or Alice? Explain why.

Solve problems which require knowing percentage and decimal equivalents of ½, ¼, 1/5, 2/5, 4/5 and those fractions with a denominator a multiple of 10 or 25
NRICH: Matching Fractions Decimals Percentages *** G**

7.
Express the yellow section of the grid in hundredths, tenths, as a decimal and as a percentage of the whole grid.
Do the same for the red section.



Misconceptions

For some children there is still confusion about the meaning of a fraction and the significance of the numerator and denominator. Some children do not fully recognise that the parts of the whole must be of equal size. They also do not see the denominator as an indicator of the number of parts in the whole and use it directly to order fractions, believing that fractions with a larger denominator are bigger.

Many children believe that you cannot have a fraction where the numerator is greater than the denominator and they may reattempt the question or alternatively turn their fraction upside down to avoid giving an answer in this form.

When finding an equivalent fraction, some children will do different things to the numerator and denominator or carry out an addition or subtraction rather than a multiplication or division to both. For instance, you may find children saying that 3/5 is equivalent to 5/7 because they have added two to both the numerator and denominator.

Children may confuse 'hundreds' and 'hundredths' or 'thousands' and 'thousandths'

Children may experience some confusion over place value headings after the decimal point - make sure these are consistent with your headings before the decimal point e.g. if you are using 1000s, 100s, 10s, 1s then you need to use 1/10s, 1/100s etc

Occasionally children may include more than one decimal point.

Teacher Guidance and Notes

- As with all fraction units in all stages, it is essential that children understand the role played by the numerator and denominator in a fraction. Specifically, that the denominator tells us the number of parts in the whole and the numerator tells us the number of parts that we are working with. Strongly model the language of part and whole throughout in order to embed these concepts.
- This is the first time that children will have encountered mixed numbers. Therefore, it is essential to use visual representations to explore the connection between improper fractions and mixed numbers e.g. 11/10 as one whole and 1/10. Gradually you will be able to move towards using division as a process to convert an improper fraction e.g. 7/3 means 7 divided by 3 which is 2 remainder 1 or 2 and 1/3.
- Children have previously encountered the processes and exercises in Stage 4 of counting in tenths. In Stage 5 this is now applied to hundredths and is key in exposing children to the idea that you can have a fraction with a numerator that is greater than the denominator e.g. 11 tenths. If this counting has been accompanied by images to support the process this will be even stronger.
- The step based on comparing and ordering fractions where the denominators are multiples is key to good fraction addition and subtraction later - therefore spend time here ensuring that children are confident to 'translate' fractions into the same language before ordering. Make use of models and images to justify their equivalent representations.
- Linguistically, try to constantly relate the symbol % with /100. Try not to get children to believe that 24% = 0.24 x 100 (which is not true!) and

As in Stage 4, some children believe that adding a zero = multiplying by 10 (e.g. 0.1 x 10 = 0.10 instead of moving the digits up a space Children do not always realise that a percentage is simply a different way of describing the proportion of the whole with a fixed size of 100. Children sometimes make errors when working with single digit percentages, conflating, for example 7% with 70%.	instead encourage them to believe that 24% = 0.24 itself. • It is useful, particularly for more able children, to relate the initial work on mixed numbers and improper fractions to percentages that are greater than 100%. • The final small step requires children to just 'know' the equivalences for common fractions - focus on speed recall here. In Stage 6 children explore the process of division within a fraction to arrive at the decimal equivalents of common fractions.

Key Assessment Checklist

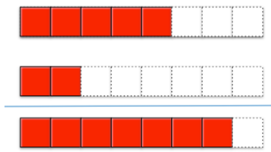
1. I can recognise mixed numbers and improper fractions and convert between the two
2. I can compare and order fractions by finding a common denominator
3. I can find equivalent fractions by multiplying or dividing the numerator and denominator by the same number
4. I can count up and down in hundredths and thousandths and can find a hundredth by dividing a number by 100
5. I can read and write decimals as fractions by looking at the lowest place value heading (e.g. 0.71 has 71 hundredths so is 71/100)
6. I can explain that % = number of parts per hundred
7. I can write percentages as fractions, showing numbers out of a hundred, and can convert these to decimals by dividing the numerator by the denominator (dividing by 100)
8. I can convert 1/2, 1/4, 1/5, 2/5, 4/5 into decimals and percentages and use these to solve problems

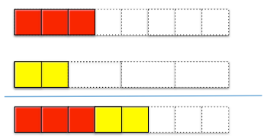
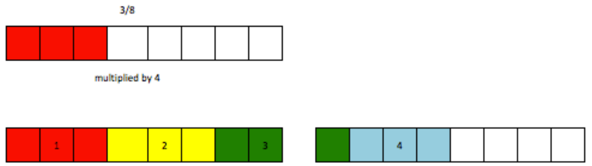
Year 5	Unit 8 : Reasoning with Fractions
8 learning hours	This unit progresses from the development of the understanding of non-whole items at the lowest end to flexibility and fluency with calculations involving fractions for older primary students. This knowledge is then applied within the secondary curriculum to the topic of probability, thus providing a clear context in which the skills of adding and multiplying fractions particularly are needed. It is critical that pupils develop confidence and security in understanding and manipulating fractions as well as flexibility in representing a number as a fraction or as a decimal, percentage, diagram etc. Note that once fraction calculations are mastered here, they should be used in following units as examples just as other numbers are in order to keep the skills fresh.

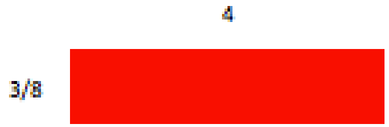
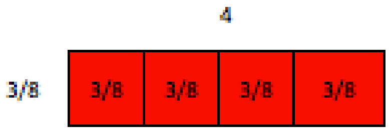
Prior Learning	Core Learning	Learning Leads to...
➢ add and subtract fractions with the same denominator ➢ solve problems involving increasingly harder fractions to calculate quantities, and fractions to divide quantities, including non-unit fractions where the answer is a whole number ➢ solve simple measure and money problems involving fractions and decimals to two decimal places	➢ **add and subtract fractions with the same denominator and denominators that are multiples of the same number** ➢ **multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams**	➢ add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions ➢ multiply simple pairs of proper fractions, writing the answer in its simplest form [for example, 1/4 × 1/2 = 1/8] ➢ divide proper fractions by whole numbers [for example, 1/3 ÷ 2 = 1/6]

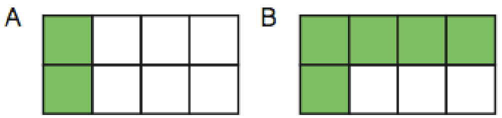
Exemplification	Vocabulary
1. Calculate a) $\frac{7}{11} - \frac{4}{11}$ b) $\frac{3}{5} + \frac{2}{15}$ 2. Calculate a) $\frac{1}{5} \times 4$ b) $1\frac{2}{3} \times 2$	proper fraction improper fraction mixed number numerator denominator equivalent reduced to cancel

Representation	Fluency	Probing Questions
Adding and Subtracting Fractions • Using the bar model to add and subtract fractions with the same denominator	1. Add and subtract proper fractions with the same denominator • add two unit fractions with the same denominator	Show me how you can use a bar model to add fractions with the same denominator

$\frac{5}{8}+\frac{2}{8}$  or $7/8$ in total	e.g. $\frac{1}{3}+\frac{1}{3}$ • add two proper fractions with the same denominator e.g. $\frac{2}{9}+\frac{3}{9}$ • subtract two proper fractions with the same denominator e.g. $\frac{6}{7}-\frac{4}{7}$ • add two proper fractions with the same denominator, simplifying the answer e.g. $\frac{5}{8}+\frac{1}{8}$ • subtract two proper fractions with the same denominator, simplifying the answer e.g. $\frac{5}{6}-\frac{1}{6}$	Show me how you can use arrays to subtract fractions with same denominator Show me two fractions with a sum of $7/9$ Show me two fractions with a difference of $2/11$
	2. Add mixed numbers and fractions with the same denominator • add a proper fraction to a mixed number with the same denominator e.g. $1\frac{1}{5}+\frac{3}{5}$ • add a proper fraction to a mixed number with the same denominator, crossing over the next whole e.g. $2\frac{2}{3}+\frac{2}{3}$ • add two mixed numbers with the same denominator e.g. $1\frac{1}{7}+2\frac{5}{7}$ • add two mixed numbers with the same denominator, where the fraction parts cross the next whole e.g. $2\frac{5}{8}+1\frac{7}{8}$	What's the same and what's different? $1\frac{1}{9}+\frac{4}{9};\ 1\frac{5}{9}+\frac{7}{9}$ Convince me that $1\frac{2}{3}+2\frac{2}{3}=4\frac{1}{3}$
	3. Subtract mixed numbers and fractions with the same denominator • subtract a proper fraction from a mixed number with the same denominator e.g. $2\frac{5}{7}-\frac{3}{7}$ • subtract a proper fraction from a mixed number with the same denominator, crossing over the next whole e.g. $3\frac{1}{5}-\frac{2}{5}$ • subtract a mixed number from another with the same denominator e.g. $3\frac{5}{6}-2\frac{1}{6}$ • subtract a mixed number from another with the same denominator, where the fraction parts cross the next whole e.g. $3\frac{1}{4}-1\frac{3}{4}$	Always, Sometimes, Never? When subtracting fractions you need to subtract both the denominators and the numerators. Convince me that $3\frac{2}{5}-1\frac{3}{5}=1\frac{4}{5}$

• Using the bar model to add and subtract fractions with one denominator that is a multiple of the other (by breaking the fraction with the smaller denominator up into the larger denominator-sized pieces) $\frac{3}{8}+\frac{1}{4}$  or $\frac{5}{8}$ in total	4. Add fractions with denominators that are multiples of the same number • add two unit fractions where one denominator is a multiple of the other e.g. $\frac{1}{3}+\frac{1}{6}$ • add two proper fractions where one denominator is a multiple of the other e.g. $\frac{3}{10}+\frac{2}{5}$ • add two fractions where both denominators are multiples of the same number e.g. $\frac{1}{6}+\frac{1}{9}$ • ext: apply to mixed number cases	Show me two fractions with a sum of $7/12$ Convince me that $1/5\ +\ 1/10\ =\ 3/10$ Always, Sometimes, Never? The denominator needs to be the same when adding or subtracting fractions.
	5. Subtract fractions with denominators that are multiples of the same number • subtract two unit fractions where one denominator is a multiple of the other e.g. $\frac{1}{3}-\frac{1}{6}$ • subtract two proper fractions where one denominator is a multiple of the other e.g. $\frac{7}{10}+\frac{2}{5}$ • subtract two fractions where both denominators are multiples of the same number e.g. $\frac{5}{6}-\frac{2}{9}$ • ext: apply to mixed number cases	Show me two fractions with a difference of $2/20$ Convince me that if 1/3 a bar of chocolate is eaten one day then 1/6 of a bar the next day then there will be 1/2 of a bar left What's the same and what's different? $\frac{3}{8}+\frac{1}{4};\frac{3}{8}-\frac{1}{4}$
Multiplying Fractions • Using the bar model to produce repeated addition of a fraction For example, $\frac{3}{8}\times\ 4$ can be represented as 4 lots of 3 squares, each of which is worth an eight, which is twelve eighths or $\frac{12}{8}=1\frac{1}{2}$  • Using an area (grid method) model For example, for $\frac{3}{8}\times\ 4$	6. Multiply a unit fraction by a whole number • multiply a unit fraction by a whole number e.g. $\frac{1}{7}\times\ 4$, no simplification • multiply a unit fraction by a whole number and simplify e.g. $\frac{1}{8}\times\ 6$ • multiply a unit fraction by a whole number to give an answer greater than 1, no simplification e.g. $\frac{1}{3}\times\ 5$ • multiply a unit fraction by a whole number to give an answer greater than 1 and simplify e.g. $\frac{1}{4}\times\ 6$	Convince me that $\frac{1}{4}\times\ 3\ \neq\frac{3}{12}$ True or False? $\frac{2}{7}\times\ 3=\frac{3}{7}\times\ 2$
	7. Multiply a proper fraction by a whole number • multiply a proper fraction by a whole number e.g. $\frac{2}{7}\times\ 3$, no simplification	Show me how you can multiply $3/4\ \times\ 12$

 which can then be broken down into 3/8s  to arrive at an answer of $\frac{12}{8} = 1\frac{4}{8}\, or\ 1\frac{1}{2}$	• multiply a proper fraction by a whole number and simplify e.g. $\frac{3}{8} \times\ 2$ • multiply a proper fraction by a whole number to give an answer greater than 1, no simplification e.g. $\frac{2}{9} \times\ 5$ • multiply a proper fraction by a whole number to give an answer greater than 1 and simplify e.g. $\frac{3}{4} \times\ 6$	Show me a fraction and a whole number with a product of $8/12$
	8. Multiply a mixed number by a whole number • multiply an improper fraction by a whole number e.g. $\frac{5}{4} \times\ 3$ • multiply a mixed number by a whole number e.g. $1\frac{1}{7} \times\ 3$, no simplification • multiply a mixed number by a whole number and simplify e.g. $2\frac{3}{8} \times\ 2$	Show me how you can multiply $\frac{11}{4} \times\ 12$ Convince me that $1\frac{2}{3} \times\ 3 = 5$ Always, Sometimes, Never? When you multiply a fraction by a whole number, you get an answer greater than 1.

Further Extension	Rich and Sophisticated Tasks
1. a) Each bar of toffee is the same. On Monday, Sam ate the amount of toffee shown shaded in A. On Tuesday, Sam ate the amount of toffee shown shaded in B. How much more, as a fraction of a bar of toffee, did Sam eat on Tuesday?  b) Sam says he ate $\frac{7}{8}$ of a bar of toffee. Jo says Sam ate $\frac{7}{16}$ of the toffee. Explain why Sam and Jo are both correct.	**NRICH:** **[Balance of Halves]()** **NRICH:** **[Route Product]()** **NRICH:** **[Forgot the Numbers]()**

2. a)

Using the numbers 5 and 6 only once, make this sum have the smallest possible answer:

$\frac{\square}{15} + \frac{\square}{10} =$

b)

Using the numbers 3, 4, 5 and 6 only once, make this sum have the smallest possible answer:

$\frac{\square}{\square} + \frac{\square}{\square} =$

3.

Graham is serving pizzas at a party. Each person is given $\frac{3}{4}$ of a pizza. Graham has six pizzas.

How many people can he serve? Draw on the pizzas to show your thinking.

Write your answer as a multiplication sentence.

Misconceptions

When adding (or subtracting) fractions children may simply add (or subtract) both the numerators and denominators. This is because they do not recognise that the denominator indicates the number of parts of the whole and so treat the fractions as 4 'whole numbers' to be added together. Stronger understanding that we are adding the numerators because these are the parts we are working with (and the denominators simply tell us how many make a whole) will support moving on from this conceptual barrier.

Children may still create equivalent fractions additively rather than multiplicatively, for example converting ¾ to eighths by adding four and wrongly obtaining $7/8$

You may find that children believe that they should simply multiply both the numerator and denominator by a whole number when multiplying a proper fraction by an integer.

Similarly, some children may believe that you simply multiply the whole number and then the fraction when multiplying a mixed number by a whole number, e.g. $2\frac{3}{5} \times 2 = 4\frac{6}{10}$

Teacher Guidance and Notes

- This unit applies the work of Unit 7 in representing fractions to the calculation process when adding, subtracting and multiplying by a whole number
- Childre may still need further development of their skills in representing a fraction in multiple ways so that they can then combine these to add/subtract fractions.
- It is strongly recommended that a school adopt a consistent approach to representing fractions using the (vertical) bar model, which can then be supplemented by additional representations as appropriate.
- As previously, ensure you model the use of language such as denominator and numerator and part and whole as much as possible to secure these concepts
- Make connections with other areas of maths where fractions are used, e.g. when describing turns, calculating measures for recipes, calculating journey times and fuel consumption, working out results of sales offers with money and comparing prices.

Key Assessment Checklist
1. I can add and subtract fractions with the same denominator.
2. I can add and subtract fractions where one denominator is a multiple of the other
3. I can recognise and solve problems involving adding and subtracting fractions
4. I can multiply proper fractions by whole numbers.
5. I can multiply proper fractions by whole numbers
6. I can multiply mixed number fractions by whole numbers
7. I can recognise and solve problems involving multiplying a proper fraction or a mixed number by a whole number

Year 5	Unit 9 : Solving Number Problems	
8 learning hours	This unit continues pupils' earlier study of arithmetic (and algebra for secondary students). At Key Stage 1 children are working on multiplication (and division in Stage 2) as a way to represented repeated addition and scaling (and repeated subtraction – grouping - and sharing) At Key Stage 2 children are developing skills in applying their arithmetic to more complex problems. At secondary level and in Stage 6, students begin to find unknown values by applying inverse operations. Equations of all types including quadratic and simultaneous are covered in later stages.	
Prior Learning	**Core Learning**	**Learning Leads to...**
➢ find the effect of dividing a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as ones, tenths and hundredths ➢ use place value, known and derived facts to multiply and divide mentally, including: ○ multiplying by 0 and 1; ○ dividing by 1; ○ multiplying together three numbers ➢ multiply two-digit and three-digit numbers by a one-digit number using formal written layout	➢ **multiply and divide whole numbers and those involving decimals by 10, 100 and 1000** ➢ **multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers** ➢ **divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context**	➢ multiply and divide numbers by 10, 100 and 1000 giving answers up to three decimal places ➢ perform mental calculations, including with mixed operations and large numbers ➢ multiply multi-digit numbers up to 4 digits by a two-digit whole number; divide numbers up to 4 digits by a two-digit whole number, interpret remainders ➢ multiply one-digit numbers with up to two decimal places by whole numbers; use written division methods in cases where the answer has up to 2dp
➢ solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects ➢ solve simple measure and money problems involving fractions and decimals to two decimal places	➢ **solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes** ➢ **solve problems involving** addition, subtraction**, multiplication and division and a combination of these, including understanding the meaning of the equals sign** ➢ **solve problems involving number up to three decimal places**	➢ solve problems involving addition, subtraction, multiplication and division ➢ use their knowledge of the order of operations to carry out calculations involving the four operations ➢ solve problems which require answers to be rounded to specified degrees of accuracy ➢ express missing number problems algebraically ➢ find pairs of numbers that satisfy an equation with two unknowns ➢ enumerate possibilities of combinations of two variables

Exemplification

1. A crate full of oranges holds 24.
   Harjit picks 148 oranges from the orchard.
   How many crates does he need to pack all the oranges?

2.
a) Find 2 square numbers that add up to make another square number.
b) Is this the only possible pair?
c) Can you find 3 square numbers that add together to make another square number?
d) Is 7 a factor of 4361? Explain your answer

3. Find the value of ∇ :

$$900 - 150 = \nabla \times 50$$

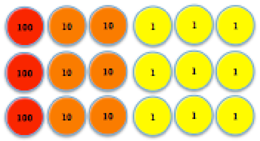
4. Bryony is measured at 1.46m tall to two decimal places.
   a) What are all the possible lengths she could be to 3 decimal places?
   b) Explain why 1.465m would not round to 1.46m?

Vocabulary

multiply	remainder
divide	factor
product	multiple
quotient	square(d)
dividend	cube(d)
divisor	equals
add	scale
sum	scale up by/scale
subtract	down by
difference	rate
place value -	per
including tenths,	
hundredths,	
thousandths and	
decimal places	

Representation

Multiplication
Use place value counters to represent large numbers in arrays e.g. 234 x 5 as 2 hundreds, 3 tens and 4 ones repeated over 5 rows.
For example: here is 123 x 3



Generalising to grid method as an 'undrawn' array.
For example, for the 123 x 3

	100	20	3
3	300	60	9

Making the link between the grid method and an expanded column method

x	200	40	3
30	6000	1200	90
6	1200	240	18

7290
1458
8748
1

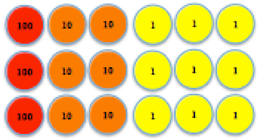
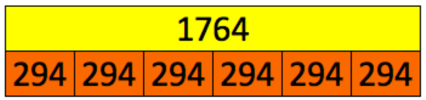
Fluency

1. Recap: multiply a whole number by a 1 or 2 digit number
   - 3-digit multiplied by 1-digit
   - 4-digit multiplied by 1-digit
   - 3-digit multiplied by 2-digits
   - 4-digit multiplied by 2-digits

Probing Questions

Convince me that 165 x 4 = 660

Convince me that 74 x 56 = 4144

Division Building a number using place value counters and grouping them into groups that are the size of the divisor, before arranging these groups as an array to explore the partitioning approach. For example, 369 ÷ 3  Repeating the above, but exchanging remaining counters for 10 counters of the next size down before continuing to group. For example, 372 ÷ 3  Recording these methods to arrive at compact division	2. Recap: divide a whole number by a 1-digit number • no exchange necessary e.g. 8484 ÷ 4 • first digit is lower than divisor requiring exchange e.g. 2196 ÷ 3 • single exchange e.g. 8476 ÷ 4 • two or more exchanges e.g. 4185 ÷ 5 • problems with a remainder e.g. 7184 ÷ 6	What's the same and what's different? 98 ÷6, 48 ÷6, 18 ÷6, 78 ÷6
Representing and Solving Problems Using the bar model to represent word problems For example, £1764 are shared between 6 people. How much does each person get?  Using the bar model to represent multi-step problems For example: Sharon and Tim each had a collection of football stickers. Tim had 5 times as many as Sharon. He had 150. How many did they have altogether?	3. Solve word problems involving a multiplication and/or division • word problem involving a multiplication (groups) e.g. There are 8 entrances to a football ground. Each entrance covers 3148 seats. How many seats are there in the football ground? • word problem involving a multiplication (scaling) e.g. The mass of a model car is 243 kg. The real car it models has a mass that is 7 times larger. What is the mass of the real car? • word problem involving a division e.g. a group of nine people win £4671 in a lottery. They share the money equally. How much money does each person receive?	Show me how you would represent this problem: Tina had a cupboard in her bedroom on which she kept her books. There were 15 books on each of 8 shelves. A friend gave her another 24 books which she put equally onto the 8 shelves. How many books were on each shelf?

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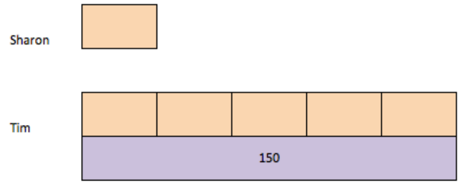
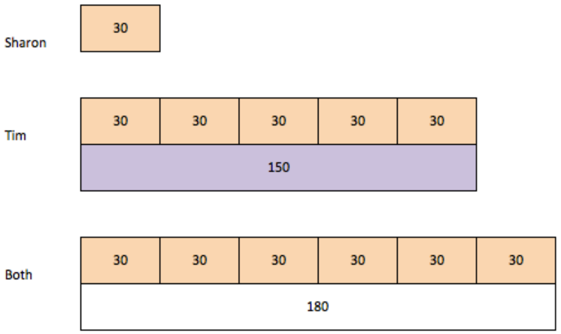
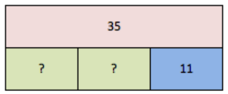
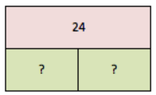
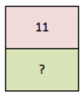


Tim has 150 stickers, so each square represents 30 stickers.
Therefore Sharon has 30 and altogether they have 180.


Using weights and scales to represent balance problems like
2 x ☐+ 11 = 35

Similarly, using the bar model for such problems






</td>
<td>
• word problem involving a division where remainder must be considered e.g. Emily is packing 795 bread rolls into packets. Each packet contains 6 bread rolls. How many complete packets can Emily produce?
• word problem involving a combination of both multiplication and division e.g. 6 friends go to a concert for one friend’s birthday. The tickets cost £89 each. The other 5 friends decide to pay for the friend who’s birthday it is. How much will each of them pay if they each pay the same amount?
</td>
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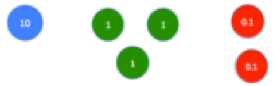
<tr>
<td>
4. Solve scaling and balance problems involving a multiplication and/or division
• correspondence problem e.g. If 12 apples weigh 168g, how much will 48 apples weigh? (note that the most efficient solution of 168 x 4 is preferred here rather than finding the weight of one apple first)
• single operation missing number problem
• $652 \times 7 = \blacksquare$
• $847 \div 7 = \blacksquare$
• $472 \times \blacksquare = 1888$
• $\blacksquare \times 6 = 2514$
• $\blacksquare \div 8 = 356$
• balancing missing number problem e.g. $243 + 316 = \blacksquare \div 7$
• form own ‘equation’ from problem and solve e.g. I am thinking of a number. When I multiply it by 3 I get the same result as when I subtract 513 from 1356. What is my number?
</td>
<td>
What’s the same and what’s different? division, finding a fraction of, scaling down
</td>
</tr>
</table>

Square Numbers and Cube Numbers Representing square numbers as square arrays (with cubes or as drawn objects). E.g. take 20 cubes – can you arrange these into a square with no gaps? What about 16 cubes? Which is a square number? Why? Experimenting with this activity to make square numbers NRICH: Picturing Square Numbers Activity Representing cube numbers as cubes (with objects e.g. cubifix). Which numbers of cubes can you arrange into a cube with no gaps?	5. Solve problems involving square and cube numbers • Recap: list the first 10 square numbers • Recap: list the first 5 (ext:10) cube numbers • Find two square numbers that add up to a given number e.g. $34 = 3^2 + 5^2$ • Find a missing number in a balance problem involving squares e.g. $61 = \Delta^2 + 5^2$ • Find two cube numbers with a given sum or difference • Relate square and cube numbers e.g. find a number that is both a square number and a cube number. How many are there less than 1000?	Show me a cube number that is also square Show me a 3-digit cube number that is also square Convince me how you would calculate15 squared? 6 cubed?
Factors, Multiples and Primes Building arrays to show all factor pairs e.g. take 24 counters and arrange as various arrays to show all the different factor pairs (what happens when the number is a square number?) (what happens when the number is a prime number?) (why can you stop trying to find arrays when you pass the half-way point or, better still, the square root?) Using arrays to show common factors (what happens when the numbers are co-prime?) Use arrays to build representations of multiples of a number (by adding an extra row each time). Practical/Visual Venn diagrams to represent common factors and multiples	6. Solve problems involving factors, multiples and primes • Recap: define factor, prime number and multiple • Recap: find the factors of a given number • Show that a given number is a factor e.g. show that 7 is a factor of 581 • Recap: find the first few multiples of a number e.g. of 16 • Find a given multiple of a number e.g. find the 8^{th} multiple of 43 • Show that a given number is a multiple of a single-digit e.g. show that 718 is a multiple of 4 • Ext: Show that a number is prime by showing it is not divisible by any prime numbers up to its square root • Solve problems involving factors/multiples and squares/cubes e.g. I am thinking of a number. It is a multiple of 3 and a square number. It is less than 100. Give all the values my number could be.	Convince me that 72 has an even number of factors
Decimals Using a hundred-grid to show why 2 tenths is the same as 20 hundredths etc Using place value counters to represent decimals (you can use unlabelled counters and given children a key) For example, here is 13.2	7. Recap: round a number to a given degree of accuracy • Nearest whole number • 1 decimal place • 2 decimal places • 3 decimal places • Nearest 10, 100, 1000, … • New: Given the rounded number, list some possible original values that could have been rounded to this	Show me a number that would be rounded 24.7 to 1 decimal place.



Using a marked number line to show the next tenth (etc) above and below to show the rounding zone of numbers that will round to this value

8. Solve problems involving 4 operations and some element of rounding
- One-step problem where trigger word for operation is clear and degree of accuracy is specified
- One-step problem where trigger word is subtle or missing for the operation but degree of accuracy is specified
- One-step problem where the degree of accuracy must be decided by the pupil
- Two-step problem with specified degree of accuracy
- Two-step problem where the degree of accuracy must be decided by the pupil
- Reverse problem where solution has been found and pupil needs to identify the (set of) possible starting numbers that could have led to that solution

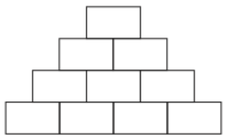
Further Extension

1.

Put the numbers 1, 2, 3 and 4 in the bottom row of this multiplication pyramid in any order you like.

What different numbers can you get on the top of the number pyramid? How can you make the largest number?

Explain your reasoning.



2.

Fill in the missing numbers:

$\square \div 120 = 117 \div 13 = 10\,800 \div \square = 234 \div \square$

3.

Sally's book is 92 pages long.

If she reads seven pages each day, how long will she take to finish her book?

4. A 50 cm piece of ribbon is cut into 6cm pieces. How many 6cm pieces would there be and how much ribbon would be left over?

Rich and Sophisticated Tasks

Solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes

NRICH: Curious Number *** P I
NRICH: Division Rules * P I
NRICH: Odd Squares * P
NRICH: Cubes Within Cubes *** P

Solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign

NRICH: Make 100 ** P I
NRICH: Multiply Multiples 1 * I
NRICH: Multiply Multiples 2 * I
NRICH: Multiply Multiples 3 * I
NRICH: Highest and Lowest * P I
NRICH: Four Goodness Sake *** P

Solve problems involving number up to 3 dp

NRICH: Route Product ** P I
NRICH: Forgot the Numbers ** I

Other Problems

1. Find the two smallest whole numbers where the difference of their

a) Fill in the blanks to represent the problem as a division:

$$\blacksquare \div \blacksquare = \blacksquare\ remainder\ \blacksquare$$

b) Fill in the blanks to represent the problem as a multiplication:

$$\blacksquare \times \blacksquare + \blacksquare = 50$$

5. I am a two digit square number. I am 17 more than the previous square number. Who am I?

6. I am a two digit multiple of 11. The product of my two digits is both a cube and a square. Who am I?

squares is a cube and the difference of their cubes is a square.

2. A 1m fence is cut into equal pieces and a piece measuring 6cm is left over.
What might the lengths of the equal parts be?
In how many different ways can the fence be cut in to equal pieces?

3. Place the digits 0 to 9 to make this calculation correct:
□□□□ – □□□ = □□□

4. Rachel is 3 years older than her sister Hannah. The sum of the squares of their ages in years is 317. How old are Rachel and Hannah?

Misconceptions

There may be confusion over the meanings of and differences between squared and cubed as well as a failure to see how these number facts (including factors and multiples) relate to multiplication and division.

Sometimes, children may struggle when a division problem has a remainder to know how to interpret this or how to represent it.

When solving problems children typically encounter the following issues:
- difficulty in pulling out the key information from any text
- difficulty deciding which calculation(s) to carry out and, where relevant, in what order (particularly where the language is not of their preferred version e.g. scaling rather than lots of)
- difficulty interpreting their answer in the context of the problem.

Children may find scaling problems challenging if they do not naturally represent these as multiplication.

Teacher Guidance and Notes

- The focus of this unit is on applying the arithmetic skills previously developed to solve a range of more complex problems.
- There is an opportunity to further embed skills of multiplication and division if required, but this may not always be necessary (hence the greyed out objectives indicating content already studied). Note that if children are not secure on these concepts and methods, you will need to address this before moving them on to the more complex skills contained in this unit. Therefore, you may choose to increase the learning hours to ensure these critical skills are mastered. See Units 2, 3 and 5 for more guidance on the greyed out sections and the prior knowledge of factors, squares and so on.
- See the AET calculation policy for more guidance on how to progress to formal methods of multiplication and division with conceptual understanding.
- It is strongly recommended that you model and encourage children to use the bar model to represent problems given and to help choose the appropriate operations. There is more guidance on the bar model at the NCETM
- Be aware that the expectation of problem solving is both within real life contexts and in mathematical contexts. Therefore you should ensure that some problems encountered are more abstract so that children are exposed to some of the technical language also. These can still be represented with the bar model but may need more unpicking. E.g. Ella is thinking of two numbers with a sum of 12. The second number is twice as large

	as the first one. What are they? • Note that the work on balancing problems and understanding the = sign is also key e.g. 14 x 5 = 85 - ? Again this can be represented by a bar and is an important precursor the algebra work of Stage 7 and above that will be needed to secure a good GCSE grade.

Key Assessment Checklist

1. I can multiply whole numbers by a 1-digit or a 2-digit number
2. I can divide whole numbers by a 1-digit number
3. I can represent and solve word problems involving multiplication and division
4. I can solve missing number problems involving the = sign and multiplication and division.
5. I can solve problems involving square and cube numbers
6. I can use my knowledge of factors and multiples to solve multiplication and division problems
7. I can work with problems involving numbers with up to 3 decimal places

Year 5	Unit 10 : Investigating Statistics
6 learning hours	In this unit children and students explore the collection, representation, analysis and interpretation of data. It covers a range of calculations of central tendency and spread as well as multiple charts and graphs to represent data. As it is the only unit directly exploring statistics, it is critical that children have time to explore the handling data cycle here and to focus sufficient time on interpreting their results.

Prior Learning	Core Learning	Learning Leads to...
➢ interpret and present discrete and continuous data using appropriate graphical methods, including bar charts and time graphs ➢ solve comparison, sum and difference problems using information presented in bar charts, pictograms, tables and other graphs	➢ **solve comparison, sum and difference problems using information presented in a line graph** ➢ **complete, read and interpret information in tables, including timetables**	➢ interpret and construct pie charts and line graphs and use these to solve problems ➢ calculate and interpret the mean as an average

Exemplification

	Bus Timetable					
Highway Rd	06:50		07:25	08:45	09:10	09:45
Rain Rd	07:00	07:25	07:41	08:55	09:19	09:53
Coldcot Rd	07:11	07:41	07:51	09:04	09:28	10:02
Westland Rd	07:18	07:59	07:59	09:11	09:38	10:11
Bod Rd	07:29	08:12	08:09	09:16	09:47	10:16
Kingswell Rd	07:33	08:15	08:14	09:20	09:53	10:21
Long Rd	07:45	08:30	08:30		10:05	10:40

1. Use the bus timetable to answer the following questions:

On the 6:50 bus how long does it take to get from Highway Rd to Westland Rd?
Can you travel to Long Rd on the 8:45 bus?
Which journey between Rain Rd and Kingswell Rd takes the longest time, the bus that leaves Rain Rd at 7:25 or the bus that leaves Rain Rd at 7:41?
Explain your reasoning.

2. Use the line graph to answer the following questions:

Vocabulary

sum	table
total	row
altogether	column
difference	heading
how many	information
more.../fewer...	timetable
line graph	start time
	end time
	duration

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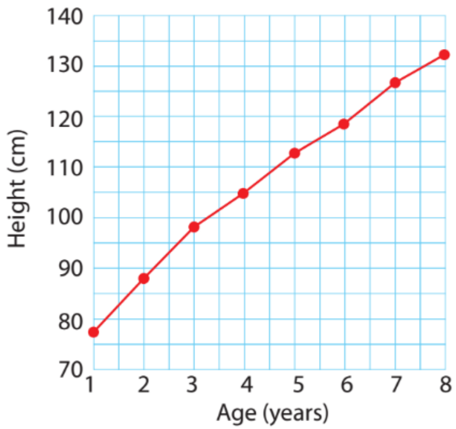
Approximately how much does the average child grow between the ages of 1 and 2?
Do they grow more between the ages of 1 and 2 or 7 and 8?

The growth of children between the ages of 1 and 8



Representation	Fluency	Probing Questions
Interpreting Timetables • Giving pupils laminated timetables that they can highlight or annotate is useful to help them identify individual journeys • Get students to explain what each row represents (stop) and what each column represents (train/bus) - this may seem trivial but is important • A fun and useful activity is to create a timetable and place 'stops' at points around your classroom. Using a clock move time forward and ask students to represent the buses moving between the stops • Relate blanks or dashes in timetables to local small stations if applicable - students should understand that faster trains don't stop at every stop	1. Timetables • List the times that a bus/train arrives at a stop • Identify what gaps in timetables mean • Calculate times between stops that don't bridge an hour • Calculate times between stops that do bridge an hour • Identify journeys that will arrive at a given stop by a certain time • Compare journey times between stops • Identify quickest journeys •	Show me a bus on this timetable that leaves before 7am Show me what time the 0815 bus gets to Crewe Show me the last train I can catch to get back home to Torquay by 1800 Convince me that this bus is quicker than the one at 1805 Always, Sometimes, Never? Timetables are read vertically

Line graphs ● Pupils need plenty of practice interpreting and reading line graphs with a variety of scales ● When solving problems such as how many more/less than a certain value, students may find it useful to draw a line across and count points above/below ● Through discussion, help students see how the gradient of a line represents the rate of change. A steep line means a greater change than a shallow line. ○ This might also be a good point to reiterate how important equal scaled increments are	2. Line graphs • Interpret value for given point • Find difference between consecutive points • Find sums/totals e.g. 'how many pieces of data were greater than 100?' • Compare differences between pairs of points	Show me how you would find the temperature in May from this graph? Show me how you would find the difference between the temperature in June and in January using this graph? Show me how you would estimate the temperature in between Sep and Oct using this graph Convince me that there are two ways to find out how many results were greater than 40 from this graph What's the same and what's different? sum, total, altogether, more, difference, how many fewer, how many more Always, Sometimes, Never? Line graphs are more useful than bar charts because they tell you values in between your data

Further Extension

	Bus Timetable					
Highway Rd	06:50		07:25	08:45	09:10	09:45
Rain Rd	07:00	07:25	07:41	08:55	09:19	09:53
Coldcot Rd	07:11	07:41	07:51	09:04	09:28	10:02
Westland Rd	07:18	07:59	07:59	09:11	09:38	10:11
Bod Rd	07:29	08:12	08:09	09:16	09:47	10:16
Kingswell Rd	07:33	08:15	08:14	09:20	09:53	10:21
Long Rd	07:45	08:30	08:30		10:05	10:40

1. Use the bus timetable to answer the following questions:

If you needed to travel from Coldcot Rd and arrive at Kingswell Rd by 8:20, which would be the best bus to catch?

Explain why.

Rich and Sophisticated Tasks

Timetables
Slow coach

Line graphs
(EXT) Graphing number patterns

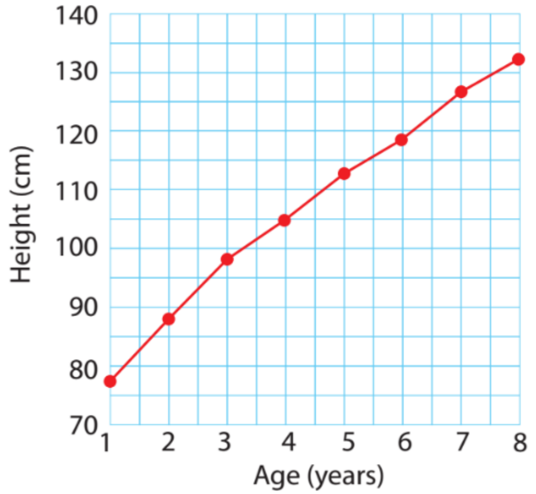
Which journey takes the longest time?

2. Use the line graph to answer the following questions:

From the graph can you predict the approximate height of an average 10 year old?
Explain how.

Consider what might be the similarities and differences between this graph and a graph of the average height of teenagers.

The growth of children between the ages of 1 and 8



Misconceptions	Teacher Guidance and Notes
Children struggle to work out which pieces of information they need to read off a graph e.g. if the question says "how many scored fewer than 11" they do not realise they need all the frequencies of the values up to 10 With timetables, children sometimes struggle to read vertically and to realise that all the times in a single column represent the same bus or train etc. Issues with 24hr clock and with time generally may also appear here. When a timetable has a blank or a dashed line to show a train or bus doesn't stop at a destination, this can confuse children.	• Give children lots of different graphs to look at with questions that progress from simply retrieving a single piece of information to those requiring the collection and addition/subtraction of multiple pieces of information. • Make use of real timetables for trains, buses and so on when looking at timetables - try to have some where a single bus has a whole column to itself and somewhere multiple trains are listed in the same column. • This is a good opportunity to revisit number bonds to 60 to find journey durations as well as to look at the 24 hour clock.

Key Assessment Checklist
1. I can find a sum or a difference to answer a question about a line graph e.g. how many pieces of data were greater than 11
2. I can make a comparison between two data points on a line graph e.g. how many more were sold on Wed than on Mon?
3. I can read a timetable to find a journey start time or end time.
4. I can read a timetable to find a journey duration
5. I can place information in the correct place in a table.
6. I can find a given piece of information from a table.

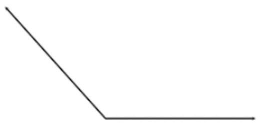
STILL IN OLD FORMAT FROM HERE ONWARDS

Year 5	Unit 11: Visualising Shape
8 learning hours	In this unit children focus on exploring shapes practically and visually. There is an emphasis on sketching, constructing and modelling to gain a deeper understanding of the properties of shapes. It is therefore necessary to secure the practical skills at the same time as using them to explore the shapes in questions. At secondary level students are developing their skills in construction and the language/notation of shape up to the understanding, use and proof of circle theorems.

Prior Learning	Core Learning	Learning Leads to...
➢ complete a simple symmetric figure with respect to a specific line of symmetry	➢ **draw given angles, and measure them in degrees (o)** ➢ **identify 3-D shapes, including cubes and other cuboids, from 2-D representations**	➢ draw 2D shapes using given dimensions and angles recognise, describe and build simple 3-D shapes, including making nets

Exemplification

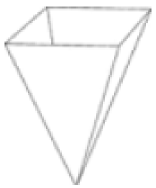
1. a) Draw an angle of 47°
b) Measure this angle to the nearest degree



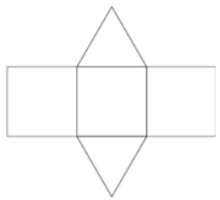
2. a) Complete the net of a cuboid:

b) Identify each of these shapes from the image

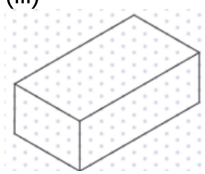
(i)



(ii) (net)



(iii)



Vocabulary

angle turn measure draw acute right angle obtuse straight line reflex degrees protractor angle measurer	2-D 3-D representation sketch image net isometric vertical horizontal parallel cube cuboid prism cross-section pyramid square based base sphere cone cylinder

<table>
<tr><th>Representation</th><th>Fluency</th></tr>
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Angles

Exploring how to use and align a protractor and an angle measurer to measure the size of an angle
Playing the angle estimation game from NRich and then estimating printed angles before checking the results by measuring to see how accurate the estimate was
Discovering how to draw a reflex angle using a 180° protractor by experimenting

3D shapes and 2D representations

Manipulating 3D shapes to view them from different orientations
Photographing the shapes in unusual orientations and producing a series of flashcards (then sorting these into categories and so on)
Sketching 3D shapes as a form of still life drawing
Exploring packaging and unfolding it to produce nets (e.g. cereal boxes, Toblerone boxes,
Making nets of shapes and testing them by cutting them out to produce the shapes
Exploring an isometric diagram already completed and then experimenting with producing own diagrams of cubes and cuboids

</td><td>

Measure an angle less than 180°

acute angle – protractor
acute angle – angle measurer
acute angle – unusual orientation
obtuse angle – protractor
obtuse angle – angle measurer
obtuse angle – unusual orientation

Measure an angle greater than 180°

reflex angle – angle measurer
reflex angle – protractor (by measuring to 180° and then and then beyond)
reflex angle – protractor (by measuring back from 360°)

Use a protractor/angle measurer to draw angles less than 180°

acute angle, unspecified location
acute angle, specified location on existing diagram
obtuse angle, unspecified location
obtuse angle, specified location on existing diagram

Use a protractor/angle measurer to draw angles greater than 180°

reflex angle, unspecified location, angle measurer
reflex angle, unspecified location, protractor
reflex angle, specified location on existing diagram

Identify 3D shapes from photographs and sketches

conventional orientations

cubes and cuboids
other prisms
pyramids
spheres, cones, cylinders

unconventional orientations

cubes and cuboids
other prisms
pyramids
spheres, cones, cylinders

Identify 3D shapes from their nets

given a net, name the shape
given a net, say whether it will ‘work’ to produce a given shape and

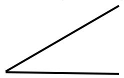
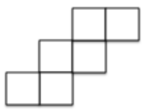
</td></tr>
</table>

identify any errors
- complete a net that has been begun for a simple shape
- produce a sketch of a net for a simple shape

7. Identify 3D shapes from isometric diagrams, stating the dimensions
   - cubes
   - cuboids
   - other prisms

8. Produce own 2D representations of 3D shapes
   - sketches
   - nets
   - isometric diagrams

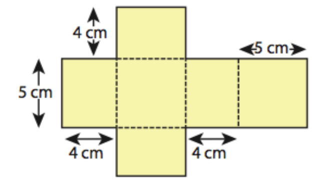
Probing Questions

Show me...	**Convince me...**	**What's the same? What's different?**	**Always, sometimes, never**
... an angle of roughly 50 degrees ... approximately 200 degrees ... approximately 300 degrees ... a right angle, ... an acute angle ... an angle larger than 140 degrees but smaller than 180 degrees ... how you line the protractor up to measure this angle ... where we are measuring from on the protractor ... the net of a cube ... a net that won't fold up to make a cube ... the net of a pentagonal prism ... a way to draw a cube ... (and another)	... that a triangle cannot have 2 obtuse angles ... that quadrilaterals have 360 degrees ... that this angle is not 140°  ... how to measure a reflex angle using a 180° protractor. ... that this is a the net of a cube  ... that this is not a correct representation of a cube 	acute, obtuse, right, reflex angle protractor, angle measurer 90, 180, 270, 360 degrees cube, cuboid, square based pyramid cylinder, triangular prism, cuboid, hexagonal prism net; sketch; isometric drawing	Triangles have 180 degrees Pyramids have triangular faces A prism has to have at least one rectangular face There are 11 possible nets for a cube.

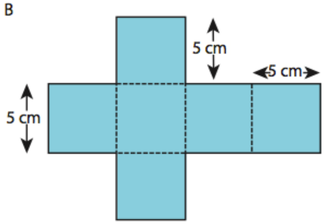
Further Extension

1.
What shapes do you make when these 2-D representations (nets) are cut out and folded up to make 3-D shapes?

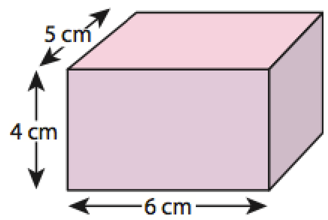
A



B



2.
Draw the 2-D representation (net) that will make this cuboid when cut out and folded up.

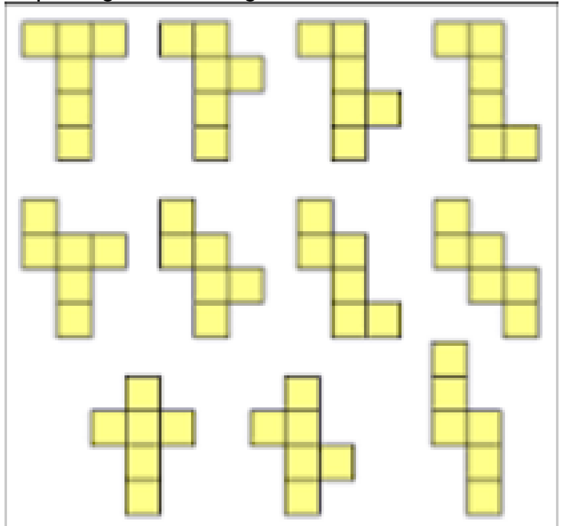


Rich and Sophisticated Tasks

Draw given angles, and measure them in degrees (°)
NRICH: **The Numbers Give the Design** *** I**
NRICH: **Six Places to Visit** *** P**
NRICH: **How Safe Are You?** *** P**
NRICH: **Olympic Turns** ***** P**

Identify 3-D shapes, including cubes and other cuboids, from 2-D representations
NRICH: **Building Blocks**
NRICH: **Cut Nets** **** P**
NRICH: **Making Cuboids** **** P I**

Exploring/discovering the 11 nets of a cube



Misconceptions

Children may still struggle with the non-metric approach of angle - they may believe that 100 degrees is a complete turn. Children therefore forget to use the basic facts of angles to help them e.g. that 90 degrees is a right angle - not using this fact to help with estimating/checking their answer.

When using a protractor some children may fail to identify 0 and thus measure the 'wrong way' round the protractor. For example, they measure an obtuse

Teacher Guidance and Notes

- Children have worked with angle estimation to the nearest ten degrees in the earlier unit, Exploring Shape.
- This unit, therefore, is focused on the specific skill in using the apparatus to measure and draw angles accurately.
- You may need to ensure that children fully understand the earlier work first. For example, they may need to 'get their eye in' in working with a scale from 0-360 degrees and develop the recall and confidence in the

angle as 50° instead of 130°. They may also fail to centre the protractor correctly on 0 or allow it to move during the measurement. There is a fundamental confusion for some children with the measurement of turn and they may intrinsically feel that angles with longer lines are bigger than those with shorter lines. Some children struggle with the duplicate language and believe that the degrees in questions are linked to temperature Many children find it very difficult to link 2D representations with the 3D shapes they represent. They may struggle to recognise shapes from pictures. They may also find it hard to imagine a net folded up solely from the 2D image. When children draw a cube on isometric paper, they may try to join dots to make a square first, and then draw horizontal and vertical lines to attempt to achieve this (rather than the diagonal lines required)	facts about half and quarter turns. • Number skills based around 360 would be a useful link to mental mathematics at this time e.g. factors of 360. • Ideally you need to expose children to both angle measurers and protractors in this work. • When sketching prisms, encourage children to start with the cross-section and then a matching (offset) cross-section that can be connected to the original with straight lines. • When working with isometric drawing, ensure children position their paper in portrait to view the isometric paper correctly. (Technically, children should not show hidden lines in an isometric drawing)

Key Assessment Checklist

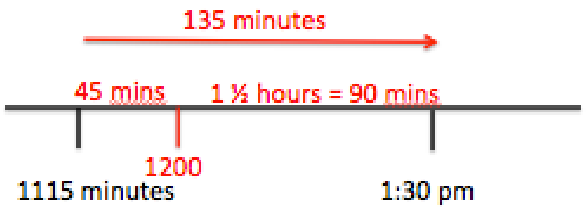
1. I can measure acute and obtuse angles in degrees using a protractor or angle measurer
2. I can measure reflex angles in degrees using a protractor or angle measurer
3. I can draw given angles using a protractor or angle measurer
4. I can identify a shape from a photograph or sketch
5. I can identify a shape from its net, specifically cubes, cuboids, prisms and pyramids; I can complete a net and say if a net works or does not.
6. I can identify a shape from an isometric diagram
7. I can produce an isometric diagram for a cube or cuboid.

Year 5	Unit 12 : Exploring Change
4 learning hours	For primary pupils this unit focuses on the measures elements of time and co-ordinates. There is a progression from sequencing and ordering through telling the time formally to solving problems involving time. The co-ordinate work flows in the secondary students' learning focused on the relationships between co-ordinates. Key objectives include the use of y=mx+c for straight lines, the use of functions and the graphing of more complex functions.

Prior Learning	Core Learning	Learning Leads to...
➢ read, write and convert time between analogue and digital 12- and 24-hour clocks ➢ solve problems involving converting from hours to minutes; minutes to seconds; years to months; weeks to days	➢ **solve problems involving converting between units of time**	

Exemplification	Vocabulary
1. a) A soldier starts an exercise at 13:00 on 24th June. The exercise finishes at 14:40 on 2nd July. Calculate the duration of the exercise in days, hours and minutes. b) How many seconds are there in 100 minutes?	convert equivalent decade century millenium millisecond arrive depart rationale prove Greenwich Mean Time British Summer Time International Date Line

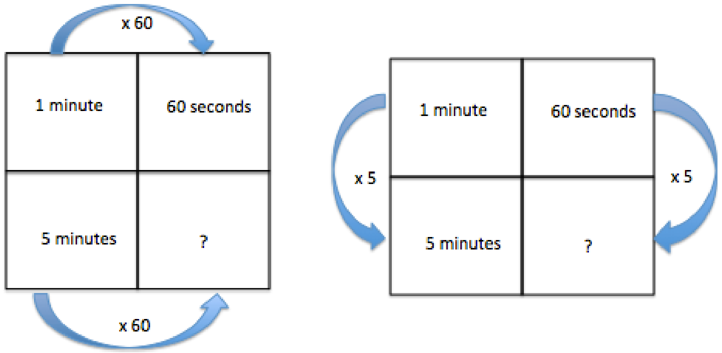
Representation	Fluency
Durations • Using a number line to find time intervals and durations For example, to find the number of minutes between 1115 and 1:30 pm	1. Recap: convert simple units of time • minutes to seconds • hours to minutes • days to hours (simple numbers) • weeks to days • years to month 2. Convert more complex units of time • decades/centuries/millennia to years • years and leap years to days



- Exploring bus or train timetables to identify durations of journeys
- Looking at TV guides to calculate durations

Converting Times

- Using 2 x 2 proportion grids to scale up and convert
  For example, to find the number of seconds in 5 minutes, either vertically or horizontally:



- years, decades etc. to months
- weeks to hours
- hours to seconds

3. Calculate the duration of an activity
   - times in same unit e.g. two 24-hour times (no bridging) e.g. 10:15 to 11:25
   - times in same unit e.g. two 24-hour times (with bridging) e.g. 10:15 to 13:08
   - times in different units (with or without bridging) e.g. 09:45 to 2:20 pm
   - times in days and hours (including dates)
   - times in days, hours and minutes (including dates)
   - more complex units e.g. years, weeks and days (including dates)

4. Solve more complex problems involving time and conversions
   - calculate and compare two durations to say which is shorter/longer
   - calculate and order three or more durations
   - use a timetable to identify start and finish times before calculating durations (including calendars, transport timetables, television guides, year planners, flight schedules)
   - problems involving leap years and/or time zone

Probing Questions

Show me...	Convince me...	What's the same? What's different?	Always, sometimes, never
... an example of something you would measure in minutes rather than hours or seconds ... and another ... an equivalent time to 900 seconds ... the number of months in a decade	... that a quarter of the day is over at 6am ... that the time in Sydney is 7pm when the time in London is 8am if Sydney is 11 hours in front of GMT ... that there are 135 minutes between 1115 and 1:30 pm.	12 years, 10 years, a decade 24 years, 100 years, a century 24 hours, 12 hours, a day 168 hours, 100 hours, a week 9:62 am and 10:02 am	... the number of Saturdays in a year is the same as the number of weeks in a year ... there will be more seconds in the same period of time than minutes

Further Extension

1. How many seconds are there in a day?
A week?
A year?!

2. Investigate the gestation period (time to grow a baby) for different animals.
Convert all the times into weeks.
Produce a timeline to show your findings.

3.
A train left London at 09:46 and arrived in Edinburgh later that day.
The clock in Edinburgh station showed this time:

How long did the train journey last?

4.
Draw a clock face, then draw the hands showing that the time is 3 p.m.

Draw a second clock face, then draw the hands showing the time 12 000 seconds later.

Rich and Sophisticated Tasks

Solve problems involving converting between units of time
NRICH: 5 On the Clock
NRICH: Watch the clock
NRICH: Two clocks
NRICH: Train timetable
NRICH: Slow coach

Misconceptions

As in Stages 3 and 4, there may be issues in working in base 12/60 when working with time for some children.

There may be confusion of am and pm, especially with noon, which should be shown as 12pm, and midnight, which should be shown as 12am. Similarly, the use of am for early morning may be an issue - some children believe that am is when it is light and pm is when it is dark.

The 24-hour clock can be problematic also. Some children find it hard to convert times because they add 10 instead of twelve e.g. they think 1pm is the same as 10 hours + 1 hour so will be 11:00 rather than 12 hours + 1 hour or 13:00.

Additionally, children may forget the 4th digit in 24-hour format writing, for

Teacher Guidance and Notes

- This unit focuses on the more complex problems involving time. It is the last direct content involving time in the national curriculum.
- The focus is on converting units and using these skills to solve problems, especially those of calculating durations and comparing/ordering time periods.
- It is recommended that you link to earlier work (Unit 10) on timetables to calculate durations or differences as well as more complex documents providing start and finish times/dates.
- As with all time work, it is recommended that you use regular opportunities in class and through cross-curricular and topic work to calculate durations and to work with different time units.

example, 2:15 instead of 02:15. When starting to work out time periods, children may revert back to addition as if they were working in base 10. Leap years can cause some confusion, particularly with the rationale. Children may also struggle to extract the key information in problems and to realise the need to work in a single unit. They may not realise that, for example, converting times to minutes for example will make the problem simpler. They may also fail to use the techniques from the calculation units when working with time - they may forget to use a bar model or to represent fraction visually to support them.	

Key Assessment Checklist

1. I can convert between units of time.
2. I can calculate durations involving two or more units (e.g. days, hours and minutes)
3. I can solve time problems using and applying my knowledge of converting of units of time.

Year 5	Unit 13: Proportional Reasoning	
4 learning hours	In this unit pupils explore proportional relationships, from the operations of multiplication and division on to the concepts of ratio, similarity, direct and inverse proportion. For primary pupils in Stages 1-3, this is focused on developing skills of division. Stages 4 and 5 revisit the whole of calculation to broaden to all four operations in a range of contexts and combination problems; the emphasis here is really on representing and then solving a problem using their calculation skills, not just calculating alone. In Stage 6 the real underpinning concepts of proportion and ratio develop. Secondary pupils begin to formalise their thinking about proportion by finding and applying scale factors, dividing quantities in a given ratio and fully investigating quantities in direct or inverse proportion, including graphically.	
Prior Learning	**Core Learning**	**Learning Leads to...**
➢ solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects	➢ **multiply and divide numbers mentally drawing upon known facts** ➢ **multiply and divide whole numbers and those involving decimals by 10, 100 and 1000** ➢ **multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers** ➢ **divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context** ➢ **solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes** ➢ **solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign** ➢ **solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates of change.**	➢ solve problems involving addition, subtraction, multiplication and division ➢ solve problems involving the relative sizes of two quantities where missing values can be found by using integer multiplication and division facts ➢ solve problems involving similar shapes where the scale factor is known or can be found ➢ solve problems involving unequal sharing and grouping using knowledge of fractions and multiples

Exemplification

1.
a) Tina had a cupboard in her bedroom on which she kept her books. There were 15 books on each of 8 shelves. A friend gave her another 24 books which she put equally onto the 8 shelves. How many books were on each shelf?

b) Sharon and Tim each had a collection of football stickers. Tim had 5 times as many as Sharon. He had 150. How many did they have altogether?

c) Find the missing number: $34 \times 15 = \blacksquare + 376$

2.
a) A tennis court is 7m wide and 24m long. A scale plan of it is drawn with a width of 3.5cm. What is its length?

b) Some children are drawing a scale drawing of their table. The drawing is scaled to one eighth of the size of the real table. The real width of the desk is 72cm. How wide should the desk be on the drawing?

c) Gordon plants 12 seeds per day.
(i) How many seeds has Gordon planted after 7 days?
(ii) After how many days has Gordon planted a total of 132 seeds?

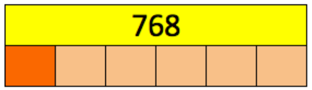
Vocabulary

addition
sum
subtraction
difference
multiplication
product
division
quotient
divisor
dividend
remainder
missing number
solve
scale up by/scale down by
rate
per
scaling
scale drawing
divisible by
remainder
inverse

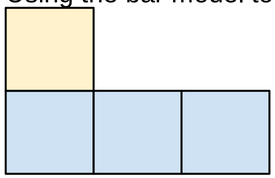
Representation

Representing problems

- Using the bar model to represent a word problem. For example, 768 shared between 6



- Using the bar model to represent and solve correspondence problems

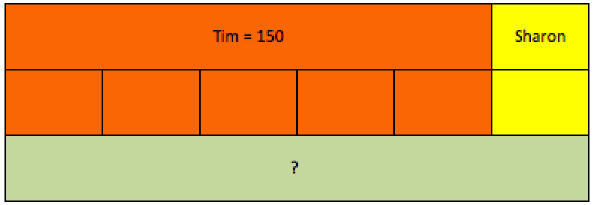


- Using the bar model to represent more complex problems
For example:
Sharon and Tim each had a collection of football stickers. Tim had 5 times

Fluency

1. Solve word problems involving a multiplication and/or division
   - word problem involving a multiplication (groups) e.g. There are 8 entrances to a football ground. Each entrance covers 3148 seats. How many seats are there in the football ground?
   - word problem involving a multiplication (scaling) e.g. The mass of a model car is 243 kg. The real car it models has a mass that is 7 times larger. What is the mass of the real car?
   - word problem involving a division e.g. a group of nine people win £4671 in a lottery. They share the money equally. How much money does each person receive?
   - word problem involving a division where remainder must be considered e.g. Emily is packing 795 bread rolls into packets. Each packet contains 6 bread rolls. How many complete packets can Emily produce?
   - word problem involving a combination of both multiplication and division e.g. 6 friends go to a concert for one friend’s birthday. The tickets cost £89 each. The other 5 friends decide to pay for the friend who’s birthday it is. How much will each of them pay if they each pay the same amount?

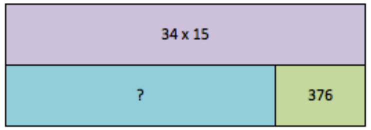
as many as Sharon. He had 150. How many did they have altogether?
can be represented by this diagram



Alternatively,

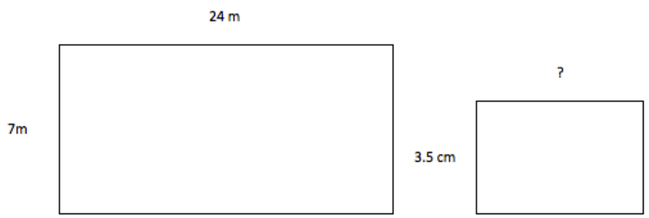
$$34 \times 15 = \blacksquare + 376$$

can be represented by



Scaling and Rates

- Producing sketches of the original and scaled item and labelling with corresponding lengths
  For example:
  A tennis court is 7m wide and 24m long. A scale plan of it is drawn with a width of 3.5cm. What is its length?
  can be represented as



- Using scaling 2x2 grids to link corresponding measurements together
  For example, for the problem above:

2. Solve balance problems involving a multiplication and/or division
   - correspondence problem e.g. If 12 apples weigh 168g, how much will 48 apples weigh? (note that the most efficient solution of 168 x 4 is preferred here rather than finding the weight of one apple first)
   - single operation missing number problem
     - $652 \times 7 = \blacksquare$
     - $847 \div 7 = \blacksquare$
     - $472 \times \blacksquare = 1888$
     - $\blacksquare \times 6 = 2514$
     - $\blacksquare \div 8 = 356$
   - balancing missing number problem e.g. $243 + 316 = \blacksquare \div 7$
   - form own 'equation' from problem and solve e.g. I am thinking of a number. When I multiply it by 3 I get the same result as when I subtract 513 from 1356. What is my number?

3. Solve scaling and problems involving rates
   - Recognise corresponding measurements on diagrams e.g. lengths on shapes or on a plan of a room
   - Scaling by an integer (larger answer)
   - Find original quantity under scaling by an integer (smaller answer – by dividing)
   - Scaling by a simple fraction e.g. ½ or ¼ or 1/3 (smaller answer – by finding fraction of amount or dividing)
   - Find original quantity under scaling by a fraction (larger answer – by multiplying)
   - Scaling problems where different units involved (e.g. m and cm)
   - Solve problems involving rates and the word 'per' (both multiplying and dividing)

4. Solve problems involving any of the four operations in a range of contexts
   - See Further Extension for examples of types of problem

	(widths)	(lengths)
(Real court)	7m	24m
(Model)	3.5 cm	?

Children can then either work horizontally to find the scale factor (not a good choice in this case as 7 is not a factor of 24) or work vertically (in this case a better choice as we just need to divide by 2 and change the units)

Example 2:
Gordon plants 12 seeds per day.
(i) How many seeds has Gordon planted after 7 days?
(ii) After how many days has Gordon planted a total of 132 seeds?

(i)

x 7

12 seeds	1 day
? seeds	7 days

x 7

or

x 12

12 seeds	1 day
? seeds	7 days

x 12

(ii)

x 11

12 seeds	1 day
132 seeds	? days

x 11

or

÷ 12

12 seeds	1 day
132 seeds	? days

÷ 12

Probing Questions			
Show me...	**Convince me...**	**What's the same? What's different?**	**Always, sometimes, never**
... the number that is 1000 times bigger than 12? 1.3? 4.02 ... the number that is 1000 times smaller than 14000, 120, 14 ... what the missing number must be: 14.3 x 100 = ☐ 2 x ☐+ 11 = 35 7 = 2 + ☐÷ 6 ... two numbers that are easy/hard to multiply ... two numbers that are easy/hard to divide ... a division with a remainder ... a division without a remainder ...how you multiply 4523 x 6 using the grid method? using partitioning? using a column method? ... how you divide 5683 ÷ 4 using place value counters? using a written method? ... the calculation you would do to find the missing numbers: 4.8 ÷ ? = 0.96 1⁄8 of ?=40 ... choose a number to put into a calculator. Add 472 (or multiply by 26, etc.). What single operation will get you back to your starting number? ... how you would represent this problem: Sharon and Tim each had a collection of football stickers. Tim had 5 times as many as Sharon. He had	. .... that 453 x 28 is the same as 453 x 20 + 453 x 8 which is the same as 400 x 28 + 50 x 28 + 3 x 28 ... that 715 x 79 cannot equal 42075 ... that I will need 8 coaches to take 375 children on a trip using coaches that seat 53 children each. ... that 3.1 x 220 = 31 x 22 = 310 x 2.2 ... that 0.05 x 32 = 0.1 x 64 = 1 x 6.4 ... that 230 ÷ 1000 = 0.23 ... that 4.5 x 1000 = 4500 ... that 72 has an even number of factors ... how you would calculate 15 squared? 6 cubed? ... if 0.4×7=2.8, then 2.4×7=16.8	1234 x 5; 123.4 x 5; 1234 x 10; 123.4 x 10 46 × 10, 460 ÷ 100, 46 × 1000 and 4600 ÷ 1000 98 ÷6, 48 ÷6, 18 ÷6, 78 ÷6 division, finding a fraction of, scaling down 125/5, 98/4, 145/9, 126/6 456x4, 312x20, 458x27, 689x50	... when you multiply you get a larger number than you started with ... when you divide a number you get a smaller number than you started with ... when you multiply a number by 100, you just add two zeroes on the end ... it is impossible to find all the multiples of 12 because there are an infinite number... ... per means divide ... numbers have an even number of factors ... a four digit number multiplied by a two number equals an eight digit number ... long multiplication is needed to multiply four digit numbers by two digit numbers ... a calculation involving division will have a remainder ... division is the inverse of multiplication

150. How many did they have altogether? ... a number sentence with add and multiply on one side of the equals sign and subtract and divid on the other (use brackets if you want)			

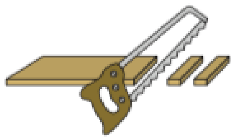
<table>
<tr><th>Further Extension</th><th>Rich and Sophisticated Tasks</th></tr>
<tr><td>
1.
Sally's book is 92 pages long.

If she reads seven pages each day, how long will she take to finish her book?

2.
A 5p coin has a thickness of 1·7 mm. Ahmed makes a tower of 5p coins worth 50p.

Write down the calculation you would use to find the height of the tower.


3.
A 50 cm length of wood is cut into 4 cm pieces.

How many 4 cm pieces are cut and how much wood is left over?

Fill in the blanks to represent the problem as division:
☐ ÷ ☐ = ☐ remainder ☐

Fill in the blanks to represent the problem as multiplication:
☐ × ☐ + ☐ = 50

4.
</td><td>
Solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign
NRICH: Make 100 ** P I
NRICH: Multiply Multiples 1 * I
NRICH: Multiply Multiples 2 * I
NRICH: Multiply Multiples 3 * I
NRICH: Highest and Lowest * P I
NRICH: Four Goodness Sake *** P
</td></tr>
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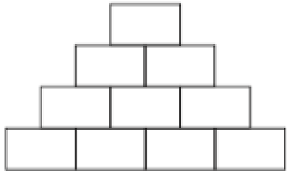
Put the numbers 1, 2, 3 and 4 in the bottom row of this multiplication pyramid in any order you like.

What different numbers can you get on the top of the number pyramid? How can you make the largest number?

Explain your reasoning.



Misconceptions

When solving problems children typically encounter the following issues:
- difficulty in pulling out the key information from any text
- difficulty deciding which calculation(s) to carry out and, where relevant, in what order (particularly where the language is not of their preferred version e.g. scaling rather than lots of
- difficutly interpreting their answer in the context of the problem.

Children may find scaling problems challenging if they do not naturally represent them as multiplication. Sometimes they simply interpret scaling as making bigger in general and do not understand the need to keep things in proportion.

Children may struggle with the idea that a rate is a division and use of the word per.

Teacher Guidance and Notes

- This unit provides an opportunity to revisit and strengthen earlier work on calculations, particularly for multiplication and division (shown in grey in objectives box)
- However, if these skills are already strong, there is no need to go through the concepts from first principles in full – rather, there can be a greater emphasis on working with solving increasingly complex problems as per the final two black objectives.
- Greater guidance on these previously covered objectives is provided in Units 3, 5 and 9 if required.
- The focus here is on solving more complex multiplication and division problems including scaling and rates. Therefore, it is expected that there will be considerable focus on the probing questions and further extension tasks for most pupils to give them access to the most challenging problems.
- Note: 4 learning hours are allocated – however, longer may be required if greyed out content is revisited.

Key Assessment Checklist

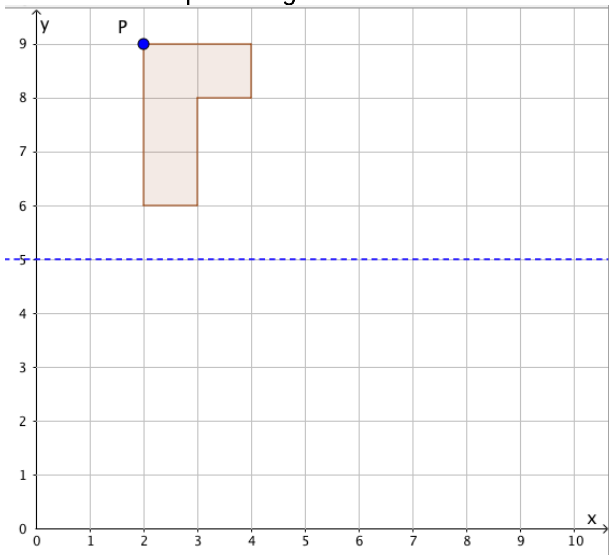
1. I can solve simple multiplication and division problems
2. I can solve balance and missing number problems involving any of the four operations
3. I can solve scaling problems and those involving rates
4. I can represent and solve complex word problems involving any combination of the four operations

Year 5	Unit 14: Describing Position
5 learning hours	In this unit pupils explore how we can communicate position and movement mathematically. They look at transformations from simple turns to reflection/rotation/enlargement/translations up to similar shapes generated by enlargements, co-ordinate systems and ultimately vectors

Prior Learning	Core Learning	Learning Leads to...
➢ describe positions on a 2-D grid as coordinates in the first quadrant ➢ describe movements between positions as translations of a given unit to the left/right and up/down ➢ plot specified points and draw sides to complete a given polygon	➢ **identify, describe and represent the position of a shape following a reflection or translation, using the appropriate language, and know that the shape has not changed**	➢ describe positions on the full coordinate grid (all four quadrants) ➢ draw and translate simple shapes on the coordinate plane, and reflect them in the axes

Exemplification

1.
Here is a L-shape on a grid.



a) Reflect the shape in the mirror line given

b) The L-shape is translated 5 units right and 2 units down. Find the new coordinates of the vertex marked P on the shape

Vocabulary

transformation
object
original
image

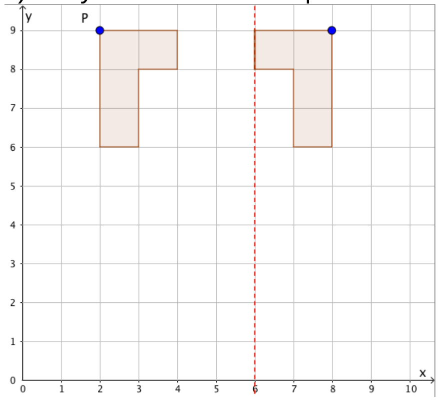
coordinate
point
vertex
axes
x-axis
y-axis
origin

reflection
mirror line
line of reflection

translation

congruent

c) Mary reflects the L-shape in the line shown. Do you agree with her answer? Explain.



Representation

Reflection

- Using mirrors to reflect the shape in the given mirror line to produce an idea of what the image will look like
- Folding paper with paint on horizontally and/or vertically along a mirror line to see where the paint ends up. Noticing that the distance to the mirror line or fold is replicated on the other side.
- Using tracing paper to draw a shape, fold along the mirror line and see where it will end up
- Using dynamic geometry software on an interactive whiteboard to predict where the image will end up OR what the mirror line was and then reveal the answer to see if they were right
- Carrying out reflections on a large grid with children as points on the shape (you can use washing line to connect them if desired). Each child tries to find their new position by standing the same distance from the mirror line on the other side.

Translation

- Carrying out translations on a large grid with children as points on the shape. Each child tries to find their new position by moving the correct number of units horizontally and vertically and the class see whether the new shape has been correctly produced. Children can also move on key person first and then stand relative to them.
- Using dynamic geometry software to carry out translations by dragging the shape to see what the new coordinates of the vertices are.

Fluency

1. Carry out and describe a translation using horizontal and vertical movements
   - translate a shape a given number of squares right or left and redraw it and give coordinates of the new vertices
   - translate a shape a given number of squares up or down and redraw it and give coordinates of the new vertices
   - translate a shape both horizontally and vertically and give coordinates of the new vertices
   - describe the translation following a horizontal movement only (using left or right)
   - describe the translation following a vertical movement only (using up or down)
   - describe the translation following a horizontal and vertical movement

2. Reflect a shape in a mirror line parallel to the one of the axes
   - reflect shape in a vertical mirror line
   - reflect shape in horizontal mirror line

3. Reflect a shape in a mirror line that touches or crosses the shape
   - vertical mirror line, touching the shape
   - vertical mirror line, crossing the shape
   - horizontal mirror line, touching the shape
   - horizontal mirror line, crossing the shape

4. Find the mirror line of a reflection
   - vertical mirror line, image and original separate

- horizontal mirror line, image and original separate
- vertical mirror line, image and original touching
- horizontal mirror line, image and original touching
- vertical mirror line, image and original crossing
- horizontal mirror line, image and original crossing

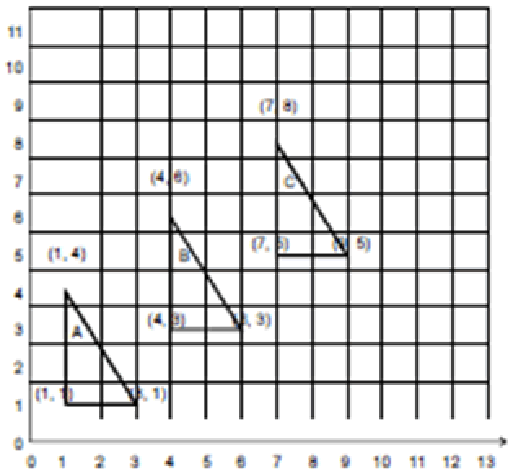
5. Understand that translation and reflections produce an image that is congruent to the original shape
   - know the meaning of congruent
   - recognise congruent shapes from a selection
   - say why two shapes are not congruent
   - know that translations produce congruent shapes
   - know that reflections produce congruent shapes

Probing Questions

Show me...	Convince me...	What's the same? What's different?	Always, sometimes, never
... where this shape will be if it reflected in this line ... if it is translated 2 units right and 1 unit up ... the line of reflection that was used to get this image ... how this shape was translated to get this image	... that the object is always the same size as the reflected image ... that the object is always the same size as the translated image ... that you can tell what the translation was from just one coordinate from the object and the image	Translate a shape and reflect the shape. Explain what is the same and what is different about the two transformed shapes translate then reflect; reflect then translate	... translated shapes will always rotate ... translated shapes must always be the same size ... translated shapes must always have the same orientation … translation takes the shape further away from the origin … mirror lines do not touch the original shape or its image … translations are easier than reflections

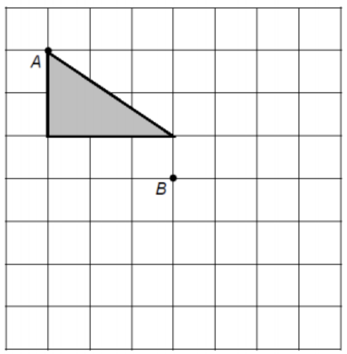
Further Extension

1.



Write the co-ordinates of the next triangle in the sequence.

2.
Translate the triangle so that point A moves to point B.



Write the translation you have done in words.

3. Emily says "if you reflect a shape in a mirror line and then reflect it back again in the same line, it ends up where it started".
Do you agree with Emily? Explain your answer?

Rich and Sophisticated Tasks

Identify, describe and represent the position of a shape following a reflection or translation, using the appropriate language, and know that the shape has not changed

NRICH: Transformations on a Pegboard * P
NRICH: More Transformations on a Pegboard ** P I

Misconceptions	Teacher Guidance and Notes
Children do not always focus on a vertex of a shape when translating shapes and may consider the difference in distance between the shapes as representing the translation, rather than the distance between corresponding points. When reflecting, children may not place the image the same distance from the line of reflection as the object. Some children think that the object and image should be an equal distance from the edge of the grid, rather than an equal distance form the mirror line. When describing the translation the pupils forget they must state the word translation and describe the translation using units left/right and up/down. They may omit the left/right and simply say 'across'. There is a tendency to reverse co-ordinates both when plotting and reading - sometimes this is because children cannot correctly identify the x-axis and the y-axis, sometimes it is due to incorrectly remembering a rule to go across first and then up. Children may not realise the importance of equal divisions between points on the axes (especially between 0 and 1) – this will be clear if they have to draw their own axes. Be aware of issues around co-ordinates on the axes themselves.	• Children have encountered coordinates and translations in Stage 4, but this is their first introduction to reflection. However, they have already seen symmetry during geometry work and so links can be made. • At Stage 5, this work should be contained within the first quadrant. Note that mirror lines will be given visually – there is no need to explore the names of these lines. All mirror lines should be parallel to either the x-axis or the y-axis. • It is generally easier for humans to see reflection when the mirror line is vertical with respect to the face. Therefore encourage children to rotate the page as necessary to help them visualise the resulting image from a reflection. • There is a conceptual progression from use of mirrors to paper folding to abstract drawing for reflection that may be helpful. • Teachers should enable pupils to discover that when translating a shape it is easier to focus on a particular vertex and then complete the image using the congruence of the image (rather than translating every point separately and then connecting them). • Children working at greater depth can begin to explore translations and reflections in combination to gain a deeper understanding of both congruence and transformations. • It is valuable to give pupils the opportunity to draw their own axes as well as providing pre-drawn axes as, whilst time-consuming, this activity may reveal issues around understanding of scale etc. • It is expected that children know and use the word 'congruent' at this stage.

Key Assessment Checklist

1. I can translate shapes horizontally and vertically
2. I can describe a translation that has been carried out
3. I can reflect shapes in a given mirror line
4. I can reflect shapes in a given mirror line where it touches or intersects the shape
5. I can describe a given reflection, showing the mirror line.
6. I can explain why reflections and translations preserve a shape and its size to produce a congruent image.