Moor Nook CP School

Year 3

Medium Term Plans

February 2021

Overview of Year

Autumn Term	**Number and Algebra**				**Geometry and Measures**	
	1. Investigating Number Systems	2. Pattern Sniffing	3. Solving Calculation Problems	4. Generalising Arithmetic	5. Exploring Shape	6. Reasoning with Measures

Spring Term	**Number and Algebra**		**Statistics**
	7. Discovering Equivalence 8. Reasoning with Fractions	9. Solving Number Problems	10. Investigating Statistics

Summer Term	**Geometry**	**Number and Algebra**		**Geometry and Measures**	
	11. Visualising Shape	12. Exploring Change	13. Proportional Reasoning	14. Describing Position	15. Measuring and Estimating

Year 3 Overview:		
Unit	**Learning Hours**	**Summary of Key Content**
1. Investigating Number Systems	8	Read and write numbers to 1000 in words and numerals; recognise place vale up to 3 digits; identify, represent and estimate numbers; compare and order numbers up to 1000.
2. Pattern Sniffing	10	Count in multiples of 4, 8, 50 and 100; find 10 or 100 more or less than a given number; recall multiplication tables for 3s, 4s and 8s. (Please refer to Moor Nook's Mental & Written Calculation Policies)
3. Solving Calculation Problems	10	Add and subtract mentally up to 3 digits plus ones, or tens or hundreds; add and subtract up to 3d using formal methods; estimate answers and use inverse operations to check a calculation. (Please refer to Moor Nook's Mental & Written Calculation Policies)
4. Generalising Arithmetic	12	Revisit mental addition and subtraction (3d + 1s or 10s or 100s) and formal methods of calculation for addition and subtraction up to 3d. Solve problems using addition and subtraction including missing number problems, number facts, place value etc. (Please refer to Moor Nook's Mental & Written Calculation Policies)
5. Exploring Shape	8	Identify horizontal and vertical lines; identify pairs of parallel or perpendicular lines; recognise angles in shapes or as turns; identify right angles; recognise multiples of right angles.
6. Reasoning with Measures	10	Add and subtract money in practical contexts to give change etc. (Please refer to Moor Nook's Mental & Written Calculation Policies) Measure perimeter of simple 2D shapes
7. Discovering Equivalence 8. Reasoning with Fractions	15	Recognise, find and write fractions of a discrete set of objects; recognise and use fractions as numbers; compare and order unit fractions; compare and order fractions with the same denominator; recognise and show equivalent fractions. Add and subtract fractions with same denominator within one whole.
9. Solving Number Problems	16	Write statements for multiplication and division using times tables, including up to 2d x 1d. (Please refer to Moor Nook's Mental & Written Calculation Policies)
10. Investigating Statistics	8	Interpret and construct simple tally charts, pictograms, block diagrams and tables; ask and answer simple questions by counting and sorting; ask and answer questions by totalling and comparing data.
11. Visualising Shape	8	Draw 2D shapes and make 3D shapes; recognise 3D shapes in various orientations and describe them.
12. Exploring Change	12	Tell and show the time to nearest minute using analogue and digital clocks, using 12 and 24 hours. Know number of seconds in a minute, number of days in each month, year and leap year. Compare durations of events.
13. Proportional Reasoning	8	Solve problems involving multiplication and division including missing numbers, scaling and correspondence problems. (Please refer to Moor Nook's Mental & Written Calculation Policies)
14. Describing Position	0	No content for Stage 3 in this strand
15. Measuring and Estimating	6	Measure, compare, add and subtract: lengths (m/cm/mm); mass (kg/g); volume/capacity (l/ml)

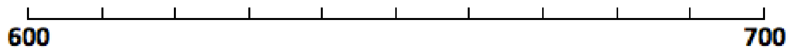
Year 3	Unit 1: Investigating Number Systems	
8 learning hours	This unit introduces the number systems and structures that we use at different levels of the curriculum. At KS1 children are working on the place value system of base 10 with the introduction of Roman Numerals as an example of an alternative system in KS2. Negative numbers and non-integers also come in at this stage and progress into KS3. At KS3 and KS4 we start to look at other ways of representing numbers, including standard form, inequality notation and so on.	
Prior Learning	**Core Learning**	**Extension Learning**
➢ read and write numbers to at least 100 in numerals and in words	➢ **read and write numbers up to 1000 in numerals and in words**	➢ read Roman numerals to 100 (I to C) and know that over time, the numeral system changed to include the concept of zero and place value
➢ recognise the place value of each digit in a two-digit number (tens, ones)	➢ **recognise the place value of each digit in a three-digit number (hundreds, tens, ones)**	➢ recognise the place value of each digit in a four-digit number (thousands, hundreds, tens, and ones)
➢ identify, represent and estimate numbers using different representations, including the number line	➢ **identify, represent and estimate numbers using different representations**	➢ identify, represent and estimate numbers using different representations
➢ use place value and number facts to solve problems	➢ **solve number problems and practical problems involving these ideas**	➢ solve number and practical problems that involve all of the above and with increasingly large positive numbers ➢ round any number to the nearest 10, 100 or 1000 ➢ round decimals with one decimal place to the nearest whole number
➢ compare and order numbers from 0 up to 100; use <, > and = signs	➢ **compare and order numbers up to 1000**	➢ order and compare numbers beyond 1000 ➢ compare numbers with the same number of decimal places up to two decimal places

Exemplification

1. a) Write this number using numerals: Eight hundred and fourteen
   b) Write this number in words: 670

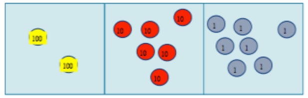
2. What is the value of the digit 7 in the number i) 475 ii) 710

3. a) Represent 630 on this number line



   b) Represent 245 using base 10

   c) State which number is shown here:



4. Here are some number cards

7	2	4	3	6

a) Choose three cards to make the smallest possible 3 digit number
b) Choose three cards to make the largest possible 3 digit number

5. Write these numbers in order from smallest to largest:
   57 570 750 75 175 751 517

Vocabulary

thousand (s)
four digit
exact
position
order
most
least
more than
greater than
less than
estimate
position
hundreds, tens, ones
estimate
position
hundreds, tens, ones

Representation

Representing Numbers

- Represent 2 digit numbers (loose and in column format) using:
  - Individual items e.g. counters, cubes
  - Individual items with some place value shown but full separation e.g. beadstrings
  - Separable to-scale tens and ones e.g. bundles of Straws, sticks of unifix
  - Inseparable to-scale tens and ones e.g. Numicon 10s and 1s, Base 10, Dienes rods
  - Not-to-scale tens and ones e.g. place value counters, money (10p and 1p coins)

Fluency

1. Convert a given representation to a number (verbal or numerals)
   - recap two digit numbers e.g. 53
   - recap two digit numbers that are multiples of 10 e.g. 80
   - three digit numbers e.g. 672
   - three digit numbers that incorporate zeroes e.g. 402 or 780

2. Convert a given number to a stated concrete or visual representation
   - recap two digit numbers
   - three digit numbers with no zero digits e.g. 456

Probing Questions

What is the same and different about these two representations?

What is the same and what is different about
1, 10, 100 and 1000?

Show me how we can represent the number 351 using
- base 10
- dienes rods
- place value counters
- other objects e.g. coins
- the number line

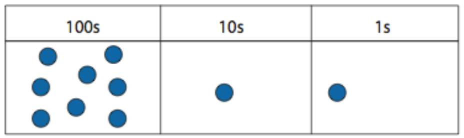
○ Overlapping place value cards ○ Numerals • Represent 3 digit numbers (loose and in column format) using: ○ To-scale inseparable hundreds, tens and ones e.g. Base 10, Dienes blocks ○ Not-to-scale hundreds, tens and ones e.g. place value counters, money (£1, 10p and 1p coins), unmarked coloured counters ○ Overlapping place value cards ○ Numerals • Exploring the idea of unitisation using double-sided counters to see how to use a different colour to represent a set amount of ones (heading towards the use of tens and ones as one of the easiest combinations) • Develop sense of size of numbers up to 1000 using paper strips and paperclips to position e.g. strip represents 0-1000, where is 234? What if the strip now represents 0-500?	○ three digit numbers that incorporate zeroes e.g. 402 or 780	Show me where 350 would be on this blank paper strip that goes from 0-1000. And now where it would be if the strip went from 0-500? 0 -400. What's the same and different? 761, 167, 176, 671, 716, 617
	3. Partition a number into hundreds, tens and ones and state the value of a given digit within a number ○ Recap two digit numbers ○ Three digit numbers ○ Reverse problem to find number from place value information ○ Partition in a non-standard way (i.e. not just H, T, U) ○ Suggest two or more different partitions of the same number	Convince me that 300 + 100 + 50 + 10 + 2 is a correct partitioning of 462 Convince me that 567 is represented by 5 hundreds, 6 tens and 7 ones in this apparatus Always, Sometimes, Never? There is one way to partition a three digit number Always, Sometimes, Never? There is a 'best way' to partition a three digit number. Convince me that 4 hundred and thirty-fourteen is worth the same as 444
Reading and Writing Numbers • Use (and make) word/numeral number cards to help convert between numerals and words	4. Convert a number written in words to numerals ○ Recap two-digit multiples of ten e.g. Eighty ○ Recap two digits e.g. Eighty-four ○ Three digits multiples of 100 e.g. four hundred ○ Three digits multiples of 10 e.g. four hundred and thirty ○ Three digits with no tens e.g. four hundred and seven ○ Three digits e.g. four hundred and thirty-seven	Show me the number six hundred and thirty-two in symbols What is the same and what is different: three hundred and five, five hundred and thirty, three hundred and fifty, fifty-three Always, Sometimes, Never? If you take three digits, there are six different three-digit numbers that you can make with them.

	5. Convert a number written in numerals to words ○ Recap two-digit multiples of ten e.g. 70 ○ Recap two digits e.g. 75 ○ Three digits multiples of 100 e.g. 700 ○ Three digits multiples of 10 e.g. 750 ○ Three digits with no tens e.g. 705 ○ Three digits e.g. 765	Show me the number 405 in words What's the same and what's different? Six hundred and seven; 670; 706; Seven hundred and sixty
	6. Recognise matching numerals, words and representations ○ Matching pairs ○ Matching three or more items ○ Matching representations without the numerals present	Show me all the different ways you can represent 416.
Comparing and Ordering • Use apparatus and then visuals and then number cards (abstract) to explore which number is greater when comparing • Use number cards to explore making different three digit numbers and finding the smallest/largest	7. Compare two numbers to say which is greater, using > or < to notate ○ Recap: two two-digit numbers ○ One three-digit number, one two-digit number ○ Two three digit numbers (unrelated) ○ Two three-digit numbers (similar digits) ○ Mixture of representations/words/numerals	Show me which of these is the greatest/the least 243, 342, 432, 234, 423, 324 Convince me that 324 is less than 342 What's the same and what's different? 654 < 765 ; 714 > 704 ; 914 < 940; 435 > 453
	8. Order numbers from smallest to largest ○ Order three numbers: ▪ Recap: (one and) two-digit numbers only ▪ Three-digit numbers (unrelated) ▪ Three-digit numbers (similar digits) ○ Order four or more numbers (as above) ○ Find a number that lies between two given numbers (2 digits, then 3 digits)	Show me a number that could complete 567 > ......; Show me a number that could complete 456 < ...... < 516 Always, Sometimes, Never? If you take a three digit number and reverse its digits, you will get a bigger number than you started with.

Further Extension

1. 674 is made of 6 hundreds, 7 tens and 4 ones.
674 is also made of 67 tens and 4 ones.
674 is also made of 6 hundreds and 74 ones.
Find different ways of expressing: 630; 704; 867

2.
Captain Conjecture says 'The number in the place value grid is the largest 3-digit number you can make using all 10 counters'.



Do you agree?

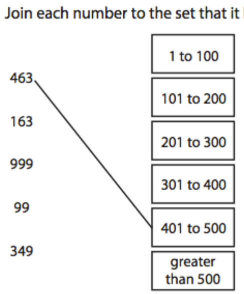
Explain your reasoning.

3.
Insert a digit into each box so that the numbers are in order from smallest to largest.

□4 6 | □3 2 | 3□1 | □6 6 | □5□

Which digits can you place in the boxes to create the largest interval between any two consecutive numbers?

4.



5. Find all the different numbers you can make from these digit cards: 3, 4 and 7

Rich and Sophisticated Tasks

Recognise the place value of each digit in a three-digit number (hundreds, tens, ones)
NRICH: Coded Hundred Square *** P**
NRICH: Which Scripts? *** P**

Solve number problems and practical problems involving these ideas
NRICH: Take Three Numbers *** I**
NRICH: Three Neighbours **** I**
NRICH: Prison Cells **** G P**
NRICH: Spot Thirteen *** G P**
NRICH: Square Subtraction ***** I**
NRICH: Planning a School Trip *** p**
NRICH: Magic Vs **** P**
NRICH: Number Differences *** G P**
NRICH: Sitting Round the Party Tables *** P**
NRICH: Dotty Six *** G**
NRICH: Nim-7 *** G**
NRICH: Number Match *** G**
NRICH: Cubes Here and There *** I**
NRICH: A Mixed-up Clock *** P**
NRICH: That Number Square! *** I**

Misconceptions	Teacher Guidance and Notes
Children sometimes write eight hundred as 8 100 Children struggle if either the tens or the units are 'missing' e.g. seven hundred and four can be miswritten as 74 or 740 Children struggle when bridging the hundreds e.g. finding ten less than 407 can lead to wrong answers such as 307 Children confuse the meaning of < and >, finding it hard to tell which is which.	When teaching place value use practical resources to expand on different base representations to emphasise the unitised structure of number ie 231 = 2 hundred squares, 3 ten rods and 1 unit/ ones in Base 10. Vary the resource used here - consider Dienes rods, (place value) counters or even coins. It is important that children develop their number sense here- they should be able to place numbers on a blank number line including where the scale changes. Try taking a blank paper strip as a scale from 0-1000 and asking children to place 200 on it. Then change the scale to 1-500 and ask them to do the same thing - they should be developing the ability to change the placement based on the scale.

Key Assessment Checklist

1. I can read and write numerals and words from 100 to 500
2. I can read and write numerals and words up to 1000.
3. I can read and write decimals with one decimal place using numerals and words.
4. I can understand place value of each digit in a 3 digit number.
5. I can partition 3-digit numbers into hundreds, tens and ones and then in different ways
6. I can position / estimate numbers up to 1000 on a number line or other representation
7. I can solve number problems using representations and known number facts.
8. I can compare two numbers up 1000 and use the signs <, > (and =) to show this comparison.

<table>
<tr><th>Year 3</th><th colspan="2">Unit 2: Pattern Sniffing</th></tr>
<tr><td>10 learning hours</td><td colspan="2">This unit explores pattern from the early stages of counting and then counting in 2s, 5s, and 10s up to the more formal study of sequences. This sequence work progresses through linear sequences up to quadratic, other polynomial and geometric for the most able older students. For children in KS1, this unit is heavily linked to the following one in terms of relating counting to reading and writing numbers.

Also in this unit children and students begin to study the properties of numbers and to hone their conjecture and justification skills as they explore odd/even numbers, factors, multiples and primes before moving onto indices and their laws.</td></tr>
<tr><th>Prior Learning</th><th>Core Learning</th><th>Extension Learning</th></tr>
<tr><td>➢ count in steps of 2, 3, and 5 from 0, and in tens from any number, forward and backward

➢ order and arrange combinations of mathematical objects in patterns and sequences
➢ recognise odd and even numbers

➢ recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables</td><td>➢ count from 0 in multiples of 4, 8, 50 and 100;
➢ find 10 or 100 more or less than a given number

➢ recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables</td><td>➢ count in multiples of 6, 7, 9, 25 and 1000
➢ find 1000 more or less than a given number

➢ recall multiplication and division facts for multiplication tables up to 12 × 12
➢ recognise and use factor pairs and commutativity in mental calculations</td></tr>
<tr><th colspan="2">Exemplification</th><th>Vocabulary</th></tr>
<tr><td colspan="2">1. a) Write the next two numbers in the number patterns:
i) 8, 16, 24, 32, 40, ..., ...,
ii) 0, 50, 100, 150, 200, 250, ..., ...,

b) Find 10 less than 74

c) Find 100 more than 456

2. Complete the missing numbers in the number sentences:

a) 3 x 8 = ...
b) 4 x .... = 36
c) 20 ÷ .... = 4
d) ...... ÷ 3 = 7</td><td>count from ...
count in ....
forwards/backwards
pattern / number pattern
10 more/less
100 more/less
tenth
missing number
times table
multiplication
division
multiple
array
row column
groups</td></tr>
</table>

<table>
<tr><th>Representation</th><th>Fluency</th><th>Probing Questions</th></tr>
<tr><td rowspan="2">Counting:
Represent counting in 4s and 8s using repeated addition with:
Numicon
Counters on a blank track
Counters in groups of 4
Bead string
Placing counter on/Colouring in 100-square
Represent counting in 50s and 100s using money
Use a counting stick to represent the first ten multiples of 4, 8, 50 and 100 – explore which values can be found by doubling.</td><td>1. Count from 0 in steps of 3, 4 and 8
work out the steps using repeated addition
work out some steps using doubling skills
count from 0 up to 10th multiple of 4 and 8 with concrete/visual aid
count from 0 up to 10th multiple of 4 and 8 without concrete/visual aid
count from 0 beyond 10th multiple of 4 and 8</td><td>What's the same and what's different?
16, 24, 32, 44

Always, Sometimes, Never?
Multiples of three have digits that add up to 3, 6 or 9

Always, Sometimes, Never?
Numbers that end in 4 or 8 are multiples of 4.</td></tr>
<tr><td>2. Count from 0 in steps of 50 and 100
work out the steps using repeated addition
work out some steps using doubling skills
count from 0 up to 10th multiple of 50 and 100 with concrete/visual aid
count from 0 up to 10th multiple of 50 /100 without concrete/visual aid
count from 0 beyond 10th multiple of 50 (and 100)</td><td>What's the same and what's different?
Counting in 50s and Counting in 100s

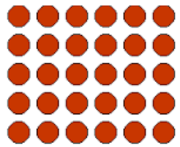
Convince me that 350 comes after 300 when counting in 50s.</td></tr>
<tr><td rowspan="2">10/100 more and less
Use a range of apparatus to count on or count back 10
Use a hundred square to discover the short-cut for adding 10 (and subtracting 10) i.e. one row difference
Represent 10 more/less using base 10/Dienes rods
Represent 10 more/less more using place value counters
Represent 100 more/less using base 10</td><td>3. Find 10 more than a number
by counting on 10 (range of apparatus: fingers, labelled number line, bead string, hundred square)
using a hundred square to jump one row down
mentally, by increasing the number of tens by one
examples beyond 100
bridging over multiples of 100 e.g. 10 more than 94</td><td>Show me 10 more than 57/97/403/999

Convince me that 267 is ten more than 257

Always, Sometimes, Never
When I find 10 more than a number, only one digit will change</td></tr>
<tr><td>4. Find 10 less than a number
by counting back 10 (range of apparatus: fingers, labelled number line, bead string, hundred square)</td><td>Show me 10 less than 43/103/1001

Always, Sometimes, Never
When I find 10 less than a number, only one digit will change</td></tr>
</table>

or Dienes • Represent 100 more/less more using place value counters	○ using a hundred square to jump up one row ○ mentally, by decreasing the number of tens by one ○ examples beyond 100 ○ bridging over multiples of 100 e.g. 10 less than 304	
	5. Find 100 more than a number ○ Using base 10 or place value counters ○ Mentally, by increasing the numbers of 100s by one ○ Examples beyond 1000 ○ Bridging over multiples of 1000 e.g. 100 more than 945	Show me 100 more than 432/709/999 Always, Sometimes, Never When I find 100 more than a number, only one digit will change
	6. Find 100 less than a number ○ 3 digits, with apparatus/visual aids (e.g. base 10 or place value counters) ○ 3 digits, mentally (by decreasing the numbers of 100s by one) ○ examples beyond 1000 ○ bridging over multiples of 1000 e.g. 100 less than 2045	Show me 100 less than 432/709/2007
Times tables • Represent a times table multiplication calculation in multiple ways: e.g. 3 x 6 as: ○ 6 groups of 3 objects ○ repeated addition using 6 numicon 3s ○ 6 lots of 3-rods (Cuisenaire) ○ **as an array** made of 6 rows of 3 counters/dots • Represent a times table division	7. Find and begin to recall times table multiplication facts (3s, 4s, 8s) ○ By representing the calculation concretely to deduce the answer ○ By representing the calculation visually to deduce the answer ○ By relating the calculation to another known calculation and counting on/back or doubling etc ○ By beginning to recall key facts	Show me a representation of the 8 times table What's the same and what's different? 4, 3, 12, 8

calculation with unknown answer: e.g. 24÷ 3 = ... as • 24 objects grouped in 3s • 24 objects grouped into an array (columns of 3) • Represent a times table division with unknown divisor in multiple ways e.g. 27 ÷ .... = 3 as ○ 27 objects shared into 3 piles ○ 27 objects grouped into an array (with rows of 3) • Represent a times table division with unknown dividend using an array e.g. .... ÷ 3 = 5 as ○ groups of 3 counters in columns until there are 5 columns altogether (i.e. 5 counters per row)	8. Find and begin to recall times table division facts (3s, 4s, 8s) ○ Unknown answer ○ Unknown divisor ○ Unknown dividend ○ Beginning to recall key facts	Convince me that 4x8 gives me the same answer as 8x4 What's the same and what's different? 3x2, 2x3, 6÷2, 6÷3
	9. Find the other related facts when given one times table multiplication or division fact: ○ Given a multiplication fact, state the equivalent multiplication fact and two related division facts ○ Given a division fact, state the equivalent division fact and two related multiplication facts	Show me the fact family for 4x3 If I know 8 x 4 = 32, then I also know …. Show me the possible calculations represented by this array 

Further Extension	Rich and Sophisticated Tasks
1. What is the relationship between these calculations? 3× 4; 4× 8; 4× 3; 8× 4 2. Complete the following: a) 83 + ... = 93 b) 104 + ... = 204 c) .... - 100 = 692 d) ...... - 10 .... = 876 3. Putting the digits 3, 4 and 8 in the empty boxes, how many different calculations can you make? [ ] x [ ] = [ ] Which one gives the largest answer? Which one gives the smallest answer?	Count from 0 in multiples of 4, 8, 50 and 100; find 10 or 100 more or less than a given number **NRICH: How Would We Count? * P** Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables **NRICH: Ordering Cards * G P** **NRICH: Music to My Ears * P I**

Misconceptions	Teacher Guidance and Notes
Pupils forget to include 0 when counting Pupils do not realise that only one digit will change when finding 10/100 more or	10 more and 10 less: begin using a 100-square to understand how 10 more is 1 row down and 10 less is 1 row up but then move towards being able to say 10 more or 10

less Pupils think that multiples of 3 will all be odd; Pupils think that numbers ending in 3 will be multiples of 3 and so on	Use the fact family concept to get children to find the associated facts e.g. for 3x4=12 you would also write 4x3=12, 12÷4=3, 12÷3=4. Use a counting stick to help children learn their times tables (multiplication and division facts) as well as to start to see how they relate to each other.

Key Assessment Checklist

1. I can count in steps of 4 and 8 from 0; I can explain how the pattern of 4s and 8s are related
2. I can count in 50s and 100s from 0; I can explain how the pattern of 50s and 100s are related.
3. I can find 10 more and 10 less than a given number
4. I can find 100 more and 100 less than a given number
5. I can count forwards and backwards in tenths, saying the whole number for every ten tenths
6. I can recall the 3 times table (multiplication and division facts)
7. I can recall the 4 and 8 times tables (multiplication and division facts)
8. I can use the 3, 4 and 8 times table facts to solve problems

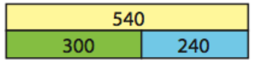
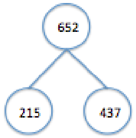
Year 3	Unit 3: Solving Calculation Problems
8 learning hours	This unit explores the concepts of addition and subtraction at KS1 building to wider arithmetic skills including multiplication at KS2. It is strongly recommended that teachers plan this unit for KS1/KS2 with direct reference to the calculation policy! At KS3 students are developing calculation into its more general sense to explore order of operations, exact calculation with surds and standard form (which have been introduced in Inv Number Systems briefly) as well developing their skills in generalising calculation to algebraic formulae. They need to substitute into these formulae and calculate in the correct order to master this strand. The formulae referenced are examples of the types of formula they will need to use, but the conceptual understanding for these formulae will be taught elsewhere in the curriculum.

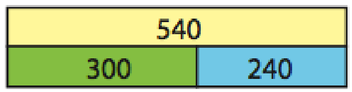
Prior Learning	Core Learning	Learning Leads to....
• recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100	• **add and subtract numbers mentally, including:** **- a three-digit number and ones** **- a three-digit number and tens** **- a three-digit number and hundreds**	
• add and subtract numbers using concrete objects, pictorial representations, and mentally, including: - a two-digit number and ones - a two-digit number and tens - two two-digit numbers - adding three one-digit numbers	• **add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction**	• add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate
	• **estimate the answer to a calculation and use inverse operations to check answers**	• estimate and use inverse operations to check answers to a calculation
• show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot		

Exemplification	Vocabulary	
1. Calculate mentally: a) 456 + 7 b) 264 – 50 c) 356 + 400 d) 342 – 6 e) 385 + 60 2. Calculate a) 267 + 518 b) 739 – 485 3. Lianne estmates the answer to 682 – 215 as 500. Do you agree with Lianne? Explain your answer	add and more make sum total altogether score double one more	how many more? take (away) leave how many left? less fewer difference between equals is the same as

	two (ten) more plus equals hundred ten one exchange column digit columnar column addition	minus number sentence order calculate column subtraction estimate inverse operation check

Representation	Fluency	Probing Questions
Addition • Representing numbers using hundreds, tens and ones equipment then combining and finding the total value (aggregation) (exchanging ten 1s for one 10 or ten 10s for one 100 as required when bridging) Progression of tens and ones equipment: ◦ **separable**: bundles of straws or sticks of multilink cubes ◦ **inseparable**: Dienes rods or Base 10 or Numicon 10s and1s ◦ **not to scale**: place value counters or money (expected at this stage) This also works for addition of a single digit, multiple of ten or multiple of 100 to a 3-digit number mentally (you will only be adding one colour of counter to the first number so making it easier to calculate the result in your head). It is a visualisation of partitioning the numbers. • Representing addition as counting or jumping on (augmentation) using: ◦ a marked number line (jumping in 100s, 10s and 1s) ◦ an unmarked number line • Representing addition problems using: ◦ the bar model	1. Add a three-digit number and ones/tens/hundreds (up to 1000) • three-digit number + 100 • three-digit number + multiple of 100 • three-digit number + one-digit number (not crossing a ten) • three-digit number + one-digit number (not crossing a ten) • three-digit number + 10 • three-digit number + multiple of 10 (not crossing a hundred) • three-digit number + multiple of 10 (crossing a hundred)	Show me ... two numbers with a sum of 220 ... two numbers with a sum of 170 ... two numbers with a sum of 500 Convince me that if I add a multiple of 100 to this number, the tens and ones digits will stay the same. Always, Sometimes, Never? Adding 5 to a number that ends in 6 will result in a number that ends in 1.
	2. Add a three-digit number and a two-digit number • No exchange required e.g. 452 + 37 • Exchange required from ones to tens e.g. 452 + 39 • Exchange required from tens to hundreds e.g. 452 + 87 • Exchange required from both ones to tens and from tens to hundred e.g. 452 + 89	Show me ... two numbers that are easy to add ... two numbers that are hard to add
	3. Add a three-digit number and a three-digit number • No exchange required e.g. 452 + 237 • Exchange required from ones to tens e.g. 452 + 239 • Exchange required from tens to hundreds e.g. 452 + 287 • Exchange required from both ones to tens and from tens to hundred e.g. 452 + 289	Always, Sometimes, Never? A three digit number add a three digit number gives a six digit number Always, Sometimes, Never? Addition makes a number larger Always, Sometimes, Never? - The sum of two odd numbers is even.

 ○ a part-part-whole model  		- The sum of three odd numbers is even.
Subtraction • Representing first number using hundreds, tens and ones equipment then removing or taking away the second number and finding the resulting value (paritioning) (exchanging one 10 for ten 1s or one 100 for ten 10s as required when bridging) Progression of tens and ones equipment: ○ **separable**: bundles of straws or sticks of multilink cubes ○ **inseparable**: Dienes rods or Base 10 or Numicon 10s and1s ○ **not to scale**: place value counters or money (expected at this stage) This also works for subtraction of a single digit, multiple of ten or multiple of 100 from a 3-digit number mentally (you will only be removing one colour of counter from the first number so making it easier to calculate the result in your head). It is a visualisation of partitioning the number. • Representing subtraction as counting or jumping back (reduction) using: ○ a marked number line (jumping in 100s, 10s and 1s) ○ an unmarked number line • Representing subtraction as a comparative difference between two sets of objects using:	4. Subtract ones/tens/hundred from a three-digit number • three-digit number - 100 • three-digit number - multiple of 100 • three-digit number - one-digit number (not crossing a ten) • three-digit number - one-digit number (not crossing a ten) • three-digit number - 10 • three-digit number - multiple of 10 (not crossing a hundred) • three-digit number - multiple of 10 (crossing a hundred)	Show me two numbers with a difference of 200 Always, Sometimes, Never? The difference of two odd numbers is odd Always, Sometimes, Never? Subtracting 8 to a number that ends in 8 will result in a multiple of 10.
	5. Subtract a two-digit number from a three-digit number • No exchange required e.g. 675 - 54 • Exchange required from tens to ones e.g. 675 - 59 • Exchange required from hundreds to tens e.g. 675 - 82 • Exchange required from both tens to ones and from hundreds to tens e.g, 675 - 89	Show me ... two numbers that are easy to subtract ... two numbers that are hard to subtract
	6. Subtract a three-digit number from a three-digit number • No exchange required e.g. 675 - 254 • Exchange required from tens to ones e.g. 675 - 359 • Exchange required from hundreds to tens e.g. 675 - 281 • Exchange required from both tens to ones and from hundreds to tens e.g, 675 - 288	Always, Sometimes, Never? A three digit number subtract a three digit number gives a double digit number Always, Sometimes, Never? Subtraction makes a number smaller

○ number lines with both numbers marked and difference found • Representing subtraction word problems using ○ the bar model ○ a part-part-whole model		
Estimation and Checking • Use place value counters or other place value equipment to represent a number and then round it to the nearest 100 (or even 10) to allow easy mental addition or subtraction. Use the bar model to represent a problem to explore inverse calculations	7. Estimate the answer to an addition or subtraction calculation • addition - numbers close to multiples of 100 e.g. 596 + 213 • subtraction - numbers close to multiples of 100 e.g. 596 - 213 • addition – by rounding to nearest 10 e.g. 449 + 219 • subtraction by rounding to nearest 10 e.g 671 - 358	What's the same and what's different? addition; subtraction Show me how you could check whether 281 + 376 = 657 using another calculation Convince me that addition and subtraction are opposites
	8. Find the inverse calculation to an addition or subtraction and use it to check an answer • give fact family for any given addition or subtraction calculation • find inverse (addition) - state checking calculation, estimate, calculate exactly • find inverse (subtraction)- state checking calculation, estimate, calculate exactly	What's the same and what's different? 285; 367; 652; 285 + 367; 367 + 285; 652 – 285; 652 – 367; 367 – 285 Show me the other calculations that you know the answer to if I tell you that 348 + 417 = 765 Show me the four number facts that this bar model shows 

Further Extension

1.

What do you notice?
Is there a relationship between the calculations?

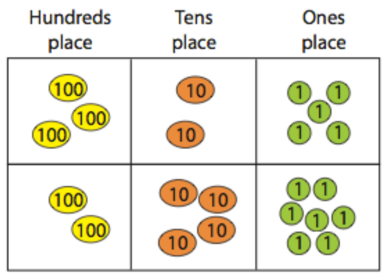
500 + 400 =	523 + 400 =	523 + 28 =
400 + 500 =	423 + 500 =	423 + 28 =
300 + 600 =	323 + 600 =	323 + 28 =
200 + 700 =	223 + 700 =	223 + 28 =
100 + 800 =	123 + 800 =	123 + 48 =

2.

Complete these calculations. What do you notice?

3 + 7 =	8 + 2 =	6 + 4 =
30 + 70 =	80 + 20 =	60 + 40 =
33 + 7 =	88 + 2 =	66 + 4 =
333 + 7 =	888 + 2 =	666 + 4 =
300 + 700 =	800 + 200 =	600 + 400 =

3.



$$\begin{array}{r} 325 \\ +\,247 \\ \hline \\ \hline \end{array}$$

Sam has completed these calculations, but he is incorrect.
Explain the errors he has made.

$$\begin{array}{r} 325 \\ +\,247 \\ \hline 581 \\ \hline \end{array} \qquad \begin{array}{r} 355 \\ -\,247 \\ \hline 112 \\ \hline \end{array}$$

Rich and Sophisticated Tasks

Add and subtract numbers mentally, including:

- a three-digit number and ones
- a three-digit number and tens
- a three-digit number and hundreds

NRICH: How Do You See it? * P
NRICH: Swimming Pool* P
NRICH: First Connect Three * G P
NRICH: A Bit of a Dicey Problem *** P
NRICH: Totality * G

4.

There are six 3-digit addition calculations shown below.

a) 124 + 233
b) 644 + 172
c) 366 + 277
d) 579 + 221
e) 791 + 163
f) 567 + 233

Which calculations have no carry digits?
Which calculations have a carrying digit only once?
Which calculations have a carrying digit twice?
Which calculation has the largest answer?
Which calculation has the smallest answer?

Check that children are looking at the numbers involved, rather than doing the calculations.

5.

☐ ☐ ☐ + ☐ ☐ ☐ =

Throw a 1 to 6 dice and each time record the digit in one of the place holders.
The aim is to get the sum as low as possible. Repeat to find different answers.
Could you have done it in a different way?
Compete against a friend and compare your answers.

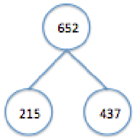
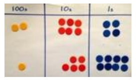
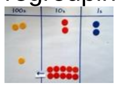
Misconceptions	Teacher Guidance and Notes
Children struggle to interpret whether to add or subtract from the language used. Children can find 'How many more/less?' particularly troublesome as it relates to ordinal values of numbers and relationships. Children struggle to add two and three digit numbers when their place value understanding is weak. If they do not read a number like '352' as 3 hundreds, 5 tens and 2 ones then they struggle to combine the ones, tens and the hundreds from two numbers appropriately. When adding/subtracting 1s, 10s or 100s mentally, children may 'change' the digit in the wrong column.	• The aim of this unit for these children is to develop security in the process of addition and subtraction and with this more fluid use of the formal methods. Simultaneously they should be developing efficiency of mental methods when appropriate. Therefore, encourage children to look at the numbers in a calculation before commencing to decide if they can do it in their head, with jottings or whether they need to use a written method. • As in Stage 2, it is crucial that children see the column addition and subtraction methods as short-hand for the practical process of addition/subtraction with objects. Therefore the representation suggestions above relating to the use of place value objects are crucial precursors to the teaching of column methods.

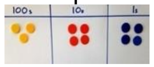
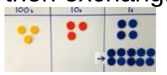
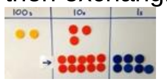
When performing columnar addition, children may forget to include the tens or hundreds they have generated from earlier exchanges. They may also fail to exchange them at all and thus end with a two-digit numbers in the 1s column. When subtracting, children will sometimes subtract the larger number from the smaller initially. When performing columnar subtraction, children may exchange from the wrong column or fail to exchange altogether (instead just finding the difference between the digits in the column, even where the second one is greater than the first). Children may also fail to correctly record the exchange and thus not reduce the tens, for example, by one so that the answer is 10 too high. Children often do not see difference as a representation of subtraction because take away is emphasised so much. They need to see subtraction represented in this way also to challenge this. When working with addition and subtraction facts, children sometime realise there is a connection e.g. 3+4=7 but then incorrectly rearrange this to make a false second fact e.g. 4 + 7 = 3. This is particularly true with subtraction facts, where children struggle to place the numbers in a correct order. The equals sign is not always correctly interpreted as 'has the same value as' by children, who may see it as 'the answer is'. Some children may use the incorrect operation when checking and fail to realise that they need to use the inverse - this is more pronounced when subtracting.	• At this level you should aim to use place value counters with children but if you need to, go back to objects where the value of the numbers is more obvious e.g. dienes or numicon. • Ensure children are going through the full exchange process when adding or subtracting i.e. picking up 10 one counters and swapping them for a ten counter or vice versa. They should then 'regroup' and ensure that the tens and ones are in the right columns to be combined. • To begin to embed the written routines of the calculation policy, it is advised that children work in pairs with one child manipulating the equipment and saying what they are doing aloud while the other child records the calculation using the column method so that they learn that the column method is just a written representation of the practical process (rather than a 'different' method) - see the videos at the NCETM for examples of this. https://www.ncetm.org.uk/resources/40532 To help with setting out calculations in columns use large squared paper or laminated grids and mini-WB pens. • The pitch of this unit is numbers up to 1000, but of course these ideas an be extended beyond 1000 for those children who are confident working with in this area. • Children need to see and use a variety of question types during this work including: oral questions 'two hundred and thirty-four add six hundred and forty-eight', written questions using symbols '471 –234', simple 'real' problems such as shopping and abstract problems such as finding related subtraction facts to an addition fact. • Try to model the wide range of language used to signify addition and subtraction – see vocabulary list above. The children ultimately need to be able to recognise that a problem is an addition problem from the language (and same for subtraction). • Use 'sum' only to mean an addition calculation – use the word 'calculations' to mean mixed operation computations • Challenge issues with the use of the = sign by looking at examples where the question is on the right e.g. ? = 514 + 288 as well as balance problems in Further Extension e.g. 143 + 614 = ? + 271 • Language is critical in this learning process - make sure you use and insist on the correct terminology for place value e.g. 123+456 would involve twenty add fifty, not two add five. Also insist on children describing their steps orally e.g. I need to add seven ones and 5 ones which makes twelve ones. So I will exchange 10 of these ones for a ten and regroup (put the ten in the right column).

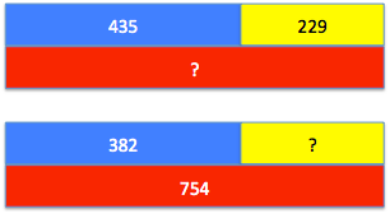
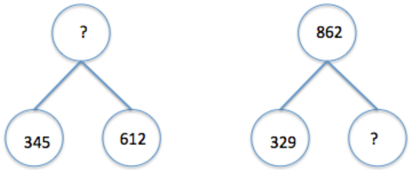
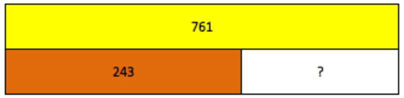
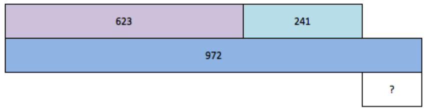
Key Assessment Checklist
1. I can add and subtract ones to and from a three digit number mentally
2. I can add and subtract tens to and from a three digit number mentally
3. I can add and subtract hundreds to and from a three digit number mentally
4. I can add two three digit numbers using a columnar method
5. I can subtract two three digit numbers using a columnar method
6. I can estimate the answer to a three digit + three digit calculation
7. I can estimate the answer to a three digit + three digit or a three digit - three digit calculation
8. I can use the inverse operation to check the answers to addition and subtraction calculations

Year 3	Unit 4 : Generalising Arithmetic	
12 learning hours	This unit is focused on developing fluency in the manipulation of number. At primary level this is focused on arithmetic itself and the methods for four operations particularly; however, this is naturally generalised to thinking about rules of arithmetic more widely at secondary level i.e. algebra. These aspects have been paired together intentionally to help teachers describe algebra as simply a generalisation of number. It is expected that teachers will go back to arithmetic to help students see where the 'rules' of algebra come from. Note that the greyed out content is covered previously and hence is not required content here unless of concern.	
Prior Learning	**Core Learning**	**Learning Leads to...**
➢ recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100 ➢ add and subtract numbers using concrete objects, pictorial representations, and mentally, including: - a two-digit number and ones - a two-digit number and tens - two two-digit numbers - adding three one-digit numbers	➢ **add and subtract numbers mentally, including:** **- a three-digit number and ones** **- a three-digit number and tens** **- a three-digit number and hundreds** ➢ **add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction**	➢ add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate
➢ Solve problems with addition and subtraction: • using concrete objects and pictorial representations, including those involving numbers, quantities and measures • applying their increasing knowledge of mental and written methods ➢ recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems. ➢ show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot	➢ **solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction**	➢ solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why

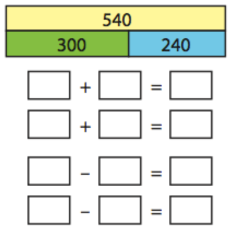
Exemplification	Vocabulary	
1. a) There are 453 people in a cinema. 152 of them are children. How many are adults? b) Find the value of the missing number: $423 + \blacksquare = 635 + 181$	ones tens hundreds mental method formal method column addition add sum of total altogether plus and more than	subtract take away minus leave fewer less than difference column subtraction fact family inverse operation check equal

Representation	Fluency	Probing Questions
Adding and Subtracting Mentally • Using a number line to jump in 100s/10s and 1s or using bonds • Partitioning and recombining numbers (using a part-part whole model) 	1. Recap: Add and subtract 2-digit and 3-digit numbers mentally • adding/subtracting single digits • adding/subtracting multiples of 10 • adding/subtracting multiples of 100 • adding/subtracting near multiples e.g. 99 or 19 • adding/subtracting combinations of 1s, 10s and 100s using number line for jottings • adding/subtracting combinations of 1s, 10s and 100s using mental partitioning (no exchange) • adding/subtracting combinations of 1s, 10s and 100s using mental partitioning (examples with an exchange)	What's the same and what's different? 234 + 100; 235 + 99; 236 + 98; 244 + 90 Convince me that 542 + 100 - 1 gives the same answer as 542 + 99
Adding 3-digit numbers • Using place value equipment to represent column method via exchanging  regrouping, leading to  and the answer of 	2. Recap: Add two 3-digit numbers together using a formal method • 3-digit add 2-digit (no exchange) • 3-digit add 3-digit (no exchange) • 3-digit add 2-digit (one exchange, ones to tens) • 3-digit add 3-digit (one exchange, ones to tens) • 3-digit add 2-digit (one exchange, tens to hundred) • 3-digit add 3-digit (one exchange, tens to hundred) • 3-digit add 2-digit (two exchanges) • 3-digit add 3-digit (two exchanges)	Show me how you can add 567 + 678 - using Dienes rods or base 10 - using place value counters - using column method
	3. Recognise, represent and solve a simple addition problem (3-digit numbers)	

	• simple one-step word problem with addition trigger word and given structure for representation (e.g. blank bar model or blank part-part-whole) • simple one-step word problem with addition trigger word (without scaffolded structure) • one-step addition word problem with subtle reference to addition • two-step problems requiring two additions	
Subtracting 3-digit numbers • Using place value equipment to represent column method via exchanging Example: 344 - 187  then exchanging a 10 for ten 1s  then removing 7  then exchanging a 100 for ten 10s  then removing 80  then removing 100 	4. Recap: Subtract a 3-digit number (or a 2-digit number) from a 3-digit number using a formal method • 3-digit subtract 2-digit (no exchange) • 3-digit subtract 3-digit (no exchange) • 3-digit subtract 2-digit (one exchange, tens to ones) • 3-digit subtract 3-digit (one exchange, tens to ones) • 3-digit subtract 2-digit (one exchange, hundreds to tens) • 3-digit subtract 3-digit (one exchange, hundreds to tens) • 3-digit subtract 2-digit (two exchanges) • 3-digit subtract 3-digit (two exchanges)	What's the same and what's different? 564 - 213; 563 - 212; 562 - 211; 554 - 203
	5. Recognise, represent and solve a simple subtraction problem (3-digit numbers) • simple one-step word problem with subtraction trigger word and given structure for representation (e.g. blank bar model or blank part-part-whole) • simple one-step word problem with subtraction trigger word (without scaffolded structure) • one-step subtraction word problem with subtle reference to subtraction • two-step problems requiring two subtractions	Convince me that order matters when you are subtracting
Recognising and Solving Addition and Subtraction • Using bar models to represent problems and to decide whether to add or subtract	6. Recognise, represent and solve two-step problems combining addition and subtraction (3-digits) • simple problems where operation is clear (e.g. people going into and out of a stadium) and structure for representation provided (e.g. blank bar model or part-part-whole model) • simple problems where operation is clear (e.g. people going into and out of a stadium) without a structure for representation provided • more complex examples involving subtle reference to addition	What's the same and different? Total; sum; difference; minus; less; fewer; altogether

 • Using part-part-whole models to represent problems and to decide whether to add or subtract  • Exploring these processes in familiar contexts e.g. shopping (total cost, change, difference between prices, baskets of equal value – what is the missing value?)	and subtraction	
	7. Find the fact family (four equivalent calculations) for a given calculation e.g. 543 + 239	Convince me that 123 + 456 gives the same answer as 156 + 423
	8. Check a calculation using inverse operations • addition calculation • subtraction calculation • two-step calculation	What's the same and what's different? addition; subtraction Convince me that 491 – 274 does not equal 223 (90 - 70 - 20, 4 - 1 =3)
Missing number problems • Using a bar model to represent the problem and show the missing number • Example 1: $761 = 243 + ■$  • Example 2: $623 + 241 = 972 - ■$ 	9. Solve an missing number addition problem using a subtraction • second number missing e.g. $323 + ■ = 545$ • first number missing e.g. $■ + 239 = 765$ • answer given first, second number missing e.g. $761 = 243 + ■$ • answer given first, first number missing e.g. $343 = ■ + 129$	Convince me that I can find the value of ■ in $■ + 239 = 765$ by calculating $765 - 239$
	10. Solve a missing number subtraction problem using an addition or a subtraction • first number missing e.g. $■ - 439 = 225$ • second number missing e.g. $863 - ■ = 345$ • answer given first, first number missing e.g. $443 = ■ - 229$ • answer given first, second number missing e.g. $661 = 883 - ■$	Always, Sometimes, Never? I can find the value of a missing number in a subtraction statement by adding.
	11. Recognise, represent and solve a missing number problem • addition problems e.g. Martha has some pennies in a jar. Her aunt gives her 218 more pennies, so now she has 442. How many did she start with? • subtraction problems (smaller amount missing) e.g. At the school fayre, the cake stall has 520 cakes to sell. 438 of the cakes are sold. How many cakes did they have left? • subtraction problems (larger amount missing) e.g. There are	Show me two numbers with a sum of 20 and a difference of 12

	624 cars in a car park. 128 of the cars are black. How many are not black? • mixture of addition/subtraction problems where children select appropriate operation • 2-step problems e.g. There are 254 children in Meadow School. There are 318 children in Acorn School. The two schools go on a trip together but 132 children cannot come. How many children go on the trip?	
	12. Solve combination missing number problems • abstract, all addition e.g. $623+241=972-\blacksquare$ • abstract, all subtraction e.g. $571-\blacksquare=452-239$ • abstract, one of each operation e.g. $434+\blacksquare=619+151$ • worded problems (and hence requiring representation) e.g. An aeroplane has 315 passengers on it. An extra 136 passengers get on. The plane now has the same number of passengers as the plane that left yesterday, which contained 281 adults. How many passengers were children on the plane yesterday? • problems where only a digit of a number is missing e.g. $852-4\blacksquare3=369$	Always, Sometimes, Never? Two numbers will have the same sum if you increase one by 7 and decrease the other by 7 Always, Sometimes, Never? Two numbers will have the same difference if you increase one by 7 and decrease the other by 7

Further Extension	Rich and Sophisticated Tasks
1. Write the four number facts that this bar model shows.  2. Flo and Jim are answering a problem: Danny has read 62 pages of the class book, Jack has read 43. How many more pages has Danny read than Jack? Flo does the calculation 62 + 43. Jim does the calculation 62–43. Who is correct? Explain how you know.	Solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction **NRICH: Buying a Balloon * P** **NRICH: GOT IT ** G** **NRICH: Make 37 ** P** **NRICH: Consecutive Numbers ** P I** **NRICH: Super Shapes * P** **NRICH: Strike it Out * G** **NRICH: Dice in a Corner *** P I** **NRICH: Domino Square ** P** **NRICH: Dicey Addition * G** **NRICH: 4 Dom *** P** **NRICH: Finding Fifteen ** P**

3. Use the digits 0-9 once only to complete these statements to make them true

4	1	□		□	8	2		□	2	□		4	□	9	
2	□	8	+	3	6	4	-	2	7	3	+	□	5	8	-
□	4	3		4	1	□		7	□	2		2	8	1	

Misconceptions

Children struggle to add and subtract from right to left in columns and hence may end up with answers that are not partitioned into hundreds, tens and ones.

Children may place the smallest number at the top of the calculation when using column subtraction method.

When numbers exchanges happen, children may forget to notate them and hence not include the extra/fewer tens, hundreds etc. in the new calculations.

Children find it hard to identify the command word in a question to decide whether to add or subtract.

When attempting missing number problems e.g. 245 + ? = 300 children may give the answer 65 as they have counted up in tens then ones

Teacher Guidance and Notes

- This unit is a chance to apply the concepts of Unit 3 to a wider range of problems and hence develop greater confidence in the methods as well as recognising situations in which they should be used.
- The pitch is numbers up to 1000.
- It is strongly recommended that you represent word and other problems using a bar model strategy as this will help children to make the key decisions about which operation to use.
- Be aware of whether a question requires a mental or a written method and encourage children to consider this before launching into a column method. If a mental method is used, children may like to use a number line or partitioning jottings to help them.
- You can refer to the Calculation Policy document for further guidance on the progression of addition and subtraction from stage to stage.
- For those children who still require apparatus to support their calculations, try to encourage them to notate a formal method alongside the practical manipulation.
- Try to model the wide range of language used to signify addition and subtraction – see vocabulary list above. The children ultimately need to be able to recognise that a problem is an addition problem from the language (and same for subtraction).
- Use ‘sum’ only to mean an addition calculation – use the word ‘calculations’ to mean mixed operation computations
- Challenge issues with the use of the = sign by looking at examples where the question is on the right e.g. ? = 514 + 288 as well as balance problems e.g. 143 + 614 = ? + 271
- Language is critical in this learning process - make sure you use and insist on the correct terminology for place value e.g. 123+456 would involve twenty add fifty, not two add five. Also insist on children describing their steps orally e.g. I need to add seven ones and 5 ones which makes twelve ones. So I will exchange 10 of these ones for a ten and regroup (put the ten in the right column).

Key Assessment Checklist

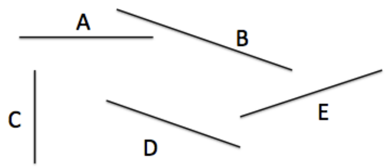
1. I can add and subtract numbers with up to 3 digits mentally
2. I can add three digit numbers using a formal written method
3. I can subtract three digit numbers using a formal written method
4. I can recognise and solve an addition word problem
5. I can recognise and solve a subtraction word problem
6. I can recognise and solve two-step addition and subtraction problems
7. I can solve a missing number problem
8. I can solve a balanced missing number problem

Year 3	Unit 5: Exploring Shape
8 learning hours	In this unit children and students explore the properties of shapes, both 2D and 3D. At KS1 this is focused on common shape names and basic features of vertices, sides etc. but this then develops to classifying quadrilaterals and triangles in KS2. Alongside this focus children begin to explore angle and turn in KS2 and develop this to more formal angle rules through Stages 5, 6, 7, 8. Older students begin to explore the field of trigonometry, encountering first Pythagoras' Theorem, then RA-triangle trig before finally looking a the sine rule and cosine rule.

Prior Learning	Core Learning	Learning Leads to...
➢ identify and describe the properties of 2-D shapes, including the number of sides and line symmetry in a vertical line ➢ identify and describe the properties of 3-D shapes, including the number of edges, vertices and faces ➢ compare and sort common 2-D and 3-D shapes and everyday objects	➢ **recognise angles as a property of shape or a description of a turn** ➢ **identify right angles, recognise that two right angles make a half-turn, three make three quarters of a turn and four a complete turn; identify whether angles are greater than or less than a right angle** ➢ **identify horizontal and vertical lines and pairs of perpendicular and parallel lines**	➢ identify lines of symmetry in 2-D shapes presented in different orientations ➢ identify acute and obtuse angles and compare and order angles up to two right angles by size ➢ compare and classify geometric shapes, including quadrilaterals and triangles, based on their properties and sizes

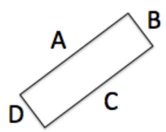
Exemplification

1. a) Here are some lines



(i) Identify a vertical line (ii) Identify a pair of parallel lines (iii) Identify a pair of perpendicular lines

b) Here is a rectangle



(i) which sides are parallel? (ii) which sides are perpendicular to side B?

2. a) Draw a triangle that contains a right angle
b) What is special about the angles of a rectangle?
c) Draw quadrilateral with four different angles

Vocabulary

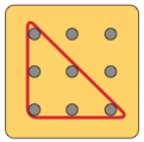
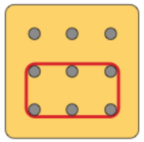
angle turn movement full turn complete whole half turn quarter turn three quarter turn direction left/right clockwise anticlockwise right angle shape vertex	greater than less than close to position horizontal line vertical line parallel lines pair perpendicular lines at right angles distance

3. a) Draw an angle that is less than a right angle b) Jamie is facing the door. He turns through four right angles. Which way is Jamie facing now?		

Representation	Fluency	Probing Questions
Understanding Angle • Using arms (or geostrips) to produce an angle and link this movement to increasing (or decreasing) the turn or the angle • Using geostrips (or just paper strips with a paper fastener attaching them) to produce angles and to move from a zero (or tiny) angle where the strips overlap to a bigger and bigger angle as one of the strips rotates away	1. Understand what an angle is • describe a turn as an angle • identify the angles in shapes • compare two (practical) turns to say which is greater • compare two (practical) angles to say which is greater • compare two angles (as images) to say which is greater • produce a right-angle using practical equipment	Show me an angle that is greater than this one  Always, Sometimes, Never A hexagon has 6 angles
Finding Right Angles • Cutting up shapes to separate the angles out • Using a right-angle finder to help search for right-angles in the everyday environment • Identifying right angles in art and paintings: a good starting point for suitable images is the The Art of Mathematics article from the NCETM	2. Identify right angles • say whether an angle is a right angle or not • if not, say whether an angle is less than a right angle or more than a right angle • recognise right angles in shapes and diagrams • recognise right angles in every day objects • notate a right-angle on a diagram using a right angle sign	Convince me that this shape has no right angles Convince me that that this shape has more than 6 right angles
Turning Right Angles • Turning through quarter turns, half turns, three quarter turns, full turns and turns close to these (i.e. just less than a right angle, just more than a half turn or 2 right angles). • Exploring how turning through 2 right angles, for example, has the same effect as turning through a half turn (and linking this to fractions later where appropriate i.e. 2 quarters make 1 half) • Using clocks to carry out quarter turns with the minute hand and associate this to right-angles as shown between the hands	3. Describe turns as a number of right angles • recognise a right angle as a quarter turn • recognise a half turn as equivalent to turning two right angles • recognise a half turn as equivalent to turning two right angles (in either direction) • recognise a three quarter turn as equivalent to turning three right angles • begin to recognise a three quarter turn as equivalent to turning a right angle in the other direction • recognise a full turn as equivalent to turning four right angles • carry out turns to objects and images when they are described in quarter turns or right angles	Show me a quarter turn clockwise/to the right Convince me that this is a quarter turn to the left What's the same and what's different? right, left, clockwise, anticlockwise

Making Right Angles • Making right angles out of sticks (or geostrips). Use the NRich activity [Right Angle Challenge] as a stimulus and a good virtual version	4. Draw right angles • horizontal and vertical to page oe • in different orientations • draw angles that are less than a right angle • draw angles that are more than a right angle • extension: draw angles that are more than two right angles etc.	Convince me that this angle is more than a right angle
Horizontal and Vertical Lines • Forming lines using sticks, metre rules, children (!) etc to fit a certain description • Playing Line Kung Fu for horizontal and vertical lines [one hand]	5. Recognise horizontal and vertical lines • vertical lines from a set a lines • horizontal lines from a set of lines • vertical and horizontal lines in a diagram or shape (i.e. the lines are connected) • draw vertical and horizontal lines • know that a right angle with a horizontal (or vertical) line will produce a vertical (or horizontal) line	Show me ... a vertical line ... a horizontal line in this picture What's the same and what's different? horizontal and vertical
Parallel and Perpendicular Lines • Measuring the distance between lines in shapes to check if they are parallel • Exploring art, designs and paintings to find parallel and perpendicular lines, for example, **Composition VII** by Kandinsky • Forming lines using sticks, metre rules, children (!) etc to fit a certain description • Playing Line Kung Fu for parallel and perpendicular lines [two hands]	6. Recognise parallel and perpendicular lines: • define parallel and perpendicular lines • parallel lines from a set of a lines • perpendicular lines from a set of lines • parallel and perpendicular lines in a diagram or shape (i.e. the lines are connected) • notate parallel lines on a diagram using arrows • begin to describe the angle that perpendicular lines meet at as a right-angle	Show me ... some parallel lines ... two lines or faces that are perpendicular to each other ... a shape which has both parallel lines and perpendicular lines Convince me that parallel lines can be curved What's the same and what's different? parallel and perpendicular Always, Sometimes, Never? 2 lines can be parallel and perpendicular at the same time Always, Sometimes, Never? Perpendicular lines are horizontal and vertical
Drawing Shapes • Using Beebots (and/or logo) to program an object to make turns (and to travel lengths)	7. Draw shapes with right angles • squares and rectangles • other quadrilaterals	Show me ... a shape with a right angle … exactly one right angle

to create a design or shape. Exploring what happens when it turns more than one right angle. • Using a geoboard to create shapes with different types of angles. The NRich task More Transformations on a Pegboard gives some good guidance on this.	• a triangle • a pentagon • larger polygons (include other features/rules e.g. a line of symmetry)	… no right angle Convince me that a triangle cannot have two parallel sides. Always, Sometimes, Never? There are no shapes with exactly one right angle

Further Extension	Rich and Sophisticated Tasks
1. Can you draw a triangle with: ▪ 1 right angle? ▪ 2 right angles? Can you draw a quadrilateral with: ▪ 1 right angle? ▪ 2 right angles? ▪ 5 right angles? ▪ No right angle? If some of these are impossible, can you explain why? 2. How many different triangles can you find on a 3x3 pin geoboard? How do you decide that they are different?  How many different quadrilaterals can you find on a 3x3 pin geoboard? How do you decide that they are different?  3. Can you find any right angles in 3D shapes? Which shapes have right angles? Can you find any parallel or perpendicular edges? Where?	Identify right angles, recognise that two right angles make a half-turn, three make three quarters of a turn and four a complete turn; identify whether angles are greater than or less than a right angle **NRICH: Square It * GAME** **NRICH: How Safe Are You?** Identify horizontal and vertical lines and pairs of perpendicular and parallel lines **NRICH: National Flags * P**

Misconceptions	Teacher Guidance and Notes
The largest misconception here is in what an angle actually is. Many children relate the distance between the lines or the length of the lines making up the angle to its size, rather than seeing it as a measure of turn from the starting line. For this reason they often say that two equal angles are not equal because one set of lines are longer than the others. Weak understanding of halves, quarters and three quarters can limit reasoning about turn. Children do not always appreciate that the direction of a turn can vary and so needs to be specified. Until they are secure with clockwise and anticlockwise some confusion may persist. Children may interpret a 'right' angle as a right hand turn and hence have their own concept of a ‘left turn’! They also believe that right angles are always flat and struggle to see them in 3D space. When a right angle is oriented off the horizontal/vertical, children may not recognise it as such. They often infer that the definition of a right angle involves one line being parallel with the bottom of the page. Children sometimes confuse horizontal and vertical directions and lines. Children sometimes think that parallel lines move away from each other/towards each other and can be confused by images showing perspective e.g. railway tracks into the distance. The definition of parallel lines as 'not meeting' may not be as helpful as the more formal 'lines that are always the same distance apart' for these children. Children sometimes believe that perpendicular lines must touch, when they simply need to be oriented at right angles to each other.	• In this Stage we are exploring the concept of angles only, rather than trying to place a numerical value on the size of an angle. Degrees as a unit of measure for angles are not introduced until Stage 5. • Right angle has its origins in the latin word for upright, hence describing the angle made between the upright walls/columns/pillars and the floor in buildings. • It is really essential to try to embed an understanding of what an angle actually is (which is difficult!). As a measure, it is utterly different to anything children have come across so far and they tend to be so consumed with measuring lengths that they cannot easily translate the idea of a different type of measure. • Practical exploration of angle as a turn of themselves, of objects etc relative to a starting line from a point is key. • Try to avoid using 'corner' interchangeably with angle or turn as they are different concepts - a corner is a point in space whereas an angle is a turn through space. • When working with turns, use directions such as 'to the right' or 'to the left' initially but be prepared to use words like clockwise and anticlockwise. Any children who can tell the time may already use this vocabulary themselves and it is more reliable as it is not dependent on where you are relative to others. • There is a significant amount of terminology in this unit and so time needs to be invested in recalling and using these words to enable pupils to access the concepts. • Explore the meaning of horizontal and vertical and provide real life examples. Link the origin of the word horizontal to the horizon. • Consider the order of learning perpendicular and right angles as children need to understand right angles in order to fully understand perpendicular. • Think carefully about how you define parallel lines - the more popular 'lines that never meet' may be more problematic than the more technical ‘lines that are always a fixed distance apart'.

Key Assessment Checklist

1. I can explain what an angle is and identify the angles in a shape
2. I can recognise a right angle
3. I can use angles to describe a turn.

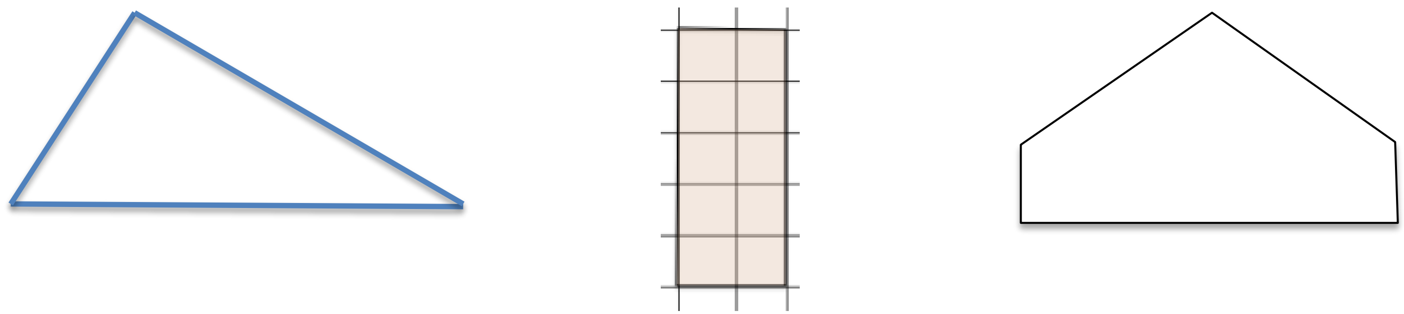
4. I can recognise right angles within a half, three quarter and complete turn
5. I can identify whether angles are greater than or less than a right angle.
6. I can identify horizontal and vertical lines.
7. I can identify pairs of perpendicular lines.
8. I can identify pairs of parallel lines.

Year 3	Unit 6 : Reasoning with Measures
10 learning hours	This unit focuses on mensuration and particularly the concepts of perimeter, area and volume. Primary children are also working on money concepts at this stage, while older secondary students develop mensuration into volume and surface area of challenging shapes, applying Pythagoras' Theorem and trigonometry also in combination with these problems. Note the focus on reasoning within this unit: it is common for children to complete routine problems involving mensuration but this unit is about the developing a secure conceptual understanding of these ideas that they can apply to a wide range of problems and contexts. The opportunity to use and build on earlier number work is built into this unit and it is expected that children apply their arithmetic skills, for example, in these problems.

Prior Learning	Core Learning	Learning Leads to...
	➢ **measure the perimeter of simple 2-D shapes**	➢ measure and calculate the perimeter of a rectilinear figure (including squares) in centimetres and metres ➢ find the area of rectilinear shapes by counting squares
➢ recognise and use symbols for pounds (£) and pence (p); combine amounts to make a particular value ➢ find different combinations of coins that equal the same amounts of money ➢ solve simple problems in a practical context involving addition and subtraction of money of the same unit, including giving change	➢ **add and subtract amounts of money to give change, using both £ and p in practical contexts**	➢ estimate, compare and calculate different measures, including money in pounds and pence

Exemplification

1. Find the perimeter of these shapes:

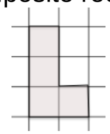
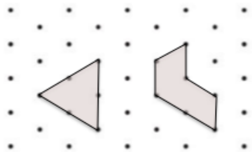


2. a) Ali buys a toy costing £2.70. He pays with a £10 note. Draw coins to show how much change should he get.

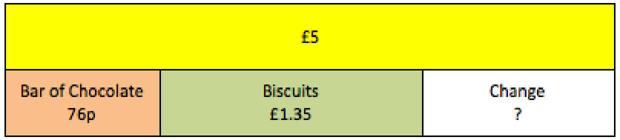
Vocabulary

measure	amount
estimate	total
perimeter	price
centimetre	balance
cm	buy
draw	cost
length	change
width	difference
height	pound, £
less	pence, p
rectangle	
ruler	
shape	
side	

b) Bella wants to buy an apple for 35p, a banana for 28p and a pen for 59p. She has £1.20. Does she have enough money to buy everything?	size square straight line unit	

Representation	Fluency	Probing Questions
Perimeter • Finding the perimeter of shapes with each side the same length (e.g. rectilinear shapes drawn on squared-paper or isometric shapes drawn on dotty paper) • Drawing over each square edge of a shape shown on a squared grid to count the number of square edges to find the perimeter. (Could do this using an acetate overlay) • Measuring the side lengths of a shape using rulers, tape measures, trundle wheels, metre rules etc. • Walking round the outside of a shape and counting aloud (or chanting the lengths before summing them) to find the perimeter.	1. Find the perimeter of a shape (not to scale) with all sides the same length by counting: • square on squared paper • rectangle on squared paper • composite rectilinear shape on squared paper e.g.  • shapes drawn on isometric paper e.g. 	Show me all the shapes with perimeter 12 on squared paper.
	2. Find the perimeter of a shape • find the perimeter of a shape given lengths of its sides (diagram) • find the perimeter of a shape given lengths of its sides (no diagram) • word problems e.g. perimeter of playground, field	Convince me that a 6cm by 6cm square has the same perimeter as a 7cm by 5cm rectangle What's the same and different? 8x2 rectangle; 5x5 square; 6 x 4 rectangle
	3. Measure the length of all sides of a shape accurately • measure the length of a horizontal line (whole number of cm) • measure the length of a vertical line (whole number of cm) • measure the length of a diagonal line (whole number of cm) • measure each length of a rectangle (whole number	Show me how you could accurately measure the size of an A4 sheet Show me how a 7cm by 5cm rectangle from a piece of card

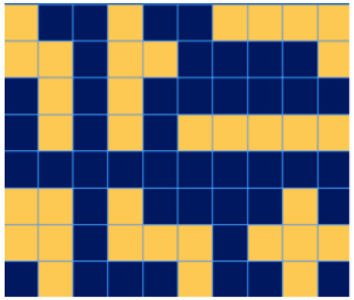
	of cm) • measure each length of a shape e.g. triangle, parallelogram (whole number of cm)	
	4. Find the perimeter by measuring and summing • find the perimeter of a shape where sides need to be measured, (whole number of cm) • find (estimate) the perimeter of a shape with curved sides using string and laying along a ruler	Show me how you can accurately measure the perimeter of an A4 sheet What's the same and different? A 10cm by 7cm rectangle; An L-shape made by cutting a 3cm square from one corner of a 10x7 rectangle If you cut a corner out of a rectangle, the perimeter of the new shape is the same as the old one.
	5. Draw/find a shape with a given perimeter • on squared paper - any shape • on squared paper – specified shape, e.g. rectangle • on isometric paper- any shape • on isometric paper- specified shape e.g. triangle • ext: on plain paper – any shape • ext: on plain paper – specified shape e.g. rectangle (difficult!)	Show me a shape/rectangle with a perimeter of 20cm Show me all the rectangles with a perimeter of 24cm (whole number sides) If I make a triangle by halving a rectangle, the triangle perimeter is half of the rectangle perimeter
Money • Using coins and notes to form prices concretely and add and subtract (using same approaches as place value counters). • Exploring catalogues, take-away menus et cetera and finding the total cost of a meal or a birthday wish-list. Using a budget to check the total falls within it. Checking whether a meal-deal is good value and how much you can save by using it. • Using the bar model to represent word problems regarding money, to help decide which operations to	6. Recognise and use all British coins/notes to solve problems • identify value given coin/note • identify coin/note given value • make a given amount in coins/notes • say which amount is greater e.g. £2 or 150p • identify the total value of a selection of coins/notes • find multiple ways to make a given amount • identify the smallest number of coins to make an amount • make two separate amounts in coins/notes and find the total	Show me all the ways I can make 48p using coins Convince me that I only need 5 coins to produce £4.57

carry out For example, "Mike spent 76p on a bar of chocolate and £1.35 on a packet of biscuits. If he had a £5 note, how much change will he need?" could be represented using this bar model  • Exploring combinations of coins that produce the same amounts. E.g. Jim needs to pay £2.32 How could he pay using the least number of coins?	7. Add amounts of money in £ and p • add two or more items in whole pounds • add two items in pence (not bridging 100) • add two items in pence (bridging 100) • add three or more items in pence • add two items in £, not bridging e.g. £1.35 and £1.21 • add two items in £, with bridging e.g. £1.35 and £1.82 • add three or more items in £ e.g. £1.35 and £2.50 and £1.89 • add items in mixed units e.g. 89p and £1.65	Always, Sometimes, Never? To find the total of ? items costing 99p, work out ? pounds - ? pence.
	8. Subtract an amount of money from another in £ and p • subtract an amount in whole pounds from another • subtract an amount in pence from another (<100) • subtract an amount in pence from another (>100) • subtract two items in £, not bridging e.g. £3.85 - £1.52 • subtract two items in £, with bridging e.g. £3.85 – £2.39 • subtract an item in pence from one in £ e.g. £2.50 – 65p or £4 – 65p	Show me how much change I would get from a £5 note for a bill costing £1.43
	9. Solve problems involving addition and subtraction of money • Find change given total cost and money paid • Find change given individual item costs and money paid • Say which total cost is greater of two sets of items • Say how much more an item costs than another item • Say if we have enough money for a set of items given budget • Say how much more money is required if short/how much money is left if over	Convince me that you can check your change using an addition Show me five different ways that you could give change for a purchase of £7.60 using a £10 note What's the same and different? addition; subtraction; total; change

Further Extension

1. Draw 4 different shapes with a perimeter of 20cm

2. Draw as many shapes as you can made of 5 squares on a piece of squared paper. Find the perimeter of your shapes.



3.

£2·60 + ☐ = £5·00

If I buy a sandwich for £2·20 and a drink for 90p, how much change do I get from £5?

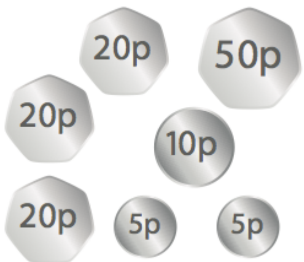
Ellie buys 2 pencils. She pays with a £2 coin and gets 70p change.
How much did each pencil cost?

4.

Sophie and Ravi have saved some money. Altogether they have saved £35. Sophie has saved £4 more than Ravi. How much have they each saved?

Sam and Tom share this money equally.
Divide the coins into two equal groups.
Could three friends share the money equally?

Explain your reasoning.



Rich and Sophisticated Tasks

Measure the perimeter of simple 2-D shapes
NRICH: Smaller and Smaller

Add and subtract amounts of money to give change, using both £ and p in practical contexts
NRICH: How Much Did It Cost? ** P

Misconceptions	Teacher Guidance and Notes
When measuring perimeter of a shape shown on a squared grid/dotty paper, some children may count border squares of a shape (ie including corners) or dots (on dotty paper) rather than counting the actual distance; this can be a failure to comprehend perimeter or a process failure. Many children may not measure from 0, instead beginning at 1 or the end of the ruler. Measuring can also be impaired by a lack of fine motor skills; inability to keep pencil hard against ruler etc. Children may ignore units when making comparisons and say, for example, that 5p > £2. Weak arithmetic skills can cause issues within this unit as fluency with number and calculation is needed to apply the measures knowledge.	• This unit covers two key (distinct) concepts linking to measures: perimeter and use of money to solve problems. • In this stage we are introducing the concept of perimeter. This is a chance for children to both develop their measuring skills and their addition of more than two numbers. • Spend time ensuring that children really understand the idea of perimeter and that they see shapes drawn on different grids as well as those on plain paper that require measuring. The focus is not so much here on problem solving with perimeter as this comes in Stage 4. • Note that perimeter is defined as the total distance around a shape. This can be obtained by measuring each side and adding but also by 'adding on the ruler' (eg for a 4 x 3 rectangle, the first side is measured as 4, the second side is measured from 4 to 7, then 7 to 11 then 11 to 14). Both should be experienced to develop understanding. • If possible try also to make time to look at perimeter in a practical context using alternative measuring equipment e.g. trundle wheels and tape measures. • The work on measurement can begin with counting the cm along the edge of a shape. The ruler speeds things up by allowing an answer without counting, but the two methods need to run side by side long enough to establish the link. In the event of difficulty, go back to counting with the ruler alongside. • The work on money builds on earlier ideas of addition and subtraction (See calculation policy or units 3 and 5 for guidance on this issue) as well as familiarity with the coins/notes of the British system gained in previous stages. The focus here is on solving problems involving addition and subtraction of amounts.

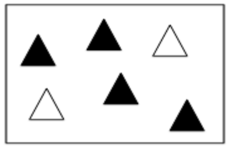
Key Assessment Checklist

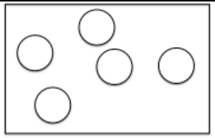
1. I can find the perimeter of a shape drawn on cm squared paper by counting or by counting and adding
2. I can measure and draw straight lines in whole numbers of cm using a ruler and apply it to practical problems, such as cutting out a rectangle of a given size
3. I can measure the perimeter of simple 2D shapes by measuring the sides and adding their lengths
4. I can solve simple perimeter problems, such as total length of fence needed for a field.
5. I can add amounts of money using £ and p.

6. I can subtract an amount of money from another using £ and p
7. I can compare different amounts of money, saying which is greater than or less than.
8. I can use £ and p in practical contexts such as finding change, checking if there is enough money etc.

Year 3	Unit 7 and 8: Discovering Equivalence & Reasoning with Fractions
15 learning hours	This unit is a combination of two units that are separated in older year groups to allow teachers extra time to master the concepts. The unit explores the concepts of fractions(decimals and percentages) as ways of representing non-whole quantities and proportions. For the youngest children, the work is focused on fractions and developing security in recognising and naming them. At KS2 this then builds to looking at families of fractions and decimals and percentages. At secondary level this is extended to more complex percentage work and equivalence with recurring decimals and surds.

Prior Learning	Core Learning	Learning Leads to...
➢ recognise, find, name and write fractions 1/3, 1/4, 2/4 and 3/4 of a length, shape, set of objects or quantity ➢ write simple fractions for example, 1/2 of 6 = 3 and recognise the equivalence of 2/4 and ½ ➢ identify, represent and estimate numbers using different representations, including the number line	➢ **recognise, find and write fractions of a discrete set of objects: unit fractions and non-unit fractions with small denominators** ➢ **recognise and use fractions as numbers: unit fractions and non-unit fractions with small denominators** ➢ **count up and down in tenths; recognise that tenths arise from dividing an object into 10 equal parts and in dividing one-digit numbers or quantities by 10** ➢ **compare and order unit fractions, and fractions with the same denominators** ➢ **add and subtract fractions with the same denominator within one whole [for example, 5/7 + 1/7 = 6/7]** ➢ **recognise and show, using diagrams, equivalent fractions with small denominators** ➢ **identify, represent and estimate numbers using different representations**	➢ solve problems involving increasingly harder fractions to calculate quantities, and fractions to divide quantities, including non-unit fractions where the answer is a whole number ➢ add and subtract fractions with the same denominator ➢ recognise and show, using diagrams, families of common equivalent fractions ➢ recognise and write decimal equivalents of any number of tenths or hundredths recognise and write decimal equivalents to 1/4,1/2, 3/4

Exemplification	Vocabulary	
1. a) What fraction of these triangles are shaded black?  b) Colour in $\frac{1}{5}$ of these circles	fraction part whole denominator numerator proportion out of	half third quarter fifth sixth seventh eighth ninth

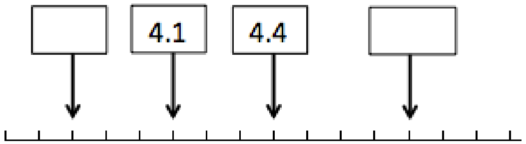


c) Find $\frac{1}{3}$ of 21

2. a) Draw an arrow on this number line to show the position of $\frac{1}{4}$

b) What fraction is the arrow pointing to on this number line?

3.
Look at this number line, which is marked in tenths.
Complete the missing numbers by counting up and down in tenths.



4. Use the >, < or = sign to fill in the empty boxes to make a true statement

a) $\frac{1}{3}$ ☐ $\frac{1}{4}$ b) $\frac{3}{5}$ ☐ $\frac{1}{5}$

5. Calculate
a) $\frac{2}{5}+\frac{1}{5}$
b) $\frac{6}{7}-\frac{3}{7}$

sharing
groups
shared between

unit fraction
non-unit fraction

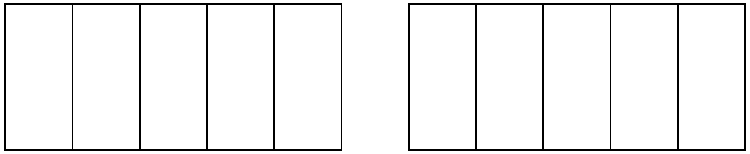
represent
bead string
number line
position
bar
array

tenth
of

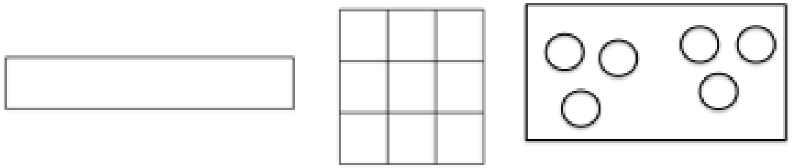
compare
greater than
less than
order

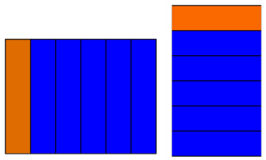
equivalent

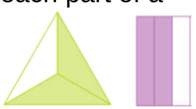
6. Explain why $\frac{1}{5} = \frac{2}{10}$. You can use these diagrams to help you if you wish.



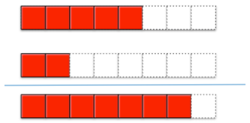
7. Show $\frac{1}{3}$ on each of these diagrams:



Representation	Fluency	Probing Questions
Fractions of a whole/shape • Folding (and colouring) paper circles to represent a unit (and then non-unit) fraction to compare two or more fractions (and hence order them). • Folding (and colouring) paper strips to represent a unit (and then non-unit) fractions to compare two or more fractions (and hence order them) • Colouring other shapes divided into parts of equal size to show fractions • Representing fractions using the bar model (vertically and horizontally) e.g. $1/6$  • Producing own fraction wall or fraction-fan to help identify equivalent fractions	1. Represent unit fractions (of an object/shape) • half • quarter • eighth • tenth • fifth • third • sixth • ninth • seventh	Convince me that $\frac{1}{4}$ is less than $\frac{1}{3}$ Convince me that finding $\frac{1}{6}$ is the same as dividing by 6 Convince me that there are at least 10 different ways to represent $\frac{1}{6}$
	2. Compare and order unit fractions • say which of two fractions is greater using given representations of the same whole • say which of two fractions is greater by producing representations of the same whole • order three or more unit fractions • realise that unit fractions can be ordered by reverse ordering their denominators	Convince me that $\frac{1}{3} > \frac{1}{5}$ What's the same and what's different? $1/2, 1/3, 1/4, 1/5$ Always, Sometimes, Never? Unit fractions cannot be greater than $1/2$
	3. Represent non-unit fractions (<1) (of an object/shape or of	Show me how you can represent $\frac{3}{4}$ in

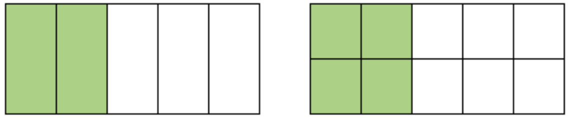
	a set of objects) • quarters • eighths • tenths, fifths • thirds, sixths, ninths • sevenths	as many different ways as possible What's the same and what's different? $1/5, 2/5, 3/5, 4/5$ True or false? $3/3$ is the same as one whole
	4. Compare and order non-unit fractions with the same denominator (<1) • say which of two fractions is greater using given representations of the same whole • say which of two fractions is greater by producing representations of the same whole • order three or more fractions with the same denominator • realise that fractions with the same denominator can be ordered by ordering their numerators	Convince me that $\frac{2}{10}$ is worth the same as $\frac{1}{5}$ What's the same and what's different? $1/3, 2/6, 1/2, 3/6$ Always, Sometimes, Never? You can order fractions with the same denominator by just putting the numerators in order Always, Sometimes, Never? The larger the denominator the bigger the fraction
Finding Fractions of an Amount/of a set of objects • Counting objects to find, for example, the fraction of set of cubes that are green (total number of objects = denominator) • Choosing objects so that a given fraction have a property e.g. $\frac{3}{8}$ of the counters are red • Laying out objects equally onto each part of a representation of a fraction e.g.  and counting the number of objects in the shaded area. • Folding a paper strip up into, for example, fifths and laying out/drawing objects onto the strip • Arranging objects into groups of size = denominator and counting the objects in the first n	5. Find unit fractions of a discrete number of objects • by sharing objects out into n groups and counting the number of objects in each group (where n is the denominator) • by forming an array to represent the objects visually • by dividing the number of objects directly by 2, 4, 8, 10 and so on	Show me $\frac{1}{3}$ of 21 Show me how you could use an array to find $\frac{1}{5}$ of 30 Show me how you could use a bar model to find $\frac{1}{5}$ of 30
	6. Find non-unit fractions of a discrete number of objects • by sharing objects out into n groups and counting the number of objects in m of these groups (where the fraction is $\frac{m}{n}$) • by forming an array to represent the objects visually	Show me how you could use an array to find $\frac{2}{3}$ of 24 Show me how you find $\frac{4}{5}$ of 35 counters

groups, where n is the numerator • Arranging objects into an array (with rows of size = denominator) and counting the objects in the first n columns, where n is the numerator. • The array representation is a useful way of showing both unit fractions but then their family of fractions too - get children to think about the 'fifth' family of 30, for example, by arranging 30 counters into an array with 5 rows (they should count out 5 counters each time before starting a new column). Then explore how the array 'shows' the value of 1/5 of 30 as well as 2/5 and so on.	(counting the number of objects in m columns of an array of width n, where the fraction is $\frac{m}{n}$) • by dividing the number of objects by 2, 4, 8, 10 and so on before multiplying by the number of parts (numerator)	
	7. Solve problems involving finding fractions of amounts (sets of objects) • word problems involving finding a fraction of whole • problems involving finding two fractions of a whole and describing the total or the remainder • reverse problems – given the fractional amount, find the whole	Convince me that $\frac{1}{4}$ of 8 pound coins is more than $\frac{1}{5}$ of 10 x 50p
Fractions as Numbers • Using a beadstring to count out the denominator and break it down into n groups, where n is the numerator. For example, finding 1/5 of 20 • Using a paper strip and a paper clip to represent the position of fractions on the number line Start by giving the strip a value e.g. the strip represents a length of 12 and ask children to show where 1/2 would be, 1/4 and so on. They will initially do this by finding 1/2 of 12 and so but will eventually begin to see that the fraction itself determines the position. You can then move on to a number line more formally. • Positioning fractions on a number line (washing line) from 0 to 1 by using pegs to divide the washing line up into equal pieces. Linking the number line to the bar model from 0-1. • Using a counting stick to count up and down in fractions (including tenths as decimals or fractions). Beginning to recognise where the wholes appear in this process.	8. Position fractions as points on a number line • unit fractions • non-unit fractions • compare two fractions to say which is greater	Show me if this strip represents 24, where $\frac{1}{2}$ would be? $\frac{1}{4}, \frac{1}{3}, \frac{1}{6}$. Convince me that a unit fraction sits on the number line between 0 and 1 Always, Sometimes, Never? A fraction is a number AND a proportion of a shape Always, Sometimes, Never? Fractions will always sit below 1 on the number line
Adding and Subtracting Fractions • Using the bar model to represent the individual fractions and then combine Example:	9. Add and subtract fractions with the same denominator • add two unit fractions with the same denominator e.g. $\frac{1}{3} + \frac{1}{3}$ • add two proper fractions with the same denominator	Show me how you can add $\frac{2}{8}$ to $\frac{3}{8}$ Show me how you can subtract $\frac{2}{6}$ from $\frac{5}{6}$

$\frac{5}{8}+\frac{2}{8}$ 	e.g. $\frac{2}{9}+\frac{3}{9}$ • subtract a unit fraction from a proper fraction with the same denominator e.g. $\frac{6}{7}-\frac{1}{7}$ • subtract two proper fractions with the same denominator e.g. $\frac{6}{7}-\frac{4}{7}$	Convince me that $\frac{3}{5}+\frac{2}{5}=\frac{5}{5}$ or 1 whole
	10. Solve problems involving adding and subtracting fractions • word problems - addition • word problems – subtraction • word problems – combinations • missing number problems (using inverse operations)	Always, Sometimes, Never? When adding or subtracting fractions you need to add both the denominator and the numerator
Representing Decimals • Using a counting stick to count up and down in tenths as decimals • Using a paper strip and a paper clip to represent the position of decimals on the number line • Linking tenths as 0.1s to tenths as sections of a bar model split into 10 equal sections	11. Represent and read decimals up to 1 decimal place • count in tenths (up and down) within one whole • read a decimal <0 with 1 decimal place as a number of tenths • interpret a diagram showing tenths as a decimal or fraction • write a decimal<0 with one decimal place as a fraction with denominator 10 • write a fraction with denominator 10 as a decimal • know that 3 tenths, for example, comes from splitting 3 (wholes) into ten equal parts • count up in tenths from any number of tenths, reading each multiple of ten tenths as a whole number e.g. *eight tenths, nine tenths, one whole* • count down in tenths from any number of tenths, reading each multiple of ten tenths as a whole number e.g. *twenty-two tenths, twenty-one tenths, two wholes*	Show me if this strip represents 0-1, where 0.3 would be. What if the strip represents 0-2?
Equivalence • Manipulating fraction pieces (circular or strips) to discover relationships between them. For example, that two $\frac{1}{6}$ pieces make a $\frac{1}{3}$ piece • Finding two given fractions of a set of objects and realising that the results is the same (so the fractions must be equivalent) • Folding paper strips vertically (rectangles) to	12. Recognise equivalent fractions; show that two fractions are equivalent • Say whether two fractions shown concretely are equivalent by manipulating the parts into the same number of groups and comparing • Say whether two fractions shown visually are equivalent by redrawing the parts in the same number of groups and comparing • Find equivalent fractions from a selection provided,	Always, Sometimes, Never? You cannot compare fractions with different denominators Convince me that you can see the whole fraction family using an array Always, Sometimes, Never? There is no equivalent fraction to 4/5

represent a fraction and then folding horizontally to discover (the family of) equivalent fractions and the proportional link between numerators and denominators

For example, for $\frac{2}{5}$ is equivalent to $\frac{4}{10}$



- Splitting the same shaded diagram up in multiple ways to show that the overall fraction shaded does not change
- NRICH: Matching Fractions (Pelmanism) http://nrich.maths.org/8283/note

matching them up

- Show and find equivalences to ½ such as $\frac{2}{4} = \frac{1}{2}$ or $\frac{5}{10} = \frac{1}{2}$ using concrete objects or a pictorial representation
- Show and find equivalences to quarters such as $\frac{2}{8} = \frac{1}{4}$ and $\frac{6}{8} = \frac{3}{4}$ using concrete objects or a pictorial representation
- Show and find equivalences to thirds such as $\frac{2}{6} = \frac{1}{3}$ and $\frac{4}{6} = \frac{2}{3}$ using concrete objects or a pictorial representation
- Show and find equivalences to fifths such as $\frac{2}{10} = \frac{1}{5}$ and $\frac{4}{10} = \frac{2}{5}$ using concrete objects or a pictorial representation

Further Extension

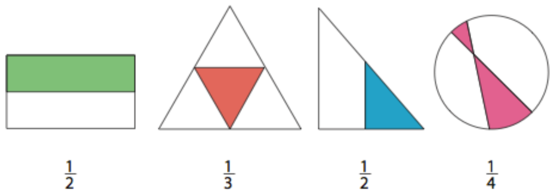
1.

Jo ate $\frac{1}{4}$ of a pizza and Sam ate $\frac{1}{2}$ of what was left. Mike ate the rest of the pizza.
Draw a diagram to show how much pizza Jo, Sam and Mike each ate.

2.

True or false?

Explain why.



3.

Shade in 0·7 of this rectangle.

Rich and Sophisticated Tasks

Recognise, find and write fractions of a discrete set of objects: unit fractions and non-unit fractions with small denominators

NRICH: Fraction Match * G

Recognise and show, using diagrams, equivalent fractions with small denominators

NRICH: Matching Fractions * G

NRICH: Fractional Triangles http://nrich.maths.org/2124/note

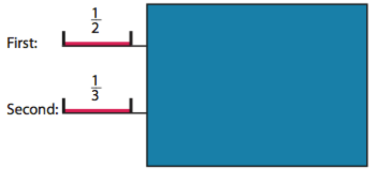
NRICH: Fractional Walls (for discovering equivalent fractions) http://nrich.maths.org/4519

Other Rich Tasks

1.

Only a fraction of each line is shown. The rest is hidden behind the blue screen.
Which whole line is the longer?

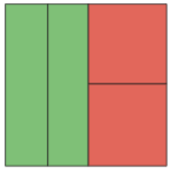
Explain your reasoning.



4.

The shape is divided into 4 equal parts. Do you agree?

Explain why.



5.

This is 0·4 or $\frac{2}{5}$ of a bag of marbles. How many marbles are in a full bag?

6.

Fill in the numerators to make the calculation correct.
How many ways can you do it?

Explain how you know you have found them all.

$$\frac{}{8} + \frac{}{8} = 1$$

7.

On a number line labelled 0 to 1, mark $\frac{1}{6}$, $\frac{1}{3}$ and $\frac{1}{2}$.

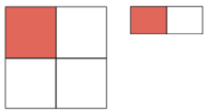
How big is the interval from $\frac{1}{6}$ to $\frac{1}{3}$?

How big is the interval from $\frac{1}{6}$ to $\frac{1}{2}$?

8.

Hamsa says the diagrams below show that $\frac{1}{4} > \frac{1}{2}$.
Do you agree?

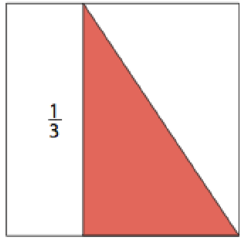
Explain why.



2.

What fraction of the square is shaded?

Explain your reasoning.



Useful Resources

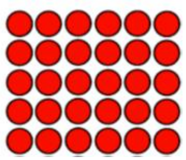
- Fractions ITP (Nat Strat) http://www.taw.org.uk/lic/itp/fractions.html
- Maths4Life Sharing Cakes https://www.ncetm.org.uk/resources/m4l_fractions_1
- Fraction manipulatives - exploring equivalence http://donnayoung.org/math/fraction.htm
- Fraction models and support questions - http://www.annery-kiln.eu/gaps-misconceptions/all-images.html
- NZ Maths TRAINS (for fractions as operators AND numbers) http://www.nzmaths.co.nz/resource/trains

Misconceptions	Teacher Guidance and Notes
Some children struggle to read a fraction from a concrete or visual representation. This is usually because they do not understand how the whole and the highlighted/shaded parts relate to each other. They may read fractions as pieces rather than as equal parts of and in the whole. For example, in a shape divided into eighths, where three parts are shaded it may be written as $3/5$ as five parts are unshaded. Some children do not fully recognise that the parts of the whole must be of equal size (although not necessarily equal shape). They also do not see the denominator as an indicator of the number of parts in the whole and use it directly to order fractions, believing that fractions with a larger denominator are bigger. For instance, they may say that 1/10 > 1/5 because 10 > 5. Children may not have a complete understanding of a fraction as both a proportion of the whole (be that one whole object or a quantity or a set of objects) as well as a number in its own right. When considering unit fractions, pupils may not understand that each 'part' needs to be the same size Pupils often interchange the denominator and numerator Look out for international language e.g. the use of a 'fourth' for a quarter. Similarly, listen to how children pronounce non-unit fractions e.g. $2/3$ is sometimes said as '2 over 3' or '2 threes' rather than a more valid alternative such as 'two thirds'. When comparing fractions and looking at equivlaence, pupils may be swayed by a numerator to determine size e.g. they may say that $2/6 > 1/3$ because 2 > 1 with no consideration for the different denominators Children tend to become attached to one or two representations of a fraction and this limits their ability to transfer their knowledge to fractions as 'proportions of' or fractions as 'numbers' or fractions as 'a process of sharing' e.g. 2/3 as 2 shared by 3. They may select an inappropriate representation to assist them with a given problem.	• Until this point, children have only really worked with halves and quarters and basic thirds. • This unit is the opportunity for them to apply these ideas to fractions with denominators of up to 10. Be aware that children's times tables at this stage cover 2s, 3s, 4s, 5s, 6s, and 10s and hence these should be the focus of any calculations involving larger numbers. • It is absolutely essential that by this stage of learning children understand the role of the denominator (and then the numerator). i.e. the denominator shows how many equal parts a number, or an object, or a set of objects, has been divided into, while the numerator (top number) tells us how many of those parts there are. Look out for children who do not divide a quantity or object into EQUAL parts as these children do not fully understand the meaning of the fraction. • Spend some significant time on representing fractions in multiple ways - always model more than one representations and ask children to develop their own. You may want to slow down fluency steps 1 and 4 to enable confidence in representing fractions to develop. • Possible representations would include: area diagrams using a range of different shapes, number lines, words, symbols, some decimal equivalents and percentages, fractions as a result of division. Pictorial representations of a particular fraction may be of different sizes and different shapes. For example, don't always use shaded sections of circles, and interesting discussions can be had from drawing half of a small square and a quarter of a larger square and asking which is the larger fraction (and this means you have to be careful when using areas to explain fractions!). • There is a key learning point that the rest of KS2 and KS3 depends on that emerges in Stage 3 - that is the difference between fractions as ordinal numbers (as numbers on a number line), fractions as being a special kind of cardinal number (the answer to 1/2 of a number depends on the quantity you are using) and fractions as operators (What is ½ of 30? What is 2/3 of 45?). Ensure that all of these are covered! • In Stage 3 we are especially developing children's understanding of a fraction as an actual number between 0 and 1 (for proper fractions). Make connections with a range of representations here to show how dividing the number line between 0 and 1 enables you to 'see; where 1/3 must sit for example. • When comparing and then ordering (which is just comparing with more than two items!), use the number line or array representations to

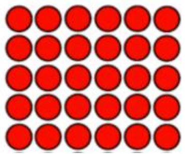
	give you best access - this way children can see which fraction is larger once they have been positioned in a comparable way. You might want to start by showing two LESS useful comparative representations to get children to realise the difficulties for themselves. • Make connections to other topics - ie time and fractions on a clock face as well as fractions linked to a range of measurements.

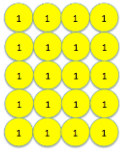
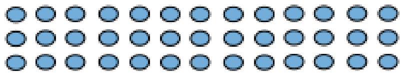
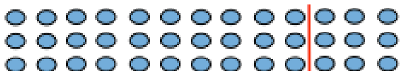
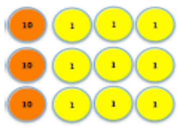
Key Assessment Checklist

1. I can find unit fractions of a discrete set of objects using a division process.
2. I can find simple non-unit fractions of a discrete set of objects.
3. I can write an amount of objects as a fraction of a whole set
4. I can recognise a unit fraction as a number between 0 and 1 and position it on a number line.
5. I can compare and order unit fractions
6. I can recognise a non-unit fraction as a number between 0 and 1 and position it on a number line.
7. I can compare and order fractions with the same denominator
8. I can recognise two equivalent fractions represented visually; I can create representations to show whether two fractions are equivalent.
9. I can use my knowledge of fractions to solve problems involving quantities (ie money)
10. I can add fractions with the same denominator within one whole representing this visually
11. I can subtract fractions with the same denominator within one whole representing this visually
12. I can solve problems involving adding and subtracting fractions with the same denominator

Year 3	Unit 9 : Solving Number Problems	
16 learning hours	This unit continues pupils' earlier study of arithmetic (and algebra for secondary students). At Key Stage 1 children are working on multiplication (and division in Stage 2) as a way to represent repeated addition and scaling (and repeated subtraction – grouping - and sharing) At Key Stage 2 children are developing skills in applying their arithmetic to more complex problems. At secondary level and in Stage 6, students begin to find unknown values by applying inverse operations. Equations of all types including quadratic and simultaneous are covered in later stages.	
Prior Learning	**Core Learning**	**Learning Leads to...**
➢ calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (×), division (÷) and equals (=) signs ➢ show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot	➢ **write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods**	➢ find the effect of dividing a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as ones, tenths and hundredths ➢ use place value, known and derived facts to multiply and divide mentally, including: o multiplying by 0 and 1; o dividing by 1; o multiplying together three numbers ➢ multiply two-digit and three-digit numbers by a one-digit number using formal written layout
Exemplification		**Vocabulary**
1. a) Represent the following problem pictorially: 3 x 7 = 21 b) Write these addition statements as multiplication statements: (i) 2 + 2 + 2 + 2 (ii) 3 + 3 + 3 c) Write a multiplication statement for this array  d) Write a division statement for this array  e) Calculate 18×4. You can use diagrams to help you.		multiplication division repeated addition scaling multiplied by lots of/groups of/sets of ... times larger/smaller product divided by shared between array row; column times table facts fact family partition

f) Calculate 52 ÷ 3 g) Find the missing number in these multiplications and divisions 3 x ☐ = 12 ☐ X 3 = 15 4 x ☐ = 20 8 x ☐ = 24	grid solve scaling commutative

Representation	Fluency	Probing Questions
Simple Multiplication • Arranging Numicon pieces end-to-end to represent groups and overlaying 10s and finally other pieces to find the total value • Using a bead string to represent groups and find the total quickly using place value • Drawing jumps on a (numbered and then blank) number line to represent groups and marking intermittent values • Building groups of single objects such as counters or cubes using hoops or other containers (or drawing these visually). • Organising groups of single objects into columns to form an array. For example, here are six groups of 5 counters shown as a 6 x 5 array 	1. Solve practical problems involving multiplication e.g. Roy has 9 buckets with 3 crabs in each. How many crabs does he have altogether? • represent the problem concretely or visually using groups or scaling (as implied by the question) • find the total by counting (or efficient counting e.g. in 3s)	Show me how you can represent this problem visually: Alice has 6 boxes. In each box there are 4 sweets. How many sweets does Alice have altogether?
	2. Explain and calculate with increasing speed abstract mathematical statements for multiplication within the times tables e.g. 7 x 4 • represent the statement concretely or visually using groups • represent the statement using a concrete or visual array • represent the statement concretely or visually using scaling • find the result by counting efficiently or portioning and recombining • record the result at the end of the number sentence • recognise that a multiplication represents a repeated addition i.e. convert 3 + 3 + 3 + 3 to 3 x 4 • write a multiplication as a repeated addition	Show me the missing number: 3 x ☐ = 18 ☐ x 2 = 24 What’s the same and what's different? 14 x 2, double 14, the number twice as big as 14, 28
Multiplying multiples of 10 • Representing a multiple of 10 multiplied by a single digit number using place value equipment to harness pupils’ unitisation skills.	3. Calculate a multiple of 10 multiplied by a single digit • Recap: 10 × single digit • 20 × single digit • 30 × single digit • 40 × single digit	Convince me that 15 x 10 = 150 Always, Sometime, Never? The 30 times table is just the 3 times table with 0s on the end.

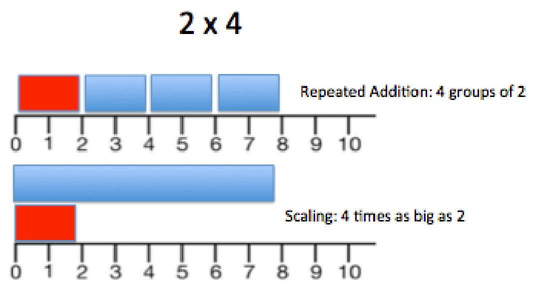
For example, 40 x 5 is 4 tens multiplied by 5 which is 20 tens or 200. Here is 4 x 5  compared to 40 x 5 	• 50 × single digit • 80 × single digit • 60/70/90/100/110/120 × 2/3/4/5/8	
Mental multiplication • Building larger arrays for numbers beyond the times tables For example, here is an array showing 13 x 3  • Partitioning these arrays to show that the total can be found by finding sub-totals first 10 x 3 3 x 3  • Using shorthand partitioning to apply this e.g. 13 x 3 = 10 x 3 + 3 x 3	4. Calculate mentally a two-digit number multiplied by a single digit (2, 3, 4, 5, 8) (with jottings) • 11-19 x single digit (no bridging) e.g. 13 x 3 • 11- 19 x single digit (bridging) e.g. 17 x 3 • two-digit x single digit (no bridging) e.g. 31 x 8 • two-digit x single digit (bridging) e.g. 36 x 4	Convince me that 23 x 4 is the same as 20 x 4 + 3 x 4 Convince me that 29 x 3 = 30 x 3 – 1 x 3
Multiplying – written methods • Building arrays using place value counters  • Using a grid method to generalise an array	5. Calculate a two-digit number multiplied by a single digit (within 2, 3, 4, 5 and 8 times tables) using any written method • 11-19 x single digit (no bridging) e.g. 13 x 3 • 11- 19 x single digit (bridging) e.g. 17 x 3 • two-digit x single digit (no bridging) e.g. 31 x 8 • two-digit x single digit (bridging) e.g. 36 x 4	Show me how you could represent 13 x 5 with an array Convince me that 38 x 4 = 152 in two different ways.

<table>
<tr>
<td>
and imagine it without all the single objects
For example, the array above would become
<table>
<tr><td></td><td>10</td><td>3</td></tr>
<tr><td>3</td><td>30</td><td>9</td></tr>
</table>

Investigating the links between the grid above and the compact formal method, that is

$$\begin{array}{rr} 1 & 3 \\ & 3 \;\times \\ \hline 3 & 9 \end{array} \qquad \begin{array}{rr} 1 & 6 \\ & 3 \;\times \\ \hline 4 & 8 \\ {\scriptstyle 1} & \end{array}$$

Looking at the difference between models of repeated addition and scaling



</td>
<td></td>
<td></td>
</tr>
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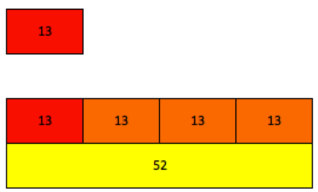
<td>
Multiplication Problems

Using the bar model to represent calculations
For example: packets of biscuits contain 13 biscuits. Anna buys four packets. How many biscuits is this?



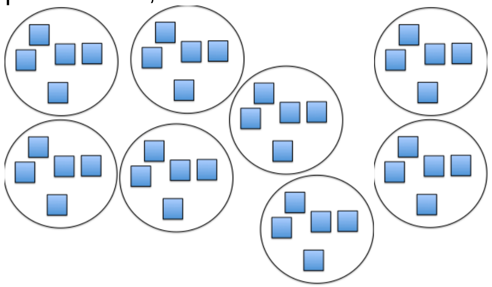
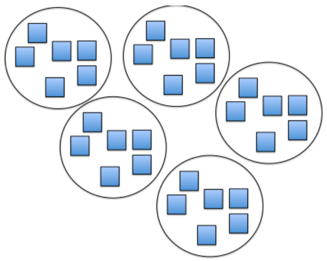
</td>
<td>
6. Recognise and solve simple word problems (out of context) involving multiplication using number sentences e.g. What is four times as many as 18?

choose an appropriate representation
solve the problem
write a number sentence to represent the problem

</td>
<td>
Show me how you can represent this problem visually: Jane is drawing a picture of a flower. Her picture is 4cm tall. The real flower is 3 times as tall as this. How tall is the real flower?

Convince me that multiplying a number by 8 is the same as doubling, then doubling again, then doubling again.
</td>
</tr>
</table>

<table>
<tr>
<td rowspan="2">Division

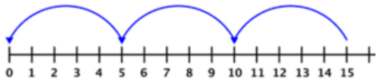
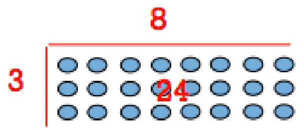
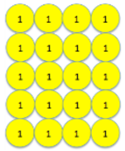
Sharing out objects equally into 2, 3, 4, 5, 6 or 10 groups
E.g. for 40 ÷ 8 there would be 40 items counted and then shared into six piles (divided into 8) and the number in each pile counted, which is 5.

Grouping objects into 2s, 3s, 4s, 5s, 8s or 10s and so on before counting the number of groups produced
e.g. 30 ÷ 6 or 30 grouped into 6s produces 5 groups

Using a bead string/Numicon for grouping/repeated subtraction

(24 beads then grouped into 3s produces 8 groups)
Using a number line to show repeated subtraction to see how many groups fit inside e.g. 15 ÷ 5
</td>
<td>7. Solve practical problems involving division e.g. Emma has 28 sweets. She shared the sweets between 4 party bags. How many sweets does she put in each bag?

represent the problem concretely using grouping or sharing (as described by the question)
find the result by counting the number of groups or number in each group as appropriate
ext: solve reverse problems e.g. A number divided by 3 gives 18. What was the number?
</td>
<td>Show me how you can represent this problem visually: Jay has 18 hats. He shares the hats equally into 3 bags. How many hats will there be in each bag?</td>
</tr>
<tr>
<td>8. Explain and calculate (abstract) mathematical statements for division within the times tables e.g. 32÷ 4

represent the statement concretely or visually by sharing
find the result by counting the number in each group
represent the statement concretely or visually by grouping
find the result by counting the number of groups
represent the statement concretely or visually using an array (organised grouping)
find the result by counting the number of columns (groups)
record the result at the end of the number sentence
</td>
<td>Show me the missing number:
20 ÷ ☐ = 5
☐ ÷ 3 = 7

What’s the same and what's different?
32÷ 2 , half of 32, the number half the size of 14, 16</td>
</tr>
</table>

 • For a calculation p ÷ q, grouping a set of p counters into groups of size q, arranging these groups as an array. For example, for 24 ÷ 3, count out 24 counters and arrange in columns of 3.... then read off the answer of 8 as the number of columns  • Finding a many different arrays as possible with a given number of counters e.g. 24 to find what it is divisible by		
Dividing multiples of 10 • Representing a multiple of 10 divided by a single digit number using place value equipment to harness pupils' unitisation skills. For example, 200 x 5 is 20 tens divided by 5 which is 4 tens or 40. Here is 20 ÷ 5  compared to 200 ÷ 5 	9. Calculate a multiple of 10 divided by a single digit (2, 3, 4, 5, 8) • Recap: multiple of 10 ÷ 2 e.g. 140 ÷ 2 • multiple of 10 ÷ 3 e.g. 90 ÷ 3 • multiple of 10 ÷ 4 e.g. 240 ÷ 4 • multiple of 10 ÷ 5 e.g. 350 ÷ 5 • multiple of 10 ÷ 8 e.g. 160 ÷ 8 • ext: multiple of 10 ÷ 6/7/9/11/12 (where the answer is 20/30/40/50/60) e.g. 360 ÷ 12	Convince me that 240 ÷ 8 = 30 What's the same and what's different? 360 ÷ 6 and 36 ÷ 6
Dividing Mentally • Using a number line to show partitioned grouping.	10. Calculate mentally a two-digit number divided by a single digit (2, 3, 4, 5, 8) (with jottings) • two-digit ÷ single digit (tens and units multiples of the	Convince me that 76 ÷ 4 is the same as 60 ÷ 4 + 16 ÷ 4 Convince me that 87 ÷ 3 is the same

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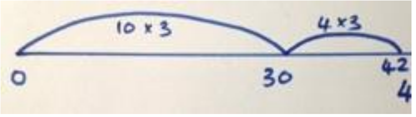
For example, 42 ÷ 3 can be found by considering known multiples of 3



Partitioning a larger number to divide each part and then recombine
For example:

42 ÷ 3
30 ÷ 3 + 12 ÷ 3
10 + 4
14
</td>
<td>
single digit) e.g. 84 ÷ 4

two-digit ÷ single digit by partitioning (chunking) e.g. 72 ÷ 3
two-digit ÷ single digit by mental skills e.g. halving and halving again to divide by 4

</td>
<td>
as 90 ÷ 3 − 3 ÷ 3
</td>
</tr>
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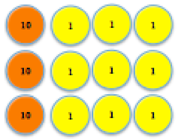
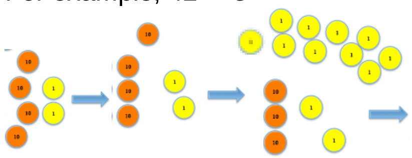
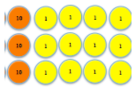
<td>
Building a number using place value counters and grouping them into groups that are the size of the divisor, before arranging these groups as an array to explore the partitioning approach. For example, 39 ÷ 3



Repeating the above, but exchanging remaining counters for 10 counters of the next size down before continuing to group. For example, 42 ÷ 3




</td>
<td>
11. Calculate a two-digit number divided by a single digit (2, 3, 4, 5, 8) using a written method

two-digit ÷ single digit (tens and units multiples of the single digit) e.g. 96 ÷ 3
two-digit ÷ single digit e.g. 72 ÷ 3 using any written method (partitioning, compact method)

</td>
<td>
Show me how you could represent 72 ÷ 3 with an array

Convince me that 87 ÷ 3 = 29 in two different ways.
</td>
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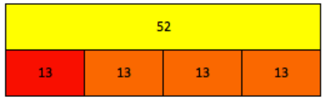
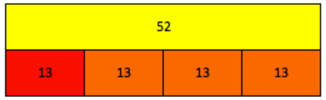
Division Problems • Using the bar model to represent calculations For example: Bob has 52 sweets. He divides them into 4 party bags. How many sweets will be in each bag?  	12. Recognise and solve word problems (out of context) involving division using number sentences e.g. What is 56 shared between 4? • choose an appropriate representation • solve the problem • write a number sentence to represent the problem • ext: solve reverse problems e.g. A number multiplied by 3 gives 54. What was the number?	Show me how you can represent this problem visually: Jane is drawing a picture of a flower. The real flower is 40cm tall, which is 8 times as tall as her picture. How tall is the drawing of the flower? Convince me that dividing a number by 8 is the same as halving, then halving again, then halving again.
Representations of Multiplication and Division • Writing statements for images of arrays, groups, scaling diagrams etc. • Writing all the statements represented by a single bar model  	13. Given a representation, suggest a (multiplication and/or division) calculation that it represents • groups diagram • scaling diagram • array	What's the same and what's different? 7 x 3, 3 x 7 , 3 x 21, 21 ÷ 3, 21 ÷ 7, 21 x 7, 7 ÷ 3, ... Always, Sometimes, Never? The opposite of multiplying by 3 is finding a third
	14. Find the fact family for a given multiplication or division • given a multiplication, find the answer and then the other three related calculations • given a division, find the answer and then the other three related calculations • from a representation e.g. an array • say whether a rearranged fact is true or false e.g. is 7 x 8 = 56 then is it true that 8 ÷ 56 = 7?	Show me the fact family for 6 x 5 Show me the fact family for 17 x 3 Convince me that when you know a times table fact, you actually know 4 facts Always, Sometimes, Never? If you know that a x b = c then you also know that a ÷ b = c
	15. Ext: Solve a mixture of multiplication and division problems • know key trigger words for multiplication and division and hence recognise whether problem is multiplication or division • find a missing number in a multiplication problem • find a missing number in a division problem	What's the same and what's different? multiply, lots of , groups of, divide, product, shared by, shared between, grouped into, quotient Always, Sometimes, Never? Multiplying is the opposite of dividing
	16. Recall times table multiplication and division facts (3s, 4s and 8s as well as 2s, 5s and 10s) ○ By representing the calculation concretely to	Always, Sometimes, Never? Every times table fact has two related division facts

deduce the answer.
- By representing the calculation visually to deduce the answer.
- By relating the calculation to another known calculation and counting on/back or doubling etc.
- By beginning to recall key facts.

Further Extension

1.

Find the missing digits.

2 □	2 □	1 □ 4
× 8	× □	× □
1 7 6	1 1 2	7 3 6

2.

□ □ × □ = ?

Putting the digits 1, 2 and 3 in the empty boxes, how many different calculations can you make?

Which one gives the largest answer?
Which one gives the smallest answer?

Rich and Sophisticated Tasks

Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods

NRICH: Andy's Marbles
NCETM: Always, Sometimes, Never:
NCETM: Pendulum Counting:
NCETM: Multiplying Numbers:
NCETM: Multiplication and Division Reasoning

Misconceptions

Children may assume that, since multiplication is commutative, division is commutative and can be done in any order! They may write sentences such as 6 ÷ 2 = 12 due to this. Therefore, they may struggle to correctly re arrange the numbers of a times table fact into the corresponding division fact, for example they may put the smaller numbers first in division calculations.

Children may not see how an array can be used to support division, only multiplication.

Poor understanding of place value may affect a child's ability to partition correctly and to calculate the result of a multiple of 10 x 1d.

Children with weak understanding of how to represent a multiplication as an array may struggle to represent and calculate a 2dx1d multiplication and fail to see why they need to be able to partition it. They may try to work with a very large array rather than sectioning it off and partitioning.

Teacher Guidance and Notes

- In this unit the focus is on developing the methods for multiplication and division in direct calculations and simple problems. The opportunity to solve more complex problems combining techniques comes later in Unit 13. Students working at greater depth can, of course, attempt more complex problems now but this is not a general expectation.
- Children have experience of calculating and recording multiplications and divisions from Stage 2, although this may have occurred using apparatus and only within the 2, 5 and 10 times tables or with very small numbers.
- The 2, 5, 10, 3, 4 and 6 times tables are suitable to be used in questions in Stage 3.
- Clearly, knowledge of times tables is essential to this unit and so regular practice of times tables all year round is recommended. Specifically, try to ensure that children do not just count forwards in multiples and that they have lots of exposure to counting back as well

Some children struggle to understand the range of language of multiplication and division e.g. they may forget the meanings of 'lots of', 'times bigger', 'product' etc. Similarly, they may find it hard to understand what operation they need to use from a word problem because there are so many ways to imply a multiplication or division. Children often fail to recognise scaling problems as multiplication (or division problems) and find it hard to represent these practically. Some children may not yet have a strong understanding that multiplication is the inverse of division and so find it hard to move between the two operations.	as forward. When teaching or rehearsing times tables make the link to division very clear. Encourage children to produce the fact family for a given times table fact (i.e. the commutative multiplication fact plus the two related division facts). • Throughout the unit, make sure that the language of calculation is both modelled and displayed - ensure you use ALL OF multiplication, times, lots of, groups of, multiply, product, times bigger, times as big etc. • In Unit 13, there is a strong focus on applying the skills above to problem solving. To be ready for this, children need to be confident and competent in calculating multiplications and divisions using both representations (repeated addition and scaling for multiplication and sharing and grouping for division). Therefore, it is key that you represent calculations in both ways now too. You could also use the bar model to show multiplications and divisions so that it can be used as a problem solving tool later.

Key Assessment Checklist

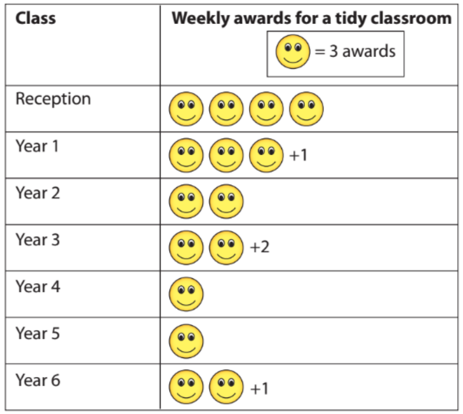
1. I can recall times table facts for 2s, 5s, 10s, 3s, 4s and 8s
2. I can represent and calculate multiplication statements
3. I can multiply a multiple of 10 by a single digit. e.g. 20 x 3.
4. I can multiply a 2d number by a 1d number using mental methods and jottings
5. I can multiply a 2d number by a 1d number using partitioning leading to more formal methods.
6. I can represent and calculate division statements
7. I can divide a multiple of 10 by a single digit e.g. 80 ÷ 4
8. I can divide a 2d number by a 1d number using mental methods and jottings.
9. I can divide a 2d number by a 1d number using partitioning or another written method
10. I can find a missing number in a multiplication or division calculations; I know that they are inverses.

Year 3	Unit 10 : Investigating Statistics
8 learning hours	In this unit children and students explore the collection, representation, analysis and interpretation of data. It covers a range of calculations of central tendency and spread as well as multiple charts and graphs to represent data. As it is the only unit directly exploring statistics, it is critical that children have time to explore the handling data cycle here and to focus sufficient time on interpreting their results.

Prior Learning	Core Learning	Learning Leads to...
➢ interpret and construct simple pictograms, tally charts, block diagrams and simple tables ➢ ask and answer simple questions by counting the number of objects in each category and sorting the categories by quantity ➢ ask and answer questions about totalling and comparing categorical data	➢ **interpret and present data using bar charts, pictograms and tables** ➢ **solve one-step and two-step questions [for example, 'How many more?' and 'How many fewer?'] using information presented in scaled bar charts and pictograms and tables**	➢ interpret and present discrete and continuous data using appropriate graphical methods, including bar charts and time graphs ➢ solve comparison, sum and difference problems using information presented in bar charts, pictograms, tables and other graphs

Exemplification

1. Transfer the information from the weekly awards table to the table below.



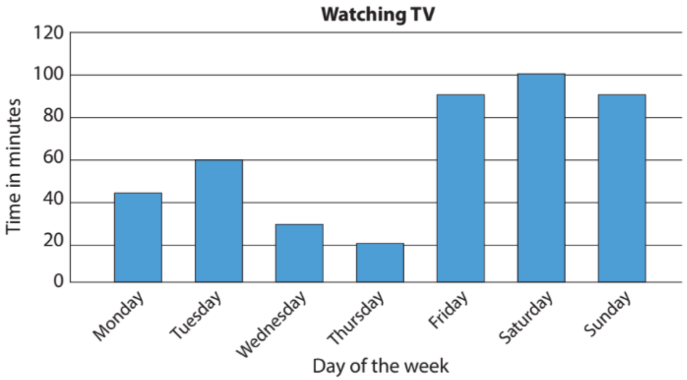
Class	Number of awards
YR	
Y1	
Y2	6
Y3	
Y4	
Y5	
Y6	

Draw a bar chart to represent the information.

Vocabulary

data	interval
category(ies)	bar chart
pictogram	how many more
key	... (less)
symbol	difference
represents	total
tally	popular
bundle	common
frequency	rare
frequency table	more than
total frequency	fewer than
scale	

2. The graph shows how many minutes Sam spent watching TV at home last week.



On which day did Sam watch the most TV?
How many minutes of TV did Sam watch on Wednesday?
How many more minutes did Sam watch on Friday than on Tuesday?
How many fewer minutes did Sam watch on Thursday compared to Sunday?

Representation	Fluency	Probing Questions
Pictograms • Students need to explore the benefit for using one symbol to represent multiple items • Get students to draw a pictogram with high frequencies (multiples of 2 or 10) using a symbol to represent one item. Then suggest representing the same data but with one symbol for 2 or 10 items so students discover the time saving benefit and how it is easier to manage • Students should discover why a key is useful by analysing pictograms without them. • Students need to develop methods for handling values that aren't multiples of the symbol representation	1. Pictograms • draw a pictogram with one symbol for one item • draw a pictogram for data in multiples of 2 or 10 where each symbol represents 2 or 10 • include a key on a pictogram • draw a pictogram for data not in multiples where each symbol represents 2 or 10 • select a suitable key for a pictogram given a data set	Show me how you would represent three objects using this symbol (symbol worth 2, 4 etc.) Show me how many these symbols represent (pictogram symbols worth more than 1 inc partial symbols) Convince me that it is not a good idea to let my symbol represent 7 objects in my pictogram Convince me that it is quicker to draw a pictogram when each symbol represents more than one object
	2. Interpreting pictograms • create a frequency table for a pictogram where symbols represent 1 • create a frequency table for a pictogram where symbols represent more than 1	Convince me that the most popular answer was …

	• identify a key given a pictogram and corresponding frequency table • find totals and make comparisons for a pictogram	
Tally charts • Recap multiple of 5 strike through • Get students to record two similar experiments; one directly into a frequency table, and one with a tally chart to see the benefit of the tally for this purpose	3. Tally charts/frequency tables • create a tally from a frequency table • record directly into a tally chart from an experiment or survey • create a frequency table from a tally chart • create an ordered frequency table from a tally chart	Show me the tally chart that this pictogram may have come from What's the same and what's different? tally and list Always, Sometimes, Never? A pictogram is easier to read than a tally chart
Bar charts • Students should explore the benefit of a scale with equal increments by analysing bar charts that don't have them and are misleading • When dealing with data with large frequencies, get students to create a bar chart with unit scale, then recreate with a scale of 2,5 or 10 etc. to see the benefit and space saving • Students should become comfortable drawing bars that are in between increments on the scale **Interpreting** • Students should identify how to construct a frequency table from a chart. A match up activity will help them discuss ideas around this. • Students should discover the benefit of using a frequency table to make numerical comparisons and charts to make value judgements (e.g. most popular etc.) • Challenge students to directly make comparisons using the scale on a chart	4. Bar charts • identify all categories and place on the x axis • label and mark values on axis • create a bar chart for data that goes up in 1s • create a bar chart for data that goes up in 2s or 10s • identify and use a suitable scale from a data set • use equal increments when creating a scale in y axis	Show me the frequency table for this data set Convince me that the best way to display the data is using a bar chart What's the same and what's different? bar chart and block graph What's the same and what's different? 2s, 5s, 7s, 10s as amounts to go up in for scale
	5. Interpreting bar charts • find totals from a bar chart with a scale that goes up in 1s • interpret a bar chart with a scale in 2s or 10s • interpret a bar chart with a scale in 5s • match-up bar charts with frequency tables	Show me the frequency table that this bar chart came from Convince me that the most popular answer was … Always, Sometimes, Never? The highest value on a bar chart is the best Always, Sometimes, Never? You can find the exact list of data that a bar chart came from

Further Extension

1. Create two separate pictograms to display the following information. The symbol used in each should have a value of more than 1.

Which value will you choose for each pictogram?

Explain your decisions.

Class	Number of merits awarded	
	Hard work	Good behaviour
YR	42	32
Y1	39	18
Y2	24	27
Y3	30	33
Y4	18	24
Y5	30	24
Y6	39	36

2. Work with two friends to collect data on how many hours each of you watch TV for a week.

Decide how you will combine and present the data using just one graph.

Rich and Sophisticated Tasks

Interpret and present data using bar charts, pictograms and tables
NRICH: Our Sports * I
NRICH: Class 5's Names * P
NRICH: Going for Gold * I
NRICH: The Domesday Project * I
NRICH: The Car That Passes * I
NRICH: Now and Then ** P
NRICH: Real Statistics *** P
NRICH: If the World Were a Village * P
NRICH: It's a Tie ** I

Solve one-step and two-step questions [for example, 'How many more?' and 'How many fewer?'] using information presented in scaled bar charts and pictograms and tables
NRICH: The Olympic Flame: Are You in the 95%? * P

Misconceptions

Children often record their data in a haphazard way and hence make counting errors when converting to tally charts or tables. This is made worse when they do not use a piece-by-piece approach to tallying the data and cross off the data as it is used.

When working with pictograms with symbols worth more than 1, children sometimes struggle to draw and/or to interpret partial symbols to represent numbers that are not exact multiples of the symbol value. They may also assume that a symbol is worth one without checking a key. Similarly, they may forget to include a key when constructing.

Children similarly struggle with bar charts where the scale does not go up in 1s to estimate the value of a given bar. This is exacerbated when the scale is in 10s or a larger unit.

When interpreting, children often fail to relate the number to the context,

Teacher Guidance and Notes

- The emphasis here is on working with data representations where there is not a 1:1 correspondence between the symbols/scales used. This is a good opportunity to link to times table practice as well as to explore why scales that go up in 2s, 4s, 5s, 10s etc are greatly preferred to those using 3s, 6s and so on.

- Make sure children have the opportunity to collect their own data before presenting it in the different formats. Discuss the advantages and disadvantages of each approach. Also allow time for analysis of data that has already been presented - what is critical here is that children can do more than read the figures from the charts and can instead INTERPRET the information in the context of the problem. E.g. if the chart shows 100m race times, which outcome is the most desirable?

assuming that a higher number is always the best and not using language related to the situation.	• It is also well worth asking children to work backwards and construct the frequency table (and maybe even the original data list) from the chart itself.

Key Assessment Checklist

1. I can record data from an experiment or survey directly in a tally chart
2. I can produce a frequency table from a list of data or a tally chart
3. I can construct a pictogram for my data where each symbol represents more than one object
4. I can construct a bar chart for my data using a scale that goes up in 1s
5. I can construct a bar chart for my data using a scale that goes up in 2s or 10s
6. I can read information from a pictogram where each symbol represents more than one object.
7. I can read information from a bar chart where the scale goes up in 2s, 5s or 10s.
8. I can answer complex questions about my data using my readings from a bar chart or pictogram.

Year 3	Unit 11 : Visualising Shape
8 learning hours	In this unit children focus on exploring shapes practically and visually. There is an emphasis on sketching, constructing and modelling to gain a deeper understanding of the properties of shapes. It is therefore necessary to secure the practical skills at the same time as using them to explore the shapes in questions. At secondary level students are developing their skills in construction and the language/notation of shape up to the understanding, use and proof of circle theorems.

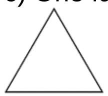
Prior Learning	Core Learning	Learning Leads to…
➢ identify 2-D shapes on the surface of 3-D shapes, [for example, a circle on a cylinder and a triangle on a pyramid]	➢ **draw 2-D shapes and make 3-D shapes using modelling materials; recognise 3-D shapes in different orientations and describe them**	➢ complete a simple symmetric figure with respect to a specific line of symmetry

Exemplification

1.
a) Draw (i) a pentagon (ii) an isosceles triangle (iii) a rectangle of length 3cm and width 2cm

b) Count out two sets of 12 cubes.
Use your cubes to make two different cuboids.

c) One face of a 3D shape looks like this:



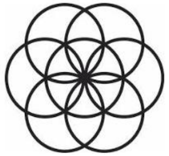
What could the shape be?

Vocabulary

shape	sides
circle	faces
square	vertices
triangle	edges
rectangle	draw
rectangular	make
pentagon	build
hexagon	construct
octagon	straight
cube	flat
cuboid	curved
pyramid	shapes
square based	dimension
triangular	dimensional
based	2D
sphere	3D
spherical	cross-section
round	base
prism	orientation
corners	

Representation	Fluency	Probing Questions
2D Shapes • Using spaghetti and marshmallows to produce polygons (from their name, from a description or with a given property) • Using geoboards and elastic bands to produce polygons	1. Recap: Describe the sides and vertices of simple 2D shapes (from diagrams or names) • sketch a named 2D shape e.g. triangle or hexagon • state the number of vertices of a 2D shape • state the number of sides of a 2D shape • use number of sides/vertices to identify	Convince me that a triangle can have a point facing down Convince me that a rectangle can be turned around to face any direction

• Drawing squares and rectangles on squared paper (or dotty paper) (can extend to other rectilinear shapes if desired)	polygons i.e. to say if a shape is a quadrilateral or not or is a pentagon or not • identify whether the sides are straight or curved • identify whether any of the sides are the same length • given a description of the sides/vertices of a shape, suggest its name or sketch the shape	Show me … a triangle cut out of paper What's the same and what's different? square, hexagon, pentagon, triangle
	2. Recap: Identify/draw (sketch) a 2D shape given its properties • given a property, suggest a possible shape e.g. four sides or all sides are equal length • given a property, suggest all possible shapes e.g. four sides or all sides are equal length • given several properties, pinpoint the exact shape e.g. three sides, one line of symmetry	Show me all the different shapes you can make from 4 straws (when you are allowed to cut them!) Always, Sometimes, Never? The bottom of a square can be horizontal, vertical or diagonal
	3. Draw more complex 2D shapes • rectangles and squares to scale on squared paper • sketch different types of triangle (equilateral, isosceles, scalene or right-angled) • sketch different types of quadrilateral (rectangle, square, parallelogram, rhombus, kite) • sketch other polygons with specific features e.g. a pentagon with a line of symmetry • visualise an image described and then sketch this e.g. a square overlapping an equilateral triangle	Convince me that a square cannot have different length sides Show me ... a square with sides of length 5 squares ... a rectangle with 2 sides of 10cm and 2 sides of 4cm ... a right angled triangle What's the same and what's different? right angled, isosceles, scalene, equilateral
Circles • Using compasses to explore how to draw circles • Drawing designs such as the Seed of Life	4. Draw a circle • by drawing round an object • using a pair of compasses • produce semicircles and other sectors • measure the width (diameter) of the circle	Always, Sometimes, Never? The distance from the centre to the edge of a circle is fixed.

	or the distance from the centre to the edge (radius)	
3D Shapes - Faces • Manipulating and turning 3D shapes to observe what they look like from different viewpoints e.g. from above or from the front • Printing and stamping with the faces of 3D shapes to explore their shapes and how they connect	5. Recap: Recognise, name and describe the faces of simple 3D shapes • cube • cuboid • pyramid (with different bases) • cone • prism (with different cross-sections) • cylinder • sphere (and hemisphere)	Show me a sphere in the room Show me all the shapes you can think of with 6 faces Convince me that there is more than one 3D shape that looks like a square from above
Modelling 3D shapes • Making shapes using clay and other mouldable resources e.g. play-doh or plasticine. (By name but also by property e.g. make a shape with a square face)	6. Make models of 3D shapes using modelling materials e.g. plasticine • spheres, cones, cylinders • cubes and cuboids • other prisms • pyramids • shape with a given property e.g. a face that is a triangle • name the shape from a model presented in a range of orientations	Show me a cuboid made out of plasticine What's the same and what's different? circle, sphere, cylinder, cube
Modelling 3D shapes - skeletons • Making skeletons of shapes using spaghetti and marshmallows (or geometric alternatives such as straws and balls of modelling clay or equivalent). Then exploring how many straws and connecting balls you need to make a cube? A pyramid? • Making shapes using Polydron or equivalent resources by connecting faces together to form a closed polyhedron	7. Make skeleton models of 3D shapes using modelling materials e.g. straws and balls of clay • cubes and cuboids • other prisms • pyramids • shape with a given property e.g. a face that is a triangle • name the shape from a model presented in a range of orientations	What's the same and what's different? pyramid, prism, cuboid, sphere

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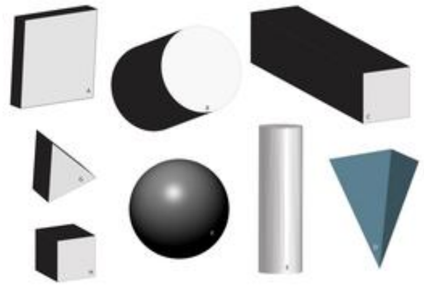
Drawing 3D shapes

- Sketching 3D shapes from models (as still life)
- Turning and re-orienting images of 3D shapes to recognise them
  For example:



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8. Identify 3D shapes with given features
   - recognise a drawn 3D shape in an unfamiliar orientation
   - name 3D shapes with a given property e.g. a square face
   - sketch a 3D shape from a given viewpoint
   - say whether a given (model/drawing/named) shape is a prism or a pyramid (and why
   - describe the features of any 3D shape in full

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Always, Sometimes, Never?
Cylinders have 2 circular faces

Always, Sometimes, Never?
Shapes with triangular faces are pyramids

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<tr><th>Further Extension</th><th>Rich and Sophisticated Tasks</th></tr>
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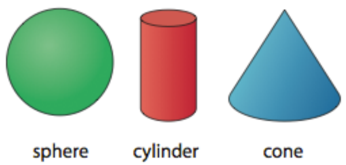
1.
True or false?
The shape of a cross section of a sphere is always a circle.
The shape of a cross section of a cylinder is always a circle.
The shape of a cross section of a cone is always a circle.

Explain your reasoning.

Can you identify a 3-D shape where the cross section is always a square?



2.

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Draw 2-D shapes and make 3-D shapes using modelling materials; recognise 3-D shapes in different orientations and describe them
NRICH: Building Blocks * P
NRICH: The Third Dimension *** P I
NRICH: Rolling That Cube * P
NRICH: Inky Cube *** P
NRICH: Triple Cubes * I
NRICH: Sponge Sections ** P
NRICH: A Puzzling Cube * P
NRICH: Arranging Cubes * G
NRICH: Board Block Challenge * G**
NRICH: Square Corners ** P
NRICH: Stick Images * G P
NRICH: Overlapping Again ** P

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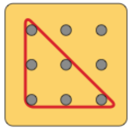
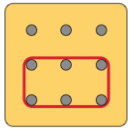
How many different triangles can you find on a 3x3 pin geoboard?
How do you decide that they are different?



How many different quadrilaterals can you find on a 3x3 pin geoboard?
How do you decide that they are different?



Misconceptions

Children may still confuse 2D and 3D shapes because of the faces on 3D shapes and the complexity of the concept of a 2D shape.

Children's understanding of surface can be weak and lead to issues understanding the difference between the shape of a face and the whole shape.

Children may interchange prisms and pyramids because of the vocabulary.

Children do not always realise that cones are not pyramids and cylinders are not prisms.

Additionally they may not fully appreciate the difference between a cube and a general cuboid (note that a cube IS a cuboid).

Within this practical exploration of shapes, there may be issues that arise due to difficulties with using a compass or a ruler as well as lack of fine motor skills to actually model the shapes.

Teacher Guidance and Notes

- Children have encountered a wide range of shapes in Stage 2 but have only really looked at shapes from an angle point of view so far in Stage 3.
- Therefore, there is a need to recap basic shape names and properties with children before beginning to look at special types of triangles and quadrilaterals and so on.
- This unit is all about the visualisation of shape and so strong emphasis should be placed on constructing, modelling, making and sketching.
- Children need to be encouraged to consider problems where there is more than one answer e.g. how many 3D shapes can you make with a square face?
- Ensure there is time for lots of practice with using drawing tools (rulers, compasses etc) - note that there is time within this unit allowed for the nature of the practical work required. Materials such as straws and unifix cubes can be very useful to children to really get the straight edges
- For those children working at greater depth, try to exploit opportunities to think about the shapes that test the definitions e.g. is a cube a prism? What about a cylinder?

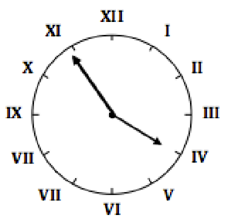
Key Assessment Checklist
1. I can draw squares and rectangles using squared paper
2. I can draw triangles (isosceles, right angled and scalene)
3. I can draw circles using a compass
4. I can make 3D shapes out of modelling materials
5. I can make outlines of 3D shapes using straws and mouldable dough (e.g. play-doh)
6. I can recognise rectangles and triangles in different orientations
7. I can recognise a range of 3D shapes in different orientations
8. I can imagine what a 3D shape looks like from above/the side/the front etc.

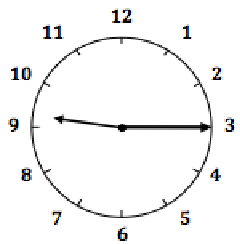
Year 3	Unit 12 : Exploring Change
12 learning hours	For primary pupils this unit focuses on the measures elements of time and co-ordinates. There is a progression from sequencing and ordering through telling the time formally to solving problems involving time. The co-ordinate work flows in the secondary students' learning focused on the relationships between co-ordinates. Key objectives include the use of y=mx+c for straight lines, the use of functions and the graphing of more complex functions.

Prior Learning	Core Learning	Learning Leads to...
➢ compare and sequence intervals of time ➢ tell and write the time to five minutes, including quarter past/to the hour and draw the hands on a clock face to show these times ➢ know the number of minutes in an hour and the number of hours in a day.	➢ **tell and write the time from an analogue clock, including using Roman numerals from I to XII, and 12-hour and 24-hour clocks** ➢ **know the number of seconds in a minute and the number of days in each month, year and leap year** ➢ **compare durations of events [for example to calculate the time taken by particular events or tasks]** ➢ **estimate and read time with increasing accuracy to the nearest minute; record and compare time in terms of seconds, minutes and hours; use vocabulary such as o'clock, a.m./p.m., morning, afternoon, noon and midnight**	➢ read, write and convert time between analogue and digital 12- and 24-hour clocks ➢ solve problems involving converting from hours to minutes; minutes to seconds; years to months; weeks to days ➢ describe positions on a 2-D grid as coordinates in the **first quadrant**

Exemplification

1.
a) Write the time shown on each of these clocks





b) Draw hands on each of these clocks to show these times:
Quarter to eight 14:40

Vocabulary

Roman Numerals
clock
second
minute
hour
week
month
year
leap year
weekend
calendar
evening
midnight
noon

12-hour clock
24-hour clock
analogue clock

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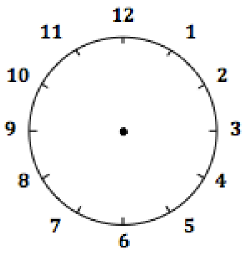
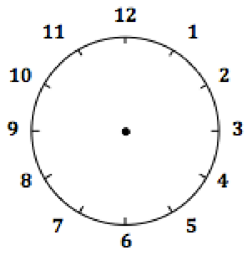

2. a) How many months are there with 30 days?
b) How many seconds are there in 2 minutes?
c) How many days are there in a year?

3. a) Jemma leaves home at 7:15 am and arrives at work at 7.55 am.
How long does it take Jemma to get to work?

b) Jack goes out at 10:45 am and gets home at 2:30 pm
For how long has Jack been out?

4. A train takes 1 hour and 20 minutes to get to London.
If the train leaves at 10:25 am, what time does it get to London?
</td>
<td>digital clock

am
pm

estimate
timetable
convert
duration
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<th>Representation</th>
<th>Fluency</th>
<th>Probing Questions</th>
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<td rowspan="2">Telling the Time
• Labelling a clock with key words, roman numerals, multiples of 5 and fractions. Then counting round the clock and moving the hands to match either in 12-hour format e.g. 1:00, 1:05, 1:10, 1:15, ... or in analogue format e.g. one o’clock, five past one, ten past one, quarter past one, twenty past one, twenty-five past one, ...</td>
<td>1. Know and use the vocabulary of time
• o’clock
• half past
• quarter past
• ... minutes past
• quarter to
• ... minutes to
• am
• pm
• morning, afternoon
• noon, midnight
• second, minute, hour
• day, week, month
• year, leap year</td>
<td>What’s the same and what’s different? year and a leap year

Always, Sometimes, Never?
Time is only measured in hours, minutes and seconds</td>
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<td>2. Know basic time conversions
• 60 minutes in an hour</td>
<td>Show me a month with more days in it than April</td>
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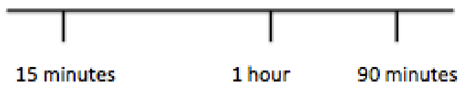
   It can be nice to do this using different overalys for the outer labels so you can shift from Roman numerals, to 24-hour clock, to past/to descriptors, to multiples of 5 and so on. • Adding labels for 24-hour clock hours alongside 1-12. • Making clocks using paper plates, card sticks and split pins for hands • Using manipulative clocks to show and read times (preferably mini-clocks for each child and a larger one for the teacher) • Making human clocks using arms • Exploring digital clocks (and especially 24-hour clocks) by comparing to analogue clock	• 24 hours in a day • 60 seconds in a minute • 7 days in a week • the number of days in each month • 365 days in a year (366 in a leap year) • solve simple problems involving these e.g. number of days in 2 years.	Show me something which would be measured better in hours than minutes Convince me that there are not 100 seconds in a minute Always, Sometimes, Never? Only one month has 28 days Always, Sometimes, Never? There are 365 days in a year
	3. Read and recognise roman numerals up to 12 • read I, V, X • read combinations from adding e.g. II, III, VI, VII, VIII, XI, XII • read combinations from subtracting e.g. IV, IX • write the numbers 1-12 as Roman numerals • label a clock with Roman numerals	Convince me that IX is smaller than VIII VI, 6, IV, 4
	4. Tell the time to 5 minutes using an analogue clock labelled with numbers or Roman numerals • recap: read times in words from a numbered clock (past the hour) • recap: read times in words from a numbered clock (to the hour) • read times in words from a clock labelled with Roman numerals (past the hour) • read times in words from a clock labelled with Roman numerals (to the hour)	Show me another way of writing 6pm ... and another
	5. Tell the time to the nearest minute using an analogue clock labelled with numbers or Roman numerals • read times in words from a numbered clock (past the hour) • read times in words from a numbered clock (to the hour) • read times in words from a clock labelled with Roman numerals (past the hour)	Always, Sometimes, Never? There are three hands on a clock

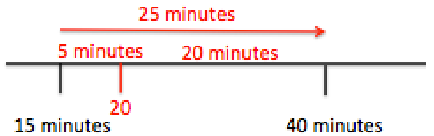
	• read times in words from a clock labelled with Roman numerals (to the hour)	
	6. Write times using 12-hour format • read a time to 5 minutes from an analogue clock and record as a 12-hour time • use am or pm to denote morning or afternoon correctly • read a time to the nearest minute from an analogue clock and record as a 12-hour time • record noon and midnight in 12-hour format	Show me how you would write the time 'midnight' using the 12 hour clock
	7. Write times using 24-hour format • convert hour times to 24-hour times e.g. 2 o'clock converts to 02:00 or 14:00 • convert other times to 24-hour times e.g. 'ten past two' converts to 02:10 or 14:10 • read a time to 1 minute from an analogue clock and record as a 24-hour time • record noon and midnight correctly in 24-hour format	Show me how you would write the time 'midnight' using the 24 hour clock What's the same and what's different? midnight; noon; 00:00; 12:00; 12pm: 12am; 12 o'clock
	8. Move or draw the hands on a clock labelled with numbers or Roman numerals to show a given time (words, 12-hour format or 24-hour format) • standard clock, time in words e.g. eight minutes past three • Roman numeral clock, time in words e.g. eight minutes past three • standard clock, time in 12-hour format e.g. 10:20 am • standard clock, time in 24-hour format e.g. 17:10 • Roman numeral clock, time in 12-hour or 24-hour format	What's the same and what's different? The big hand at III and the little hand at XI, 11:03 and quarter past eleven
Estimating Time • Predicting how long it might take to do simple activities eg walking to the hall. Then setting a stopwatch and all walking to hall to time it precisely and compare. • Estimating lengths of time by putting heads on desk and sitting up when the time has elapsed. Discussing strategies and comparing to the real time.	9. Estimate times • estimate a minute • estimate a multiple of a number of seconds e.g. 10 seconds • estimate a multiple of a number of minutes e.g. 5 minutes • have a sense of an hour, a quarter of an hour	Show me something that takes longer than one minute ... and another Convince me that it would not take you an hour to eat a sandwich

Time Sequencing/Durations

- Representing a time (including those in excess of 60 minutes) by counting in 5s and using a clock rotation to convert to hours and minutes.
- Using a number line to represent and position times to support ordering



- Making timelines to show events in sequence and with more accurate time positioning
- Using a number line to find time intervals and durations



- Exploring bus or train timetables to identify durations of journeys
- Looking at TV guides to calculate durations

10. Compare times given in hours, minutes or seconds.
    - order times given in the same unit
    - find the difference between two times (in same unit) e.g. 15 minutes and 38 minutes
    - order times given in mixed units
    - find the difference between two times in mixed units e.g. 1 hour and 30 minutes and 3 hours and 20 minutes
    - order times in different units e.g. 3 hours, 150 minutes, 2 hours and 25 minutes
    - find the difference between two times in different units

Show me how many seconds there are in 3 minutes

What's the same and what's different? 1200 minutes and 12 hours and half a day

11. Calculate a duration from the start and end time
    - find the difference duration of an activity where the start time and finish time are given (12-hour format, both morning or both afternoon)
    - find the difference duration of an activity where the start time and finish time are given (12-hour format, one morning and one afternoon)
    - find the difference duration of an activity where the start time and finish time are given (24-hour format, both morning or both afternoon)
    - find the difference duration of an activity where the start time and finish time are given (from clock faces)

Show me which of these bus journeys takes the longest

Convince me that a programme starting at 17:10 and finishing at 17:40 lasts longer than a programme starting at quarter to four and finishing at five past four.

Further Extension

1.

Match the two clocks that show the same time.

Rich and Sophisticated Tasks

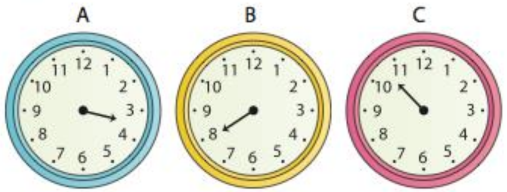
Tell and write the time from an analogue clock, including using Roman numerals from I to XII, and 12-hour and 24-hour clocks

NRICH: **Two Clocks** **** P**
NRICH: **Clocks** *** P**
NRICH: **The Time Is …** **** P**
NRICH: **How Many Times?** *** I**
NRICH: **5 on the Clock** ***** I**

Estimate and read time with increasing accuracy to the nearest minute; record and compare time in terms of seconds, minutes and hours; use vocabulary such as o'clock, a.m./p.m., morning, afternoon, noon and midnight

NRICH: **Wonky Watches** **** P**
NRICH: **Watch the Clock** ***** P**

2.
These clocks have only one hand, but can you suggest a time that each could be showing?



Explain your reasoning.

Misconceptions

Some children may have insecure knowledge of reading the time and number, particularly counting in 5s. Similarly, there may be a misconception of working in base 10 with time that leads to issues around the use of 60 minutes in an hour, for instance. Thus they may believe that there are, for example, 100 seconds in a minute, 100 minutes in an hour and so on.

There may be some misunderstanding of Roman Numeral symbols, particularly where numerals are combined and in numbers such as 4 or 9 where they are shown as one less than a symbol..

Many children will have unrealistic estimations and concept of time periods and actually how long it is. This is often because they count seconds too quickly.

When starting to work out time periods, children may revert back to addition as if they were working in base 10.

There may be confusion of am and pm, especially with noon, which should be shown as 12pm and midnight, which should be shown as 12am. Similarly, the use of am for early morning may be an issue - some children believe that am is when it is light and pm is when it is dark.

The 24 hour clock can be problematic also. Some children find it hard to convert times because they add 10 instead of twelve e.g. they think 1pm is the same as 10 hours + 1 hour so will be 11:00 rather than 12 hours + 1 hour or 13:00.

Leap years can cause some confusion, particularly with the rationale.

Teacher Guidance and Notes

- In Key Stage 1 children have used standard clock faces to tell time to 5 minute intervals.
- The new content here lies in the use of roman numerals on clock faces, the official use of the 12-hour and the 24-hour clock.
- Note that children who followed the AET medium term plans in Stage 2 will have used 12-hour format to record times alongside the word format as a way into the concepts.
- Children only need to look at Roman numerals to 12 in Stage 3.
- Note that occasionally a clock face (particularly historically) 4 is shown using the Roman numerals IIII. This is the only situation where such a use of 4 repeated symbols is acceptable.
- You may wish to explain the historic origins of Roman numerals as signals using the human body. For example, I represents a finger, V represents a hand (5 fingers) and X represents two crossed hands (10 fingers)
- Ensure you are consistent in notating (and expectations when children are notating) times. Specifically, it is recommended that times are shown using a colon e.g. 10:15.
- As in Stages 1 and 2, it is good practice to use all opportunities to discuss and work out times all through the day regardless of current mathematical unit. Link times to the daily routine/class schedule.
- Similarly, having a visible daily timetable for class with analogue and digital alternatives beside each landmark in the day can assist.

Key Assessment Checklist

1. I can read the time from an analogue clock showing in word format.
2. I can tell and show the time using a clock with Roman Numerals on it.
3. I can tell the time and show a time using the 12-hour clock.
4. I can tell the time and show a time using the 24-hour clock.
5. I can use the terms am, pm, noon and midnight accurately.
6. I can solve problems where I convert seconds to minutes, days to months, years including a leap year etc.
7. I can estimate and record the amount of time a task or journey may take.
8. I can read time accurately to the nearest second, minute or hour.
9. I can calculate a duration of an activity given the start and end time.

Year 3	Unit 13: Proportional Reasoning
8 learning hours	In this unit pupils explore proportional relationships, from the operations of multiplication and division on to the concepts of ratio, similarity, direct and inverse proportion. For primary pupils in Stages 1-3, this is focused on developing skills of division. Stages 4 and 5 revisit the whole of calculation to broaden to all four operations in a range of contexts and combination problems; the emphasis here is really on representing and then solving a problem using their calculation skills, not just calculating alone. In Stage 6 the real underpinning concepts of proportion and ratio develop. Secondary pupils begin to formalise their thinking about proportion by finding and applying scale factors, dividing quantities in a given ratio and fully investigating quantities in direct or inverse proportion, including graphically.

Prior Learning	Core Learning	Learning Leads to...
➢ recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables ➢ solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts	➢ **recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables** ➢ **solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects**	➢ recall multiplication and division facts for multiplication tables up to 12 × 12 ➢ solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects

Exemplification	Vocabulary	
1. Complete the missing numbers: a) $7 \times 8 = \cdots$ b) $4 \times \ldots = 20$ c) $36 \div 3 = \cdots$ d) $\ldots \div 4 = 9$ 2. a) A teacher puts the children in a class into groups of 3. Altogether there are 27 children in the class. How many groups will the teacher make? b) Matt is bottles of cola for a large party. They come in packs of 8. Matt needs 35 bottles of cola. How many packs must he buy? c) A baby lizard measures 7cm. The caterpillar grows until it is four times as long. How long is the caterpillar now? d) Paper cups cost 3p each. Abigail buys 28 cups. How much does she pay? e) A café sells 3 different drinks and 4 different sandwiches. How many different combinations of drinks and sandwiches are there?	multiplication division repeated addition scaling multiplied by lots of/groups of/sets of ... times larger/smaller product divided by shared between array	partition grid solve scaling commutative grid solve problem scaling correspondence operation prove

	row; column times table facts fact family	

Representation

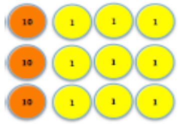
Multiplication:

- Building larger arrays for numbers beyond the times tables
  For example, here is an array showing 13 x 3

- Partitioning these arrays to show that the total can be found by finding sub-totals first

10 x 3 3 x 3

- Building arrays using place value counters



- Using a grid method to generalise an array and imagine it without all the single objects
  For example, the array above would become

	10	3
3	30	9

- Investigating the links between the grid above and the compact formal method, that is

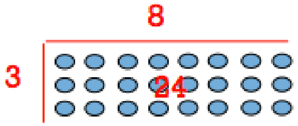
$$\begin{array}{r} 1\ 3 \\ 3\ \times \\ \hline 3\ 9 \end{array} \qquad \begin{array}{r} 1\ 6 \\ 3\ \times \\ \hline 4\ 8 \\ {\scriptstyle 1}\ \ \end{array}$$

Division

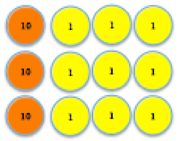
- For a calculation p ÷ q, grouping a set of p counters into groups of size q, arranging these groups as an array. For example, for 24 ÷ 3, count out 24 counters and arrange in

Fluency	Probing Questions
1. Recall and use multiplication and division facts for the three, four and eight times table • Count in 3s, 4s and 8s from 0 (forwards and backwards) • Complete a missing answer for a multiplication instantly e.g. $7 \times 3 = ■$ • Complete a missing answer for a division instantly e.g. $12 \div 4 = ■$ • Find a missing number in a multiplication statement e.g. $■ \times 8 = 72$ • Find a missing number in a division statement e.g. $■ \div 4 = 7$ • Give the fact family for a multiplication or division by 3, 4 or 8	Show me. a number that is in the 3 and 4 times tables but not in the 8 times tables ... and another ... and another What's the same and different? multiples of 4; multiples of 8; multiples of 2
2. Create mathematical statements for multiplication and division by (2, 5, 10) 3, 4 and 8 • Given number cards and symbols, create correct multiplication and division statements • Say if a given multiplication or division statement is true or false and justify this • Find missing numbers in written calculations	Convince me that if you know that a x b = c then you also know that a ÷ b = c
3. Recap: Multiply and divide 2-digit numbers by 1-digit numbers • Multiply a 2-digit number by 2, 3, 4, 5, 8 or 10 mentally • Multiply a 2-digit numbers by 2, 3, 4, 5 or 8 using a written (formal) method • Divide a 2-digit number by 2, 3, 4, 5, 8 or 10 mentally • Divide a 2-digit numbers by 2, 3, 4, 5 or 8 using a written (formal) method	Convince me that 25 x 3 = 20 x 3 + 5 x 3 = 10 x 3 + 10 x 3 + 5 x 3 Convince me that if I can calculate 10 x 3 and 1 x 3 I can calculate 19 x 3

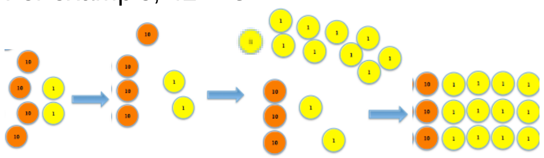
columns of 3.... then read off the answer of 8 as the number of columns



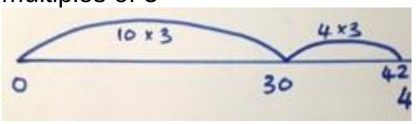
- Building a number using place value counters and grouping them into groups that are the size of the divisor, before arranging these groups as an array to explore the partitioning approach. For example, 39 ÷ 3



- Repeating the above, but exchanging remaining counters for 10 counters of the next size down before continuing to group. For example, 42 ÷ 3



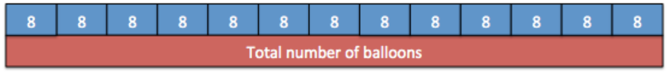
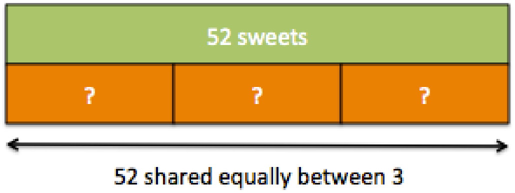
- Using a number line to show partitioned grouping. For example, 42 ÷ 3 can be found by considering known multiples of 3



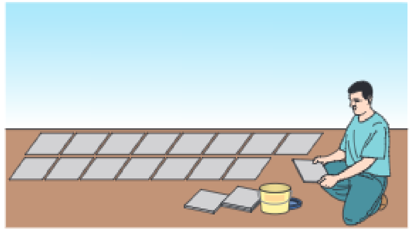
- Partitioning a larger number to divide each part and then recombine
For example:

42 ÷ 3
30 ÷ 3 + 12 ÷ 3
10 + 4
14

Solving Problems • Using the bar model to represent multiplication and division problems to help decide what calculation to complete.	4. Represent and solve multiplication word problems • Represent a 'lots of' word problem	Show me how you can represent this problem visually: Jane is drawing a picture of a flower. Her

• For example: Matt is buying balloons for a party. The balloons come in packs of 8. Matt buys 13 packets. How many balloons will he have for the party?  • For example: Bella shares 52 sweets between 3 children fairly. 	concretely or visually (e.g. with groups or a bar model) • Represent a 'scaling' word problem concretely or visually (e.g. with scaling or a bar model) • Write a mathematical statement to show the calculation needed • Solve the problem and complete the mathematical sentence to show the answer	picture is 4cm tall. The real flower is 3 times as tall as this. How tall is the real flower? Show me how you can represent this problem visually: Alice has 6 boxes. In each box there are 4 sweets. How many sweets does Alice have altogether?
	5. Represent and solve correspondence problems of n objects being connected to m objects • List the possible combinations of objects • Find the total number of combinations of such problems	Convince me that there are 12
	6. Represent and solve division word problems • Represent a word problem concretely or visually (e.g. with groups or sharing or a bar model) • Write a mathematical statement to show the calculation needed • Solve the problem and complete the mathematical sentence to show the answer	Show me how you can represent this problem visually: Jay has 18 hats. He shares the hats equally into 3 bags. How many hats will there be in each bag? Always, Sometimes, Never? Any number divided by 2 = one half of its original value
	7. Identify the operation required to solve a multiplication or division problem • know key trigger words for multiplication and division • recognise whether problem is multiplication or division • represent the problem concretely or visually • solve the problem • record the problem using a number sentence	What's the same and different? multiply, lots of, groups of, divide, product, shared by, shared between, grouped into, quotient

	8. Solve multi-step problems • Represent 2-step problems visually • Carry out 2 operations in the correct order (e.g. multiplication then subtraction) • Solve the problem in context	Show me a number sentence that includes the numbers 3, 4, 8 and 20 Always, Sometimes, Never? ... dividing a multiple of 10 by 2 will result in a multiple of 5

Further Extension	Rich and Sophisticated Tasks
1.  Roger has 96 patio slabs. Using all of the slabs find three different ways that he can arrange the slabs to form a rectangular patio. 2. Sam is planting onions in the vegetable plot in his garden. He arranges the onions into rows of 4 and has two left over. He then arranges them into rows of 3 and has none left over. How many onions might he have had? Explain your reasoning. 3. The following problems can be solved by using the calculation 8 ÷ 2. True or false? ▪ There are 2 bags of bread rolls that have 8 rolls in each bag. How many rolls are there altogether? ▪ A boat holds 2 people. How many boats are needed for 8 people? ▪ I have 8 pencils and give 2 pencils to each person. How many people receive pencils? ▪ I have 8 pencils and give 2 away. How many do I have left?	Solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects **NRICH: A Square of Numbers * G P** **NRICH: What do you Need? * P** **NRICH: This Pied Piper of Hamelin ** P** **NRICH: Follow the Numbers * P I** **NRICH: What's in the Box? * P** **NRICH: How Do You Do It? * P** **NRICH: Ip Dip * I** **NRICH: Journeys in Numberland * I**

Misconceptions	Teacher Guidance and Notes
Children may struggle to correctly rearrange the numbers of a times table fact into the corresponding division fact, for example they may put the smaller numbers first in division calculations. Some children may not yet have a strong understanding that multiplication is the inverse of division and so find it hard to move between the two operations. Children with weak understanding of how to represent a multiplication as an array may struggle to represent and calculate a 2dx1d multiplication and fail to see why they need to be able to partition it. They may try to work with a very large array rather than sectioning it off and partitioning. Poor understanding of place value may affect a child's ability to partition correctly and to calculate the result of a multiple of 10 x 1d. Some children struggle to understand the range of language of multiplication and division e.g. they may forget the meanings of 'lots of', 'times bigger', 'product' etc. Children often fail to recognise scaling problems as multiplication (or division problems) and find it hard to represent these practically. Issues with recall of multiplication facts may lead to some children making errors in calculations.	• This unit builds on the earlier work on multiplication and division of Unit 9. While the focus of Unit 9 is on carrying out the calculations of multiplication and division, the emphasis shifts here to applying these skills to word problems and other contexts. • The statement regarding the recall of 3, 4 and 8 multiplication tables/facts is repeated in this unit as it is expected that once again this will be a focus. However, by this stage of the year children should be increasingly confident and rapid in their recall and use of these facts. • It is important to present a variety of problems to children throughout this unit, encompassing the full range of language of multiplication and division. It is common for scaling problems to be minimised at the expense of repeated addition problems for multiplication, so you need to actively reverse this imbalance. • Children working at greater depth can begin to combine techniques and operations to solve multi-step problems.

Key Assessment Checklist

1. I can use times table facts (3, 4, 8) to instantly find missing numbers in multiplication and division calculations.
2. I can multiply and divide a 2-digit number by a single digit
3. I can solve multiplication problems involving 2d numbers.
4. I can solve division problems involving 2d numbers
5. I can solve scaling problems involving either multiplication or division.
6. I can solve correspondence problems where n objects are connected to m objects.